

Data Structure - Homework 2: Transpose

n is number of row

m is number of col k is number of none-zero elements

Analysis Traditional matrix

```
void transpose() override {
    vector<vector<int>> transpose(m, vector<int>(n, 0));
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < m; ++j) {
            transpose[j][i] = mat[i][j];
        }
    }
    mat = transpose;
    swap(n, m);
}
```

For all element in mat.

Move $mat[i][j]$ to $mat[j][i]$

The complexity is $O(n*m)$

spare matrix transpose

```
void transpose() override {
    vector<tuple<int, int, int>> transpose;
    for(int i=0;i<=m;++i){
        for(auto& [a, b, c]: mat){
            if(b == i){
                transpose.push_back({b, a, c});
            }
        }
    }
    mat = transpose;
    swap(n, m);
}
```

The total iteration of the inner loop is k.

The outer loop runs $m+1$ times.

The complexity is $O(m*k)$.

spare matrix fast transpose

```
void transpose() override {  
    vector<tuple<int, int, int>> transpose(mat.size());  
    vector<int> count(m+1, 0);  
    for(auto& [a, b, c]: mat){  
        count[b]++;  
    }  
    vector<int> pos(m+1, 0);  
    for(int i=1;i<=m;++i){  
        pos[i] = pos[i-1] + count[i-1];  
    }  
    for(auto& [a, b, c]: mat){  
        transpose[pos[b]++] = {b, a, c};  
    }  
    mat = transpose;  
    swap(n, m);  
}
```

The number frequency in column. The complexity is $O(k)$.

Initializing the position. The complexity is $O(m)$.

Filling the transposed matrix is also $O(k)$.

Therefore, the overall time complexity of this method is $O(k + m)$.