CS/ECE 374 P12

Junquan Chen, Pengxu Zheng, Jiawei Tang

TOTAL POINTS

100 / 100

QUESTION 1

1 Problem 12.A. 20 / 20

✓ **- 0 pts Correct (i.e., grammar is well-defined and correct)**

QUESTION 2

2 Problem 12.B. 80 / 80

✓ **- 0 pts Correct**

Make sure to add all appropriate pages to this problem.

HW Solution

CS/ECE 374: Algorithms & Models of Computation, Spring 2019 Version: **1.0**

Submitted by:

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10

Solution:

12.A.

$$
\begin{aligned}
S &\to aSc|X & \{a^i b^j c^{k_1+k_2} | k_1 + k_2 = i + j\} \\
X &\to \epsilon|bXc & \{b^j c^{k_1} | k_1 = j\}
\end{aligned}
$$

We first concatenate b and c together as many times as we want so that we have $k_1 = j$. Then we can concatenate a with bc and every time we add an a in the front, we add a c at the end so that $k_2 = i$. Therefore, we have $k_1 + k_2 = i + j$, which is $k = i + j$. Also, we can have ϵ when $i = j = k = 0$. We can also have either ac when $j = 0, i = k > 0$, or bc when $i = 0, j = k > 0$.

12.B.

In order to prove my grammar described in 12.A. is correct, I need to prove $L = L(G)$. Therefore, I need to prove $L \subseteq L(G)$ and $L(G) \subseteq L$ respectively. To simplify notation, because we have $L = \{a^i b^j c^k | k = i + j\}$, let $i = \#(a, x), j = \#(b, x), k = \#(c, x)$ for some string x.

First, we want to prove $L(G) \subseteq L$, which means every string in $L(G)$ should be in L. Let w be an arbitrary string in $L(G)$. Assume x is in the set $\{a^i b^j c^k | k = i + j\}$ for every string $x \in L(G)$ that can be derived with fewer than $|w|$ productions. There are 2 cases to consider based on the first production in the derivation of w.

- If $w \in s_1$, where s_1 is the set of strings X produces and we want to prove $s_1 = \{b^j c^k | k = j\}$ by induction. Assume x_1 is in the set $\{b^j c^k | k = j\}$ for every string $x_1 \in s_1$ that can be derived with fewer than $|w|$ productions. There are 2 subcases to consider.
  - If $w = \epsilon$, then $k = j = 0$. Therefore, $w \in s_1$.
  - Suppose the derivation begins $X \to bXc \to^* w$. Then $w = bx_1c$ for some string $x_1 \in s_1$. Our induction hypothesis implies that $x_1 \in s_1$. Also, since $k + 1 = j + 1$ and it has the correct order(b followed by c), $w \in s_1$.

  When i=0, $L = s_1$. Therefore, s_1 is a subset of L. Thus, for all $w \in s_1$, $w \in L$.
- Suppose the derivation begins $S \to aSc \to^* w$. Then $w = axc$ for some string $x \in L(G)$. Our induction hypothesis implies that $x \in L$. Also, since $k + 1 = (i + 1) + j$ and it has the correct order(a followed by c), $w \in L$.

In all 2 cases, we have $w \in L$ as required.

Second, we want to prove $L \subseteq L(G)$. Let w be an arbitrary string in L. Assume G generates every string x that is shorter than w, where $x \in L$. There are 2 cases to consider.

- If $w = \epsilon$, then $\epsilon \in S$ because of the production $S \to \epsilon$.
- Suppose w is non-empty. It means that at least one of i and j must be greater than 0. There are two subcases to consider.

1 Problem 12.A. **20 / 20**

✓ **- 0 pts** **Correct (i.e., grammar is well-defined and correct)**

HW Solution

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10

Solution:

12.A.

$$S \to aSc|X \qquad \{a^i b^j c^{k_1+k_2} | k_1 + k_2 = i + j\}$$

$$X \to \epsilon|bXc \qquad \{b^j c^{k_1} | k_1 = j\}$$

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2 Problem 12.B. **80 / 80**

✓ **- 0 pts** **Correct**

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