CS/ECE 374 P01

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TOTAL POINTS

100 / 100

QUESTION 1

1 Problem 1.A. **30 / 30**

✓ **- 0 pts** **Correct**

QUESTION 2

2 Problem 1.B. **30 / 30**

✓ **- 0 pts** **Correct**

QUESTION 3

3 Problem 1.C. **40 / 40**

✓ **- 0 pts** **Correct**

HW Solution

CS/ECE 374: Algorithms & Models of Computation, Spring 2019 Version: **1.0**

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Solution:

1.A. We will prove it by contradiction. First we assume there is an edge uv in G such that f(u)=f(v). According to the algorithm, initially all the vertices are not colored. Then for each iteration i, we assign v_i a color that none of its neighbours have, until we have assigned all of the assignable vertices. If we have an edge uv such that f(u)=f(v), it means we have assigned the same color to vertex u and vertex v, where u and v are neighbours. It is against our algorithm so that it is false. Therefore, there is no edge uv in G such that f(u)=f(v).

1.B. We prove the claim that for a vertex v which is colored by color k, there is a simple path in the graph $u_1, ..., u_k = v$ such that for $i = 1, ..., k$, we have $f(u_i) = i$ and $u_i u_{i+1} \in E(G)$ for $i = 1, ..., k-1$. The proof is by induction on k.
Base case: if vertex v is colored by color k=1, we have a path that has only one vertex $u_1 = v$ and $f(u_1) = 1$.
Induction hypothesis: Assume for any number k', where $1 < k'$, if a vertex v is colored by color k', there is a simple path in the graph, $u_1, ..., u_{k'} = v$ such that for $i = 1, ..., k'$, we have $f(u_{k'}) = k'$ and $u_i u_{i+1} \in E(G)$ for $i = 1, ..., k' - 1$.
Induction step: for every vertex v is colored by color k where k = k'+1, it has at least k' neighbours which are colored 1,2,...,k'. The reason is that in order to color a vertex with the color k, according to the coloring algorithm, none of this vertex's neighbours is colored with color k and we want to color this vertex with the smallest color. According to the induction hypothesis, for the vertex $u_{k'}$ colored with color k', there is a simple path in the graph, $u_1, ..., u_{k'} = v$. Therefore, since the vertex v has the neighbour, vertex $u_{k'}$ where $f(u_{k'}) = k'$, v is connected with $u_{k'}$. Then there forms a path, $u_1, ..., u_{k'}, u_k = v$ where k = k'+1. Thus, the claim is true.

1.C. In order to prove (i) or (ii) to be true, we have to prove (i) to be true and when (i) is false, (ii) has to be true. There are two cases.
First, assume there are totally k colors, where $k \geq \lfloor \sqrt{n} \rfloor$. According to 1.B.(last question) as proved above, if a vertex v is colored by color $\lfloor \sqrt{n} \rfloor$, then there is a simple path in the graph, $u_1, ...u_{\lfloor \sqrt{n} \rfloor} = v$, for $i = 1, ..., \lfloor \sqrt{n} \rfloor$. This simple path has $\lfloor \sqrt{n} \rfloor$ vertices.
Second, if there are totally k colors, where $k < \lfloor \sqrt{n} \rfloor$. (i) will not hold as there won't be a simple path with $\lfloor \sqrt{n} \rfloor$ vertices according to the contrapositive of 1.B. We want to prove (ii) holds in this case. There will always be a case that number of vertices in each color is $\frac{n}{k} > \lfloor \sqrt{n} \rfloor$. In this case, it consists of the vertices that have the same color. Therefore, this set of vertices will not connect each other due to the algorithm. Then, this set is an independent set by its definition. Also, any subset of independent set is an independent set. Therefore, this set has size of $\lfloor \sqrt{n} \rfloor$ and (ii) is true in this case.

1 Problem 1.A. **30 / 30**

✓ **- 0 pts** Correct

2 Problem 1.B. 30 / 30

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3 Problem 1.C. **40 / 40**

✓ **- 0 pts** Correct