# CS/ECE 374 P08

Junquan Chen, Pengxu Zheng, Jiawei Tang

TOTAL POINTS

**93 / 100**

QUESTION 1

1 Problem 8.A. **30 / 30**

✓ **- 0 pts** Correct

QUESTION 2

2 Problem 8.B. **55 / 60**

✓ **- 5 pts** **(vi) Correct justification, but error in expression.**

QUESTION 3

3 Problem 8.C. **8 / 10**

✓ **- 2 pts** **Missing or wrong case for Kleene star.**

# HW Solution

**CS/ECE 374: Algorithms & Models of Computation, Spring 2019** Version: **1.0**

Submitted by:

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## 8

## Solution:

**8.A.**

When r = ∅,

$$even_0(L(r)) = \emptyset$$

$$odd_0(L(r)) = \emptyset$$

When r = $\epsilon$,

$$even_0(L(r)) = (00)^*$$

$$odd_0(L(r)) = 0(00)^*$$

When r = 0,

$$even_0(L(r)) = 0(00)^*$$

$$odd_0(L(r)) = 00(00)^*$$

When r = 1,

$$even_0(L(r)) = (00)^*(010 + 1)(00)^*$$

$$odd_0(L(r)) = (00)^*(10 + 01)(00)^*$$

**8.B.** (i)

$$even_0(L(r_1 + r_2)) = e_1 + e_2$$

Since we have $L(r_1 + r_2) = L(r_1) \cup L(r_2)$, the input can be $L(r_1)$ or $L(r_2)$. Also we are asking for inserting even number of 0s. Therefore, we want to take the union of $e_1$ and $e_2$, which is $e_1 + e_2$.

(ii)

$$odd_0(L(r_1 + r_2)) = o_1 + o_2$$

Same reason as above but in the case of inserting odd number of 0s. So we will take the union of $o_1$ and $o_2$, which is $o_1 + o_2$.

(iii)

$$even_0(L(r_1 r_2)) = e_1 e_2 + o_1 o_2$$

In this case of $r_1$ concatenated with $r_2$, we want to insert even number of 0s. To insert even number of 0s, we have to concatenate $e_1$ with $e_2$ or $o_1$ with $o_2$ because only an even number adds to another even number or an odd number adds to another odd number can be an even number.

(iv)

$$odd_0(L(r_1 r_2)) = e_1 o_2 + o_1 e_2$$

In this case of $r_1$ concatenated with $r_2$, we want to insert odd number of 0s. To insert odd number of 0s, we have to concatenate $e_1$ with $o_2$ or $o_1$ with $e_2$ because only an even number adds to an odd

1 Problem 8.A. 30 / 30

✓ - 0 pts Correct

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Since we have $L(r_1 + r_2) = L(r_1) \cup L(r_2)$, the input can be $L(r_1)$ or $L(r_2)$. Also we are asking for inserting even number of 0s. Therefore, we want to take the union of $e_1$ and $e_2$, which is $e_1 + e_2$.

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$$odd_0(L(r_1 + r_2)) = o_1 + o_2$$

Same reason as above but in the case of inserting odd number of 0s. So we will take the union of $o_1$ and $o_2$, which is $o_1 + o_2$.

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$$even_0(L(r_1r_2)) = e_1e_2 + o_1o_2$$

In this case of $r_1$ concatenated with $r_2$, we want to insert even number of 0s. To insert even number of 0s, we have to concatenate $e_1$ with $e_2$ or $o_1$ with $o_2$ because only an even number adds to another even number or an odd number adds to another odd number can be an even number.

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## 2 Problem 8.B. **55 / 60**

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number can be an odd number.
(v)

$$even_0(L(r_1^*)) = (o_1 e_1^* o_1 + e_1)^*$$

In this case of Kleene star of $r_1$, we want to insert even number of 0s. There are two cases. The first one is we can have $e_1^*$ because it will always add even number of 0s. The second one is we can have two $o_1$ concatenated together to add even number of 0s. We can also put any number of $e_1$ in the middle of $o_1 o_1$ so that it will still insert even number of 0s. We can have any combination of these two cases so that we take the Kleene star of the union of both. Besides, this expression holds when we insert zero 0. Also, if $L(r_1^*) = \{\epsilon\}$, it still holds as it can be equal to $e_1$ where $e_1$ can be $even_0(L(\epsilon))$, which is a base case in 8.A.
(vi)

$$odd_0(L(r_1^*)) = (o_1 e_1^* o_1 + e_1)^* o_1 e_1^*$$

In this case of Kleene star of $r_1$, we want to insert odd number of 0s. Then we can concatenate it with the insertion of any even number because any odd number is in the form of $2y + 1$ where $y \geq 0$. Therefore, we must have $o_1$ in the expression. And we can then have $even_0(L(r_1^*))o_1$. At last, we may end it with $e_1$ or $o_1$ by adding $e_1^*$ at the end, which leads to the final answer above. If $L(r_1^*) = \{\epsilon\}$, it holds as $odd_0(L(r_1^*)$ can be equal to $o_1$, where $o_1 = odd_0(L(\epsilon))$, which is a base case in 8.A.

**8.C.** We can have recursive definition by using the conclusions from previous questions. Here, we use the same notation as 8.B. that for $j \in \{1, 2\}$,

$$e_j = even_0(L(r_j))$$

$$o_j = odd_0(L(r_j))$$

$even_0(L(r))$ is recursively defined as below:

$$even_0(L(r)) = \begin{cases} even_0(L(r)) \text{ as defined in 8.A.} & \text{if L(r) is in base case, } r \in \{\emptyset, \epsilon, 0, 1\} \\ e_1 + e_2 & \text{if } r = r_1 + r_2 \\ e_1 e_2 + o_1 o_2 & \text{if } r = r_1 r_2 \\ (o_1 e_1^* o_1 + e_1)^* & \text{if } r = r_1^* \end{cases}$$

$odd_0(L(r))$ is recursively defined as below:

$$odd_0(L(r)) = \begin{cases} odd_0(L(r)) \text{ as defined in 8.A.} & \text{if L(r) is in base case, } r \in \{\emptyset, \epsilon, 0, 1\} \\ o_1 + o_2 & \text{if } r = r_1 + r_2 \\ e_1 o_2 + e_2 o_1 & \text{if } r = r_1 r_2 \\ (o_1 e_1^* o_1 + e_1)^* o_1 e_1^* & \text{if } r = r_1^* \end{cases}$$

## 3 Problem 8.C. 8 / 10

✓ **- 2 pts** **Missing or wrong case for Kleene star.**