CS/ECE 374 P33

Abhay Varmaraja, Pengxu Zheng, Jiawei Tang

TOTAL POINTS

72 / 100

QUESTION 1

1 33. **72 / 100**

- **0 pts** Correct
- **24 pts** Incorrect Reduction
- **24 pts** One direction of the proof of correctness of the reduction is incorrect.
- **48 pts** Both directions of the proof of correctness of the reduction are incorrect.

✓ **- 8 pts** **Not mentioning that the reduction runs in polynomial time.**

✓ **- 10 pts** **Not sketching a correct verifier to show that the problem is in NP.**

✓ **- 10 pts** **Not mentioning that the verifier runs in polynomial time.**

- **75 pts** IDK

HW Solution

CS/ECE 374: Algorithms & Models of Computation, Spring 2019 Version: **1.0**

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Solution:

33. (Collaborated with Bo Wang's group.)
We want to first prove the FAN is NP.
certificate = G' is a subgraph of G
certifier C=(G' is 3-blade fan) && ($|V(G') = 3k - 2$) Because we can see that C has polynomial time complexity, we know that FAN is NP accordingly.
Second, we want to prove that FAN is NP-hard. We can give a polynomial-time reduction from a known NP-Complete problem. Here the NP-Complete problem would be Hamiltonian Cycle.
$G = (V, E)$ is an arbitrary graph. Let $G' = (V', E')$ be obtained by rotating G twice. Therefore, G can have 3 subgraphs which share a same vertex. Then we want to prove that G has a Hamiltonian cycle iff G' is a 3-blade fan. First, prove that if G has a Hamiltonian cycle, then G' is a 3-blade fan. We can know that the 3 subgraphs in G' have Hamiltonian cycles. Therefore, G' has 3 cycles sharing one vertex and are of the same length. Thus, G' is a 3-blade fan. Second, prove that if G' is a 3-blade fan, then G has a Hamiltonian cycle. Since here we know G' is a 3-blade fan, which means G' has 3 cycles sharing the same vertex, we also know that G' is obtained from another graph by twice-rotation. We have the information that $|V'| = 3|V| - 2$. Thus, we can see that all 3 cycles in G' are of length $|V|$. Then we know that G must have a Hamiltonian cycle. Therefore, the claim we want to prove is true.

Thus, we prove that FAN problem is NP and also NP-hard. Therefore, we can conclude that FAN problem is NP-complete.

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