CS/ECE 374 P10

Junquan Chen, Jiawei Tang, Pengxu Zheng

TOTAL POINTS

73 / 100

QUESTION 1

1 Problem 10.A. 25 / 25

✓ **- 0 pts Correct**

- Don't write braces around strings x, y, z, etc. It makes them into sets of strings instead of just strings.

QUESTION 2

2 Problem 10.B. 0 / 25

✓ **- 25 pts Wrong fooling set**

- yz is always in L because it contains two runs of length j. Also, if F is a fooling set of L' and L' is a subset of L, it does NOT follow that F is a fooling set of L (consider that every language is a subset of sigma*. By your logic, sigma* is not regular because every fooling set is a fooling set for it.)

QUESTION 3

3 Problem 10.C. 25 / 25

✓ **- 0 pts Correct**

QUESTION 4

4 Problem 10.D. 13 / 15

- 2 Point adjustment

- You should've also explained how L U L' is regular, to fulfill the requirements of the counter example.

QUESTION 5

5 problem 10.E. 10 / 10

✓ **- 0 pts Correct**

HW4 Solution

CS/ECE 374: Algorithms & Models of Computation, Spring 2019 Version: **1.0**

Submitted by:

- **≪Pengxu Zheng≫**: **≪pzheng5≫**
- **≪Junquan Chen≫**: **≪junquan2≫**
- **≪Jiawei Tang≫**: **≪jiaweit2≫**

10

Solution:

10.A.

Proof: Let F be fooling set for L, F = $\{a^*b\}$.
Let x, y ∈ F be arbitrary strings, x = $\{a^ib\}$, y = $\{a^jb\}$, i, j ∈ Z, i ≠ j.
Let z = $\{a^ib\}$, then
xz = $\{a^iba^ib\}$ ∈ L,
yz = $\{a^jba^ib\}$ ∉ L because i ≠ j.
Therefore, F is a fooling set of L.
Because F is infinite, L is not a regular language.

10.B.

Proof: Let $L' = \{a^nb^n\} \subseteq$ L, and let F be the fooling set of L', F = $\{a^ib^i\}$.
Let x, y ∈ F be arbitrary strings, x = $\{a^ib^i\}$, y = $\{a^jb^j\}$, where i, j ∈ Z and i ≠ j.
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Hence, F is a fooling set for L'. Since L' ⊆ L, we say that F is also the fooling set for L.
Because F is infinite, L is not a regular language.

10.C.

Proof: Using set operations, we obtained L ∪ L' \ (L' \ L) = L.
Suppose L ∪ L' is regular. Since:
a. L' \ L is regular
b. By closure properties of regular languages, L ∪ L' \ (L' \ L) is regular
Therefore, L should also be regular.
However, given that L is not regular, we obtained a contradiction.
Thus, we conclude that L ∪ L' is not regular.

10.D.

Let L = $\{a^nb^n$ | n ≥ 0$\}$, L' = $\{a, b\}^*$, then L ∩ L' = L, which is not regular and thus contradicts with the claim.

10.E.

Proof: Let F be the fooling set for L, F = $\{0^{n^4}\}$.
Let x, y ∈ F be arbitrary strings, x = $\{0^i\}$, y = $\{0^j\}$, i, j ∈ Z, i > j
Let z = $\{0^{i^4+4i^3+6i^2+3i+1}\}$,
then
xz = $\{0^{i^4+4i^3+6i^2+4i+1}\} = \{0^{(i+1)^4}\} \in L$,

1 Problem 10.A. **25 / 25**

✓ **- 0 pts** **Correct**

Don't write braces around strings x, y, z, etc. It makes them into sets of strings instead of just strings.

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3 Problem 10.C. **25 / 25**

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4 Problem 10.D. **13 / 15**

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$yz = \{0^{i^4+4i^3+6i^2+3i+j+1}\}$
Since i > j ≥ 3, $i^4 < i^4 + 4i^3 + 6i^2 + 3i + j + 1 < (i+1)^4$, yz ∉ L.
Therefore, F is a fooling set of L.
Because F is infinite, L is not a regular language.

5 problem 10.E. 10 / 10

✓ - 0 pts Correct