CS/ECE 374 P06

Pengxu Zheng, Junquan Chen, Jiawei Tang

TOTAL POINTS

95 / 100

QUESTION 1

1 Problem 6.A. 30 / 30

✓ - 0 pts Correct

QUESTION 2

2 Problem 6.B. 30 / 30

✓ - 0 pts Correct

Proof by contradiction here is not needed (may be extra work) - but is still correct, good work.

QUESTION 3

3 Problem 6.C. 35 / 40

✓ - 5 pts **Minor notational/syntax mistakes**

S2 is not a set, it is incorrect to say that q \in S2.

HW Solution

CS/ECE 374: Algorithms & Models of Computation, Spring 2019 Version: **1.0**

Submitted by:

- **≪Jiawei Tang≫**: **≪jiaweit2≫**
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6

Solution:

6.A. First for $L_3 = \{\epsilon\}$, we construct its DFA $M_3(Q_3, \Sigma, \delta_3, s_3, A_3)$. $Q_3 = \{q, e\}$ where $q \in s_3, e \in s_3$ and $q \in A_3$. $s_3 = \{q, e\}$. $A_3 = \{q\}$. $\delta : Q \times \Sigma \to Q$ is defined as $\delta(q, a) = e$.
Since $L = L_1 \cup \overline{L_2} \cup \{\epsilon\} = L_1 \cup \overline{L_2} \cup L_3$, we use product construction in the formal description of $M(Q, \Sigma, \delta, s, A)$ as below.

$$Q = Q_1 \times Q_2 \times Q_3$$

$$s = (s_1, s_2, s_3)$$

$\delta : Q \times \Sigma \to Q$ is defined as follows: for each $(q_1, q_2, q_3) \in Q$ and $a \in \Sigma$,

$$\delta((q_1, q_2, q_3), a) = (\delta(q_1, a), \delta(q_2, a), \delta(q_3, a))$$

$$A = (A_1 \times Q_2 \times Q_3) \cup (Q_1 \times (Q_2 - A_2) \times Q_3) \cup (Q_1 \times Q_2 \times A_3)$$

6.B We claim that $L(M')$ is the set of prefix of the set of strings that are accepted by DFA M_1. To formally prove the claim, let $\{x | y = xw \text{ where } y \in L_1 \text{ and } x, w \in \Sigma^*\} = L$. We prove $L(M') = L$ by proving $L(M') \subseteq L$ and $L \subseteq L(M')$.
To prove $L(M') \subseteq L$, for all $x' \in L(M')$, $x' \in L$. Because $L(M') \subseteq \Sigma^*$ by definition, $x' \in \Sigma^*$. Since we have $\delta_1^*(q, w) \in A_1$, where $q \in H_1, w \in \Sigma^*$, it means that every accepting state q in DFA M' can take some input string w to go to an accepting state in DFA M_1. Therefore, for all x' that are accepted by DFA M', all $y = x'w$ will be accepted by DFA M_1, which means $y \in L_1$. Therefore, for all $x' \in L(M')$, $x' \in L$ by the definition of L.
To prove $L \subseteq L(M')$, for all $x' \in L$, $x' \in L(M')$. According to our definition of L, for all $x' \in L$, $y = x'w$ where $y \in L_1$ and $x', w \in \Sigma^*$. Let's prove $x' \in L(M')$ by contradiction. We assume that there is a x' is in L, but it is not in $L(M')$. Therefore, $\delta^*(s, x') \notin H_1$ where $s \in s_1$. Since x' is in L and $y = x'w$ where y is a set of strings accepted by DFA M_1 and $w \in \Sigma^*$, $\delta^*(\delta^*(s, x'), w) = \delta^*(q, w) \in A_1$ where $q \in H_1$. It means that $\delta^*(s, x') \in H_1$. However, it contradicts to the previous statement that $\delta^*(s, x') \notin H_1$. Therefore, the claim that there is a $x' \in L$ that $x' \notin L(M')$ is false. Thus, we can prove that for all $x' \in L$, $x' \in L(M')$.
Thus, $L = L(M')$.

6.C. To prove $\epsilon \in L_2$ if and only if $L_2 = \Sigma^*$, we want to first prove $\epsilon \in L_2$ implies $L_2 = \Sigma^*$, and then prove $L_2 = \Sigma^*$ implies $\epsilon \in L_2$.
In the first case, assuming $\epsilon \in L_2$, in order to prove $L_2 = \Sigma^*$, we want to prove $L_2 \subseteq \Sigma^*$ and $\Sigma^* \subseteq L_2$. Since by definition, $L_2 \subseteq \Sigma^*$ holds. Also, we want to prove $\Sigma^* \subseteq L_2$ that for all $w \in \Sigma^*$, $w \in L_2$. Because $\epsilon \in L_2$, there is a $q \in A_2$ that $q \in S_2$. Due to the assumption given in the prompt, we have $\delta(q, a) = q$ where $a \in \Sigma$, $q \in A_2$. Therefore, we have $\delta^*(q, w) = \delta(\delta^*(q, x), a)) = q$ where $x \in \Sigma^*, w = xa$ by definition of δ^*. Any symbol $a \in \Sigma$ can append to ϵ or x where $x \in \Sigma^*$, to form a w where $w \in \Sigma^*$. Thus we have all $w \in \Sigma^*$, $w \in L_2$, so $\Sigma^* \subseteq L_2$ holds. Therefore, assuming $\epsilon \in L_2$, $L_2 = \Sigma^*$ is true.
In the second case, assuming $L_2 = \Sigma^*$, we want to prove $\epsilon \in L_2$. Since $\epsilon \in \Sigma^*$ and L_2 accepts all $w \in \Sigma^*$, L_2 accepts ϵ. Therefore, $\epsilon \in L_2$.
Thus, according to above two cases, we can conclude that $\epsilon \in L_2$ if and only if $L_2 = \Sigma^*$.

1 Problem 6.A. **30 / 30**

✓ **- 0 pts** Correct

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$$Q = Q_1 \times Q_2 \times Q_3$$

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6.B We claim that $L(M')$ is the set of prefix of the set of strings that are accepted by DFA M_1. To formally prove the claim, let $\{x | y = xw \text{ where } y \in L_1 \text{ and } x, w \in \Sigma^*\} = L$. We prove $L(M') = L$ by proving $L(M') \subseteq L$ and $L \subseteq L(M')$.
To prove $L(M') \subseteq L$, for all $x' \in L(M')$, $x' \in L$. Because $L(M') \subseteq \Sigma^*$ by definition, $x' \in \Sigma^*$. Since we have $\delta_1^*(q, w) \in A_1$, where $q \in H_1, w \in \Sigma^*$, it means that every accepting state q in DFA M' can take some input string w to go to an accepting state in DFA M_1. Therefore, for all x' that are accepted by DFA M', all $y = x'w$ will be accepted by DFA M_1, which means $y \in L_1$. Therefore, for all $x' \in L(M')$, $x' \in L$ by the definition of L.
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