Top-quark effective field theory

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Abstract

This essay is mainly based on Cen Zhang and Scott Willenbrock's work[6] in 2010. It employs effective field theory to dimension six to describe process in top-quark physics such as top-quark decay and top-quark production. Effective field theory is believed to be simpler and better at describing BSM phenomena compaired with traditional vertex function approach.

1 Introduction

The Standard Model is the most powerful phenomenologic model in partical physics. It unifies three of the four known fundamental interactions and classifies all the known elementary particles. However, despite of its theoretical self-consistency and successful experimental predictions, there are still many phenomena unexplained: It can neither incorporate general relativity nor explain matter–antimatter asymmetry and neutrino oscillations. The Standard Model is by no means the nature of the world and theories beyond the Standard Model is needed.

A theorist has two way to construct theories beyond the Standard Model. One is put forward a more fundamental theory, such as supersymmetry, string theory, and extra dimensions. The other is to use the philosophy of effective field theory: write down all the allowed operators and calculate without knowing the exect theory. In this review, we mainly focus on the latter.

Here we can write down an effective field theory lagrangian in a general form[4]:

$$\mathscr{L}_{\mathrm{EFT}} = \mathscr{L}_{\mathscr{D}\leq 4} + \frac{\mathscr{L}_5}{\Lambda} + \frac{\mathscr{L}_6}{\Lambda^2} + \ldots$$

where Λ is a large cutoff and also the energy scale where new physics occurs. $\mathscr{L}_n$ represents for n dimension operator, which satisfies the constraint of $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ invariance. If the BSM physics lies at low energy scale when $\mathscr{L}_{n>4} \ll \Lambda^n$, higher order terms $\frac{\mathscr{L}_n}{\Lambda^n}$ can be ignored. If it lies at higher energy scale, the high energy effect can be described by higher order terms $\frac{\mathscr{L}_n}{\Lambda^n}$

There is only one dimension-five gauge invarient operators:

$$L_{eff} = \frac{c^{ij}}{\Lambda}\left(L^{iT}\epsilon H\right) C\left(H^{T}\epsilon L^{j}\right) + \text{ h. c.}$$

where L is the lepton doublet and its upper index represents for its generation. H is the higgs doublet. However, it has nothing to do with our top quark and there is no need for for us to take it into consideration.

In contrast to the dimension-five operator, we have tons of possible dimension-six operators. Buchmüller and Wyler [1] first enumerated 80 independent gauge-invarient dimension-six operators, while redundancy were found in later articles [3]. As is listed in Table 1 and 2, we only consider those terms contribute to the top-quark-relevent process.

operator	process
$O_{\phi q}^{(3)} = i\left(\phi^{+}\tau^{I}D_{\mu}\phi\right)\left(\bar{q}\gamma^{\mu}\tau^{I}q\right)$	top decay, single top
$O_{tW} = \left(\bar{q}\sigma^{\mu\nu}\tau^{I}t\right)\tilde{\phi}W_{\mu\nu}^{I}$ (with real coefficient)	top decay, single top
$O_{qq}^{(1,3)} = \left(\bar{q}^{i}\gamma_{\mu}\tau^{I}q^{j}\right)\left(\bar{q}\gamma^{\mu}\tau^{I}q\right)$	single top
$O_{tG} = \left(\bar{q}\sigma^{\mu\nu}\lambda^{A}t\right)\tilde{\phi}G_{\mu\nu}^{A}$ (with real coefficient)	single top, $q\bar{q}, gg \to t\bar{t}$
$O_{G} = f_{ABC}G_{\mu}^{A\nu}G_{\nu}^{B\rho}G_{\rho}^{C\mu}$	$gg \to t\bar{t}$
$O_{\phi G} = \frac{1}{2}\left(\phi^{+}\phi\right)G_{\mu\nu}^{A}G^{A\mu\nu}$	$gg \to t\bar{t}$
7 four-quark operators	$q\bar{q} \to t\bar{t}$

Table 1: CP-even dimension-six operators related to top-quark[6]

operator	process
$O_{tW} = \left(\bar{q}\sigma^{\mu\nu}\tau^{I}t\right)\tilde{\phi}W_{\mu\nu}^{I}$ (with imaginary coefficient)	top decay, single top
$O_{tG} = \left(\bar{q}\sigma^{\mu\nu}\lambda^{A}t\right)\tilde{\phi}G_{\mu\nu}^{A}$ (with imaginary coefficient)	single top, $q\bar{q}, gg \to t\bar{t}$
$O_{\tilde{G}} = f_{ABC}\tilde{G}_{\mu}^{A\nu}G_{\nu}^{B\rho}G_{\rho}^{C\mu}$	$gg \to t\bar{t}$
$O_{\phi\tilde{G}} = \frac{1}{2}\left(\phi^{+}\phi\right)\tilde{G}_{\mu\nu}^{A}G^{A\mu\nu}$	$gg \to t\bar{t}$

Table 2: CP-odd dimension-six operators related to top-quark[6]

With the effective lagrangian, we would mainly discuss the application of the effective field theory approach on top quark decay and production. For top quark production, we include both top-quark pairs and single top-quarks.

2 Top Quark Decay

The only know way of top quark is through weak interaction. A top quark tends to decay into a W boson and a bottom quark, while decay into strange or down quarks are also possible but supressed by the corresponding small CKM elements[5]. The only two operators relevant to single top decay is

$$O_{\phi q}^{(3)} = i\left(\phi^{+}\tau^{I}D_{\mu}\phi\right)\left(\bar{q}\gamma^{\mu}\tau^{I}q\right)$$
$$O_{tW} = \left(\bar{q}\sigma^{\mu v}\tau^{I}t\right)\widetilde{\phi}W_{\mu v}^{I}$$

After spontaneous symmetry broken, replace the higgs field with its vaccum expection value,and then we get the corresponding lagrangian:

$$L_{eff} = \frac{C_{\phi q}^{(3)}}{\Lambda^2}\frac{gv^2}{\sqrt{2}}\bar{b}\gamma^\mu P_L t W_\mu^- + \text{h. c.}$$
$$L_{eff} = -2\frac{C_{tW}}{\Lambda^2}v\bar{b}\sigma^{\mu v}P_R t\partial_\nu W_\mu^- + \text{h.c.}$$

where C_i is the corresponding coefficient of O_i. Assuming W boson would further decay into $e^+$and v, we can write down the squared root amplitude based on the above discussion:

$$\frac{1}{2}\Sigma|M|^2 = \frac{V_{tb}^2 g^4 u\left(m_t^2-u\right)}{2\left(s-m_W^2\right)^2} + \frac{C_{\phi q}^{(3)}V_{tb}v^2}{\Lambda^2}\frac{g^4 u\left(m_t^2-u\right)}{\left(s-m_W^2\right)^2} + \frac{4\sqrt{2}\,\mathrm{Re}\,C_{tW}V_{tb}m_t m_W}{\Lambda^2}\frac{g^2 su}{\left(s-m_W^2\right)^2}$$

where s, t, u are Mandelstam variables: $s = \left(p_t - p_b\right)^2, t = \left(p_t - p_v\right)^2, u = \left(p_t - p_{e^+}\right)^2$.

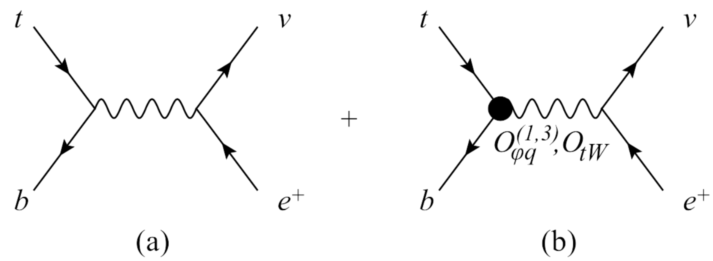

Figure 1: Feynman diagrams of top decay with EFT correction[6]

We can calculate the decay rate under narrow width approximation:

$$\frac{d\Gamma}{d\cos\theta} = \left(V_{tb}^2 + \frac{2C_{\phi q}^{(3)}V_{tb}v^2}{\Lambda^2}\right)\frac{g^4}{4096\pi^2 m_t^3 m_W \Gamma_W}\left(m_t^2-m_W^2\right)^2\left[m_t^2+m_W^2+\left(m_t^2-m_W^2\right)\cos\theta\right]$$
$$(1-\cos\theta) + \frac{\mathrm{Re}\,C_{tW}V_{tb}g^2}{128\sqrt{2}\pi^2\Lambda^2 m_t^2\Gamma_W}m_W^2\left(m_t^2-m_W^2\right)^2(1-\cos\theta)$$

Both operator contributes to the differential decay rate, while $C_{\phi q}^{(3)}$ only leads to a rescaling effect of the SM Wtb vertex.

3 Top Production in a hadron collider

3.1 Single Top Production

Single production can happen in three different process: T-channel, S-channel and tW channel. In s-channel and t-channel, a W-boson decay into a top and anti bottom quark. In tW channel, a bottom quark decay into a top-quark and a W boson.

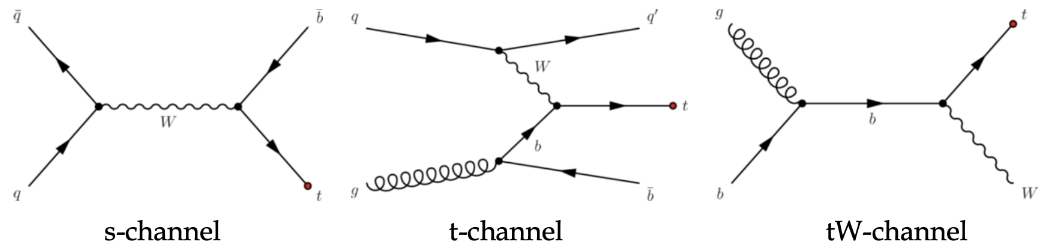

Figure 2: SM Feynman diagram of single top production

For s-channel and t-channel:

$$\begin{aligned}
O_{\phi q}^{(3)} &= i\left(\phi^{+}\tau^{I}D_{\mu}\phi\right)\left(\bar{q}\gamma^{\mu}\tau^{I}q\right)\\
O_{tW} &= \left(\bar{q}\sigma^{\mu v}\tau^{I}t\right)\tilde{\phi}W_{\mu v}^{I}\\
O_{qq}^{(1,3)} &= \left(\bar{q}^{i}\gamma_{\mu}\tau^{I}q^{j}\right)\left(\bar{q}\gamma^{\mu}\tau^{I}q\right)
\end{aligned}$$

$O_{\phi q}^{(3)}$ simply renormalizes the standard model interaction, since when we replace the higgs field with its vev, it would yields $\mathcal{L}_{eff} = \frac{C_{\phi q}^{(3)}v^{2}}{2\sqrt{2}\Lambda^{2}}\left(\bar{b}\gamma^{\mu}\left(1-\gamma_{5}\right)t\right)W_{\mu}^{-}$+h.c. $O_{qq}^{(1,3)}$ is the four fermion interaction.

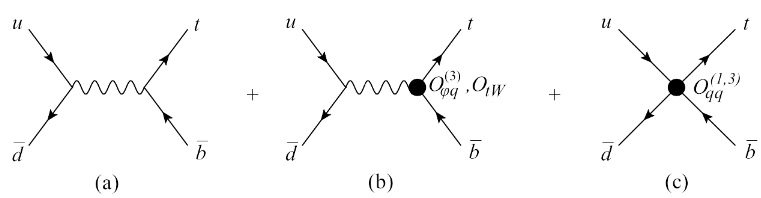

Figure 3: s-channel single top pair production. t-channel is simmilar and only need to turn around the diagram.

Now consider the tW channel, we have the following relevant operator:

$$\begin{aligned}
O_{\phi q}^{(3)} &= i\left(\phi^{+}\tau^{I}D_{\mu}\phi\right)\left(\bar{q}\gamma^{\mu}\tau^{I}q\right)\\
O_{tW} &= \left(\bar{q}\sigma^{\mu v}\tau^{I}t\right)\tilde{\phi}W_{\mu v}^{I}\\
O_{tG} &= \left(\bar{q}\sigma^{\mu v}\lambda^{A}t\right)\tilde{\phi}G_{\mu v}^{A}
\end{aligned}$$

Here we have a new operator: the "chromomagnetic moment" O_{tG}, which modifies the gtt coupling. We can similarily draw the corresponding Feynman diagram (4) and calculate the corresponding amplitude(which won't be done here.)

3.2 Top pair production

In a hadron collider, the largest production mechanism is pairs of top quarks through the strong interaction, while production through a virtual Z boson is

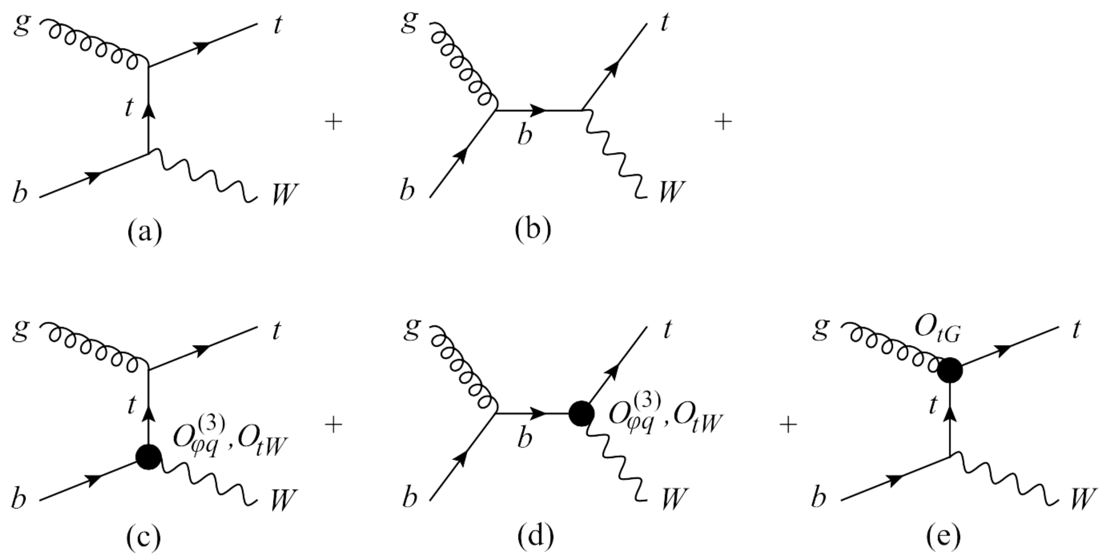

Figure 4: tW-channel single top production, where (a)(b) is SM Feynman diagram and the others are from EFT operators[6]

much smaller. The latter is easier to see in lepton colliders[2]. At leading order, there are gluon-gluon and quark anti-quark initial states.

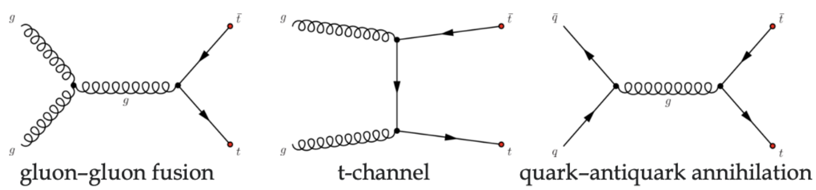

Figure 5: SM Feynman diagram of top pair production

At the LHC, top pair production is dominated by $gg \to t\bar{t}$, which is influenced by

$$
\begin{aligned}
O_{tG} &= \left(\bar{q}\sigma^{\mu v}\lambda^{A}t\right)\tilde{\phi}G_{\mu v}^{A} + \text{ h.c.} \\
O_{G} &= f_{ABC}G_{\mu}^{Av}G_{v}^{B\rho}G_{\rho}^{C\mu} \\
O_{\phi G} &= \frac{1}{2}\phi^{\dagger}\phi G_{\mu\nu}^{A}G^{A\mu\nu}
\end{aligned}
$$

Among these operators, O_G modifies the triple-gluon vertex and the O_{tG} couples the higgs field to gluon. The corresponding feynman diagram is shown in fig6.

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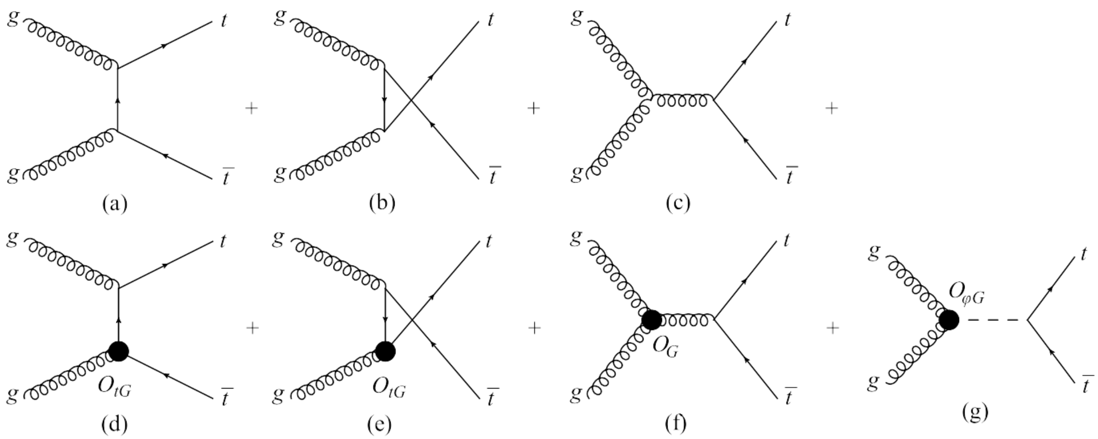

Figure 6: Top-quark pair production via gg collision [6]

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