CS/ECE 374 P03

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TOTAL POINTS

100 / 100

QUESTION 1

1 Problem 3 **100 / 100**

✓ **- 0 pts** **Correct**

HW Solution

CS/ECE 374: Algorithms & Models of Computation, Spring 2019 Version: **1.0**

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Solution:

Proof: We claim that there is some constant c that $T(n) = O(n) \leq cn$ for all natural number n. Consider c as 20. We prove this by induction on n.

Base case:
At $n = 1, n = 2, n = 3, n = 4, n = 5$, when $c = 20$.
$T(1) = 1 \leq 20 \times 1$,
$T(2) = 1 \leq 20 \times 2$,
$T(3) = 1 \leq 20 \times 3$,
$T(4) = 1 \leq 20 \times 4$,
$T(5) = 1 \leq 20 \times 5$.
hence the claim holds for $n = 1, n = 2, n = 3, n = 4, n = 5$.

Induction hypothesis:
Let k be an arbitrary natural number and $k \geq 6$. Suppose that $T(n) = O(n) \leq cn$ where c = 20 for $n = 1, 2, 3, \ldots\ldots, k - 1$. we will prove $T(k) \leq ck$ at n=k.

Induction step:

At $n = k$, T(k) = $T(\left\lfloor\frac{k}{3}\right\rfloor)+T(\left\lfloor\frac{k}{4}\right\rfloor)+T(\left\lfloor\frac{k}{5}\right\rfloor)+T(\left\lfloor\frac{k}{6}\right\rfloor)+$k

Since $k/3, k/4, k/5, k/6 \leq k$, by induction hypothesis, $T(\left\lfloor\frac{k}{3}\right\rfloor) \leq 20 * \frac{k}{3}$, $T(\left\lfloor\frac{k}{4}\right\rfloor) \leq 20 * \frac{k}{4}$, $T(\left\lfloor\frac{k}{5}\right\rfloor) \leq 20 * \frac{k}{5}$, $T(\left\lfloor\frac{k}{6}\right\rfloor) \leq 20 * \frac{k}{6}$, so $T(k) = T(\left\lfloor\frac{k}{3}\right\rfloor)+T(\left\lfloor\frac{k}{4}\right\rfloor)+T(\left\lfloor\frac{k}{5}\right\rfloor)+T(\left\lfloor\frac{k}{6}\right\rfloor)+$k $\leq 20 * \frac{k}{3} + 20 * \frac{k}{4} + 20 * \frac{k}{5} + 20 * \frac{k}{6} + k = 20 * k$,
which establishes the claim for k, where c = 20. Therefore, $T(n) = O(n) \leq cn$ in all cases

1 Problem 3 **100 / 100**

✓ **- 0 pts** Correct