CS/ECE 374 P02

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TOTAL POINTS

100 / 100

QUESTION 1

1 Problem 2 **100 / 100**

✓ **- 0 pts** **Correct**

HW1 Solution

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Solution:

Proof: for any $w \in L$ and any prefix u of w, we have $\#_0(u) \geq \#_1(u)$.

Base Case: Let w = ε, the prefix u of w is also ε by definition. $\#_0(u) = \#_1(u) = 0$.
Hence, the statement holds for w = ε.

Inductive Hypothesis: Let w be an arbitrary string in L, and let u be its prefix.
There exists $\#_0(u) \geq \#_1(u)$.

Induction (w = 0y1):
Let u be the prefix of w, w = 0y1, where y is not empty and $y \in L$.
Case 1: If u = ε, then $\#_0(u) = \#_1(u) = 0$.
Case 2: If 0 < |u| < |w|, then let u = 0x, $x \in L$,

$\#_0(u) = \#_0(0) + \#_0(x) + \#_0(1) = 1 + \#_0(x)$
$\#_1(u) = \#_1(0) + \#_1(x) + \#_1(1) = \#_1(x)$
$\#_0(u) - \#_1(u) = 1 + \#_0(x) - \#_1(x)$
By inductive hypothesis, $\#_0(x) \geq \#_1(x)$, thus
$\#_0(u) - \#_1(u) \geq 1 > 0$

Case 3: If |u| = |w|, then u = w = 0y1.

$\#_0(u) = 1 + \#_0(y)$,
$\#_1(u) = 1 + \#_1(y)$,
$\#_0(u) - \#_1(u) = \#_0(y) - \#_1(y)$.
By inductive hypothesis, $\#_0(y) > \#_1(y)$.
Hence, $\#_0(u) > \#_1(u)$.

The inductive hypothesis holds for w = 0y1.

Induction (w = xy):
Let u be the prefix of w, w = xy, where x, y are not empty and $x, y \in L$. Let a be the prefix of x, b be the prefix of y, $a, b \in L$.
Case 1: If $|u| = |a| < |w|$, u = a, by inductive hypothesis, $\#_0(a) \geq \#_1(a)$.
Case 2: If $|a| = |x| < |u| \leq |w|$, $u = xb$, by inductive hypothesis, $\#_0(x) \geq \#_1(x)$.

$\#_0(u) = \#_0(x) + \#_0(b)$; $\#_1(u) = \#_1(x) + \#_1(b)$.
By inductive hypothesis, $\#_0(u) \geq \#_1(u)$. Hence, the inductive hypothesis holds for $w = xy$.

Conclusion: The inductive hypothesis holds true for any $w \in L$.

1 Problem 2 **100 / 100**

✓ **- 0 pts** Correct