CS/ECE 374 P27

Jiawei Tang, Pengxu Zheng

TOTAL POINTS

90 / 100

QUESTION 1

1 27.A. **10 / 10**

- ✓ **+ 10 pts** Correct
- **+ 5 pts** Correct idea but hand-wavy
- **+ 2.5 pts** IDK
- **+ 0 pts** Incorrect

QUESTION 2

2 27.B. **80 / 90**

- **+ 0 pts** Incorrect
- **+ 22.5 pts** IDK
- **+ 90 pts** Correct graph reduction (reduction + algorithm + time analysis)
- ✓ **+ 40 pts Correct graph construction (V+E+graph problem)**
- **+ 10 pts** Correct vertex set for new graph, including weights if necessary
- **+ 15 pts** Correct edge set for new graph, including weights if necessary
- **+ 15 pts** Correct graph problem to solve on new graph
- ✓ **+ 40 pts Correct algorithm to solve the graph problem**
- **+ 10 pts** Correct running time analysis
- **- 10 pts** Correct but slower algorithm
- **- 5 pts** Typo or minor mistake
- **+ 90 pts** Correct modified Floyd Warshall/DP version
- **+ 9 pts** variable definition
- **+ 9 pts** recurrence: base case
- **+ 27 pts** recurrence: recursive case
- **+ 9 pts** final answer
- **+ 9 pts** data structure
- **+ 18 pts** evaluation order
- **+ 9 pts** running time analysis

Where is the flloyd warshall algorithm?

HW Solution

CS/ECE 374: Algorithms & Models of Computation, Spring 2019 Version: **1.0**

Submitted by:

- **≪Jiawei Tang≫**: **≪jiaweit2≫**
- **≪Pengxu Zheng≫**: **≪pzheng5≫**

27

Solution:

27.A.

Because every two edge has different weights, $|w(e_1) - w(e_2)|$ where e_1, e_2 are edges and $e_1 \neq e_2$, must be larger than 0. If a cycle, $f(C)$, has repeated edges or vertices, this cycle must contain an internal cycle, $f(C')$, that uses less edges or vertices than the previous cycle and doesn't have repeated edges or vertices. If $f(C)$ has repeated vertices, then it must have more edges than $f(C')$. $f(C)$ will be equal to ($f(C')$+absolute difference in weight of extra edges). Thus,

$$f(C) > f(C')$$

Therefore, the minimum fluctuation of a cycle will not have any repeated vertices or edges.

27.B.

(discussed with Yiqing's group) As we have proved in 27.A, the minimum fluctuation must be a simple cycle. Define the original graph as G. We want to construct a new graph G' such that each edge from G is a vertex in G'. For every vertex v in G, let u be this vertex's in-neighbors and let v' be this vertex's out-neighbors. Then we will have edge $e = ((u, v), (v, v'))$ in G', and $w(e) = |w(u, v) - w(v, v')|$. We want to first use Flyod-Warshall algorithm to find all-pairs shortest path for G'. Then we can get a cycle's fluctuation by adding a path from a vertex to an another vertex to the path from the other vertex to this vertex.

Construct a new graph G'
for each edge $e = (u, v)$ in G **do**
  add vertex e in G'
end for
for each vertex v in G **do**
  for each out-neighbors v' of v **do**
    for each in-neighbors u of v **do**
      add edge $e = ((u, v), (v, v'))$
      $w(e) = |w(u, v) - w(v, v')|$
    end for
  end for
end for
V =number of vertices in G'
$M = V \times V$ array of shortest path(Every entry is initialized as ∞)
$M' = V \times V$ array of shortest cycle
$M = FloydWarshall(G')$//run Floyd-Warshall algorithm to get M
//Here in Floyd-Warshall algorithm, we keep track of each vertex when we calculate every shortest path. Then we are able to get the complete cycle once we find the shortest cycle.
for each pair of vertices u, v in G' **do**
  $M'[u][v] = M[u][v] + M[v][u]$

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