Semester 2 Notes

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Spring Semester 25

Contents

1 Analysis II

1.1 Propositional Logic

27/1/25

- $\neg(p \cup q) = (\neg p) \cap (\neg q)$
- $\neg(p \cap q) = (\neg p) \cup (\neg q)$
- $\neg(\exists p) = \forall p$
- $p \Rightarrow q = (p \cap q) \cup \neg q$
- so $\neg(p \Rightarrow q) = (\neg p \cup \neg q) \cap \neg\neg q = \neg p \cup q$

1.2 Continuous Functions

31/1/25

Continuity

- For $f : D \to \mathbb{R}$, f is continuous at a point $a \in D$ if:
  $\forall \epsilon > 0, \exists \delta > 0 : \forall x \in D, |x - a| < \delta \Rightarrow |f(x) - f(a)| < \epsilon$
- f is uniformly coninuous if continuous $\forall a$, or 3/2/25
  $\forall \epsilon > 0, \exists \delta > 0 : \forall x, y \in D, |x - y| < \delta \Rightarrow |f(x) - f(y)| < \epsilon$
- In general, continuity $\not\Rightarrow$ uniform continuity
- A function $f : D \to \mathbb{R}$ is continuous at $a \in \mathbb{R}$ iff:

$$\forall (x_n)_{n\in\mathbb{N}}, x_n \in D, \lim_{n\to\infty} x_n = a, \lim_{n\to\infty} f(x_n) = f(a)$$

- An alternative statement of this is:

$$\lim f(x_n) = f(\lim(x_n))$$

  if f not continuous at a:

$$\exists \epsilon > 0 : \forall \delta > 0, \exists x \in D :$$

$$|x - a| < \delta \Rightarrow |f(x) - f(a)| \geq \epsilon$$

  Theorem 1 (Intermediate Value Theorem) *for* $f : D \to C \subseteq \mathbb{R}$ *then* $u \in C \Rightarrow \exists x \in D : u = f(x)$

- A function is also continuous if:
- $\delta > 0 : \forall x \in D \backslash a : |x - a| > \delta$
- $\forall (x_n)_{n\in\mathbb{N}}$

2 waves and fields

2.1 Periodic Motion

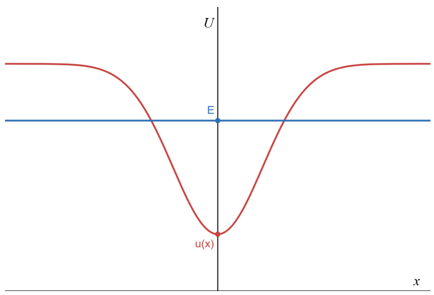
Periodic Motion In 1D

28/1/25

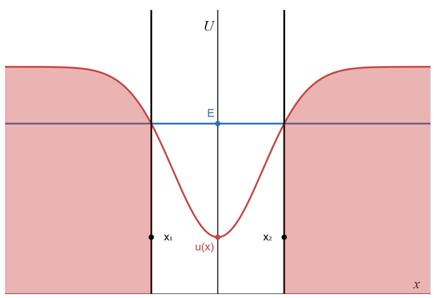
- Periodic motion is one in which the position repeats values regularly
- It can be characterised like so:

$$r(t + nT) = r(t), \forall t \in \mathbb{R}, n \in \mathbb{N}, T = period$$

- If we consider a situation in a consevative field, with a potential $u(x)$, and a system with a total energy E:

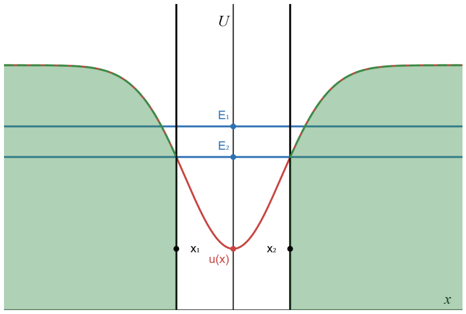


- We can see that the system would be bound within the region $U < E$:

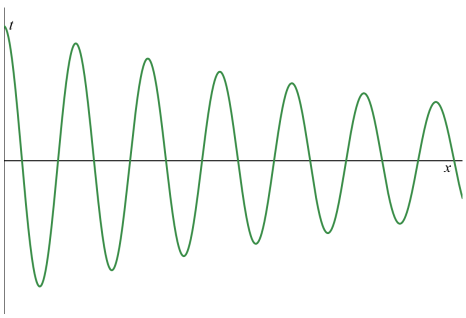


- This naturally leads to periodic motion, as at $x = x_1, v = 0$
- At this point, the energy is not minimised, and so the system tends towards $x = 0$, where $v \neq 0$, causing periodic motion. 30/1/25
- One way to achieve this is through simple harmonic motion (SHM)
- SHM is any system where $\ddot{x} = -x$
- We can see here that $u(x) = -\int f dx = \int \frac{\ddot{x}}{m} dx$, or $f = -\frac{du}{dx} \Rightarrow m\ddot{x} = -\frac{du}{dx}$
- We can then add another term to take energy out of the system:
- $m\ddot{x} = -\frac{du}{dx} - \gamma\dot{x}$ in general.

- This describes a frictional force, causing damped harmonic motion (DHM)
- This will lead to the energy gradually decreasing:



- Which will result in the following motion:

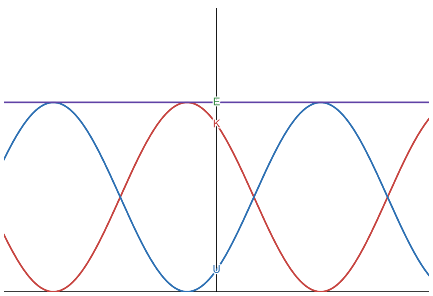


Periodic Motion In 3D

- Gravity is a conservative force so $u = u(\vec{r})$, and angular momentum is conserved
- Evidently, from $\vec{L} = \vec{r} \times \vec{p}$, the position $\vec{r}$ is perpendicular to the constant angular momentum $\vec{L}$, and thus the system is confined to a single plane.
- Let v_ϕ =the component of v perpendicular to $\vec{r}$, and v_r = the component of v parallel to $\vec{r}$
- Then $L = m\vec{r} \times (r\dot{\phi})$
- Since $K = \frac{1}{2}m\dot{\vec{r}} = \frac{1}{2}m(\dot{\vec{r}}_\phi^2 + \dot{\vec{r}}_r^2) = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2)$
- $\dot{\phi} = \frac{L}{mr^2} \Rightarrow K = \frac{1}{2}m(\dot{r}^2 + r^2(\frac{L}{mr^2})^2)$
- So $E = K + u = \frac{1}{2}m(\dot{r}^2 + (L/mr)^2 + u(\vec{r}))$

Simple Harmonic Motion 31/1/25

- Simple harmonic motion is any motion of the form $x(t) = Acos(\omega t + \phi)$
- From this, we can see that $\ddot{x}(t) = -A\omega^2 \cos(\omega t + \varphi) = -\omega^2 x$
- This differential equation, $\ddot{x} \propto x$, characterises simple harmonic oscillation(SHO)
- Then, $f = m\ddot{x} = -m\omega^2 x$, is only dependant on position
- Then, the potential energy: $U(x) = -\int f dx = m\omega^2 \int x dx = \frac{1}{2}m\omega^2 x^2$
- Then, $E = K + U = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}mA^2\omega^2$
- E, K, and U will look like this over time, with $\overline{U} = \overline{K}$ over one period:



Simple Pendulums and Small oscillators

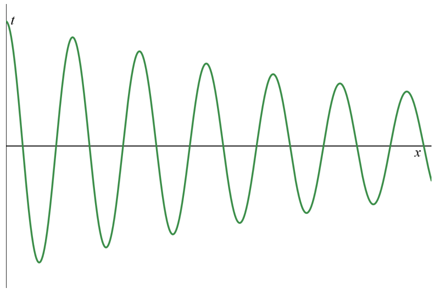
- For a simple pendulum, $|\tau| = I\ddot{\theta} = -f_{\perp} \cdot r = -f_g \sin(\theta) \cdot l$
- Then, using the small angle approximation $\sin(\theta) = \theta$:
  $I\ddot{\theta} = -f_g l\theta$
- Another example of simphe harmonic motion is the torsion pendulum
- This is a mass on a string that can twist around its axis:
  $\tau = -c\theta \Rightarrow I\ddot{\theta} = -c\theta$

Damped Harmonic Motion 6/2/25

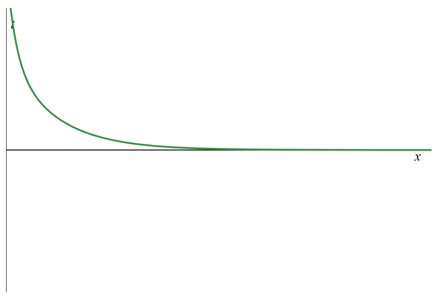
- We expect the amplitude A of a damped harmonic oscillator to decrease over time.
- For DHM, The 2nd law of motion looks like this:
- $m\ddot{x} = -kx - b\dot{x}$
- Setting b=0, we recover the SHM characteristic equation
- For $b^2 - 4km < 0$, This is called underdamping

- Underdamping will have the following form of motion:

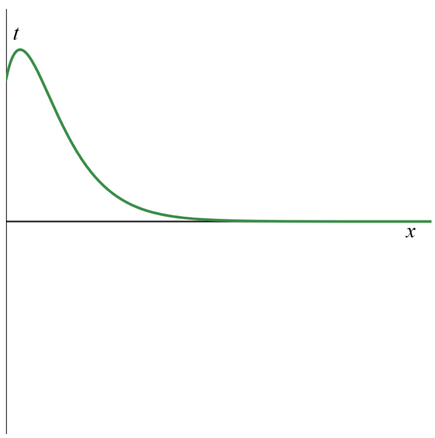
$$x(t) = Ae^{-\frac{b}{2m}t}\cos(\omega t + \varphi) \tag{2.1}$$



- Then considering the quantity $\frac{2m}{b}$, we see $|x(t + \frac{2m}{b})| = e^{-1}|x(t)|$
- If $b^2 > 4mk$, we get overdamped motion:
- $x(t) = A_1 e^{-\lambda_1 t} + A_2 e^{-\lambda_2 t}$



- and if $b^2 = 4mk$, critically damped Motion:
- $x(t) = (A_1 + A_2 t)e^{-\frac{b}{2m}t}$



Forced Harmonic Oscillation & Resonance

7/2/25

- A forced harmonic oscillator (FHO) is one subject to an external driving force F_d
- $\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m} = \frac{1}{m}F_d$
- If the driving force is also periodic, the motion will oscillate at the driving frequency f_d
$$\omega_r = \sqrt{\frac{k}{m} - \frac{b^2}{2m^2}} \tag{2.2}$$
- Where A is maximised at $\omega_d = \omega_r$

Coupled Harmonic Oscillators

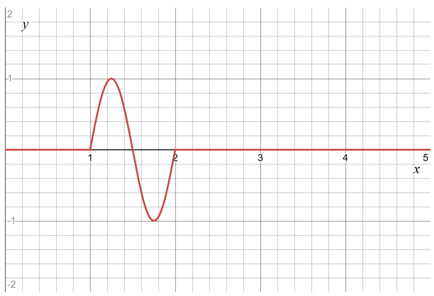
- for coupled springs with masses $m_{1,2}$ at positions $x_{1,2}$
- Clearly, $m\ddot{x}_1 = -kx_1 - k'(x_1 - x_2)$
- And a similar result for x_2
- If we then consider the case $x1 = x2$, the middle spring's force vanishes and we get 2 uncoupled harmonic motions with frequency $\omega_1 = \sqrt{\frac{k}{m}}$
- This is called a normal mode of the system, as the $x_1(t) \perp x_2(t)$: They are independant
- Considering the alternate normal mode: $x_1(t) = -x_2(t)$
- And the motion $\omega_2 = \sqrt{\frac{k+2k'}{m}}$
- In n dimensions, there are n normal modes, and all normal frequencies are normal frequencies
- All modes of a system are superpositions of the normal modes

Waves

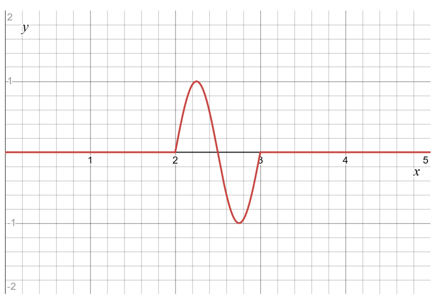
Wave Basics

13/2/25

- Waves are a form of harmonic motion
- Waves have an amplitude, wave speed, and carry energy and momentum
- There are travelling waves and standing waves
- The simplest consideration is a string moving in one dimension:



- After some time, the wave will now look like so:



- The function for a 1d wave will be $y(x,t) = f(x - vt)$ as it moves in time but the shape doesnt move in time
- The actual particles causing the wave always return to rest once the wave has left the space
- For sinusoidal waves:
- $y(x,t) = A\cos(k[x \mp vt] + \varphi) = A\cos(kx \mp \omega t + \varphi) = A\cos(2\pi[\frac{x}{\lambda} \mp ft] + \varphi)$
- where $k =$the wave number or spacial frequency, and $k = \frac{1}{\lambda}$

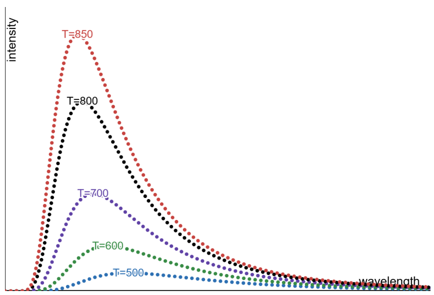
3 Modern Physics

3.1 Black Body Radiation

31/1/25

Observations

- Black bodies satisfy 3 criteria:
  - absorbs all incident radiation
  - emits as much radiation as possible at any wavelength
  - emits radiation isotropically
- By studying the spectra of black bodies at different temperatures:



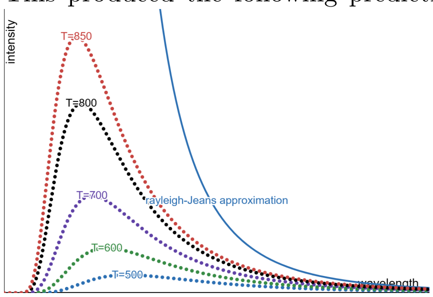
- We can observe 2 main laws:
  - Wein's displacement law: $\lambda_{max} = \frac{b}{T}$
  - Stephans law: $P(T) = \sigma A T^4$

Rayleigh-Jeans Law

- In June 1900, John Rayleigh discovered that the spectral raiance $\propto \lambda^{-4}$
- In 1905, James Jean derived the full Rayleigh-Jeans law including constants
- The Rayleigh-Jeans law is a classical approximation of the radiance of a black body
- Rayleigh-Jeans law can be expressed as so:

$$B(\lambda, T) = \frac{2ck_bT}{\lambda^4} \tag{3.1}$$

- This produced the following prediction for spectral radiance at T=800:



- This presents many problems, mainly that it doesnt match for $\lambda < \lambda_p$, and $I \to \infty$ as $\lambda \to 0$

Planck's Law

- Rayleigh-Jeans law, as seen in the derivation, can be expressed as:
- $R(\lambda) \propto \rho(f) \cdot E_{avg}$
- Where $\rho and E$ are the number of waves and average energy of each wave
- In the Rayleigh-Jeans law, $E_{avg} = kT$
- This was due to the equipartition theorem
- Max Planck's great insight was to abandon the equipartition theorem, instead writing:
- $E = n\epsilon \Rightarrow \overline{E} = \frac{hf}{e^{\frac{hf}{kT}}-1}$
- And so:
$$R(\lambda) = \frac{2hc}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda kT}}-1} \tag{3.2}$$

3.2 The Photoelectric Effect

6/2/25

- In 1887, Heinrich Hertz discovered that light rays incident on a metal plate produces an emf across the material
- This is due to electrons being liberated by the light
- This works with metals due to the delocalised electron bonds
- The electrons would be liberated with a range of kinetic energies up to a maximum
- There was also an observed threshold frequency, below which a ray can't liberate electrons

classical Predictions

- $V_s \propto I$
- $K_{max} \propto A$
- $\frac{dN}{dt} \propto f$

Einstein's Explanation

- In 1905, Einstein published his explanation of the photoelectric effect
- He explained that the light interacts in a 1:1 mannor, with $E = hf$
- This is insightful as Max Planck had only quantised matter oscillation
- Einstein qunatised light
- This explains the intensity profile, as the particle light must interact with the electrons 1:1
- It also explained the threshold frequency phenomenon, as light with insufficient energy just reflects off of the metal
- $E_\gamma = hf = \varphi + \frac{1}{2}m_e v_{max}^2$ or $hf = \varphi + K_{max}$

3.3 Rutherford Experiment

13/2/25

- In 1814, Joseph von Fraunhofer observed line spectra
- When light shone through a gas is split through a prism, you observe a full spectrum with line gaps
- In 1885, Johann Balmer discovered the Balmer series, which described where these line gaps would occur
- Neils Bohr described these balmer seires by quantising electrons' angular momentum, meaning they can only occupy specific orbitals
- Then, $E_\gamma = hf = E_i - E_f$
- Bohr's model worked well only for hydrogenic atoms
- We can obtain:
$$E_\gamma = -(13.6\ eV)Z^2[\frac{1}{n_i^2} - \frac{1}{n_f^2}] \tag{3.3}$$
- We call the ionisation energy $E_i = E_{infty} - E_1$, the energy required to leave the atom

4 Intro to Quantum Physics

4.1 A Brief History

Ultraviolet Catastrophe

29/1/25

- In 1860, Gustav Kirchhoff discovered what is now called black body radiation
- This is where an object with internal energy radiates electromagnetic waves correlated with its thermal properties.
- Classical predictions, using namely the Rayleigh-Jeans law, predicted an infinite ammount of low-frequency light waves:
- In 1990, Max Planck produced a soloution to this problem which correctly predicted the spectral radiance of black body radiation
- This theory worked by quantising light, which he expected to disapear in the final solution
- He still did not believe that light was acually quantised
- He earned the Nobel Prize in 1918

Photoelectric Effect

- In 1887, Heinrich Hertz discovered the photoelectric effect
- This is when you expose a metal plate to high intensity light and observe a voltage over the plate
- The contemporary theories predicted that the energy transfered should be proportional to the frequency
- Instead, there is an observed threshold frequency, below which no energy is transfered
- In 1905, Albert Einstein explained this by employing Max Plancks quantum light
- He won the Nobel Prize in 1921

Atomic Models

- In 1911, Ernst Rutherford discovered positively charged atomic nuclei
- The presence of the electron in the atom, then, predicted that atoms were unstable, decaying in $10^{-}11s$ as the electron would fall into the nuclei
- In 1913, Neils Bohr predicted that angular momentum was quantised, restricting electrons to specific "orbits" and stopping atomic decay

- Neils Bohr could not come up with a mechanism for angular momentum quantisation, however
- He won the Nobel Prize in 1913

DeBroglie's Electron Strings

- In 1924, Louis DeBroglie suggested wave particle duality
- This suggests that sufficient restrictions upon an electrons position force it to exhibit wave like behaviour
- When employed on an electron in orbit, the electron waves superpose, producing a standing wave
- Standing waves can only exist with an integer multiplier on the fundamental frequency
- This multiplier corresponds to the energy the electron can have, quantising orbital angular momentum
- He won the Nobel Prize in 1929

4.2 Rutherford And Compton Scattering

30/1/25

Rutherford Scattering

- In 1987, Thompson devised the Plum Pudding model.
- In 1906 in Manchester, Ernst Rutherford devised the α scattering experiment
- He shot α particles at a thin sheet of gold and observe the distribution of scattering angles
- The Plum Pudding model predicts the following distribution:
- It predicted little to no scattering due to the diffusity of the positive charge
- Whereas the distribution Rutherford found was the following:
- In response to this, rutherford devised the nuclear model
- This explained that the nucleus repels the α particles due to the like charges, sometimes up to a full π^c

Compton Scattering

- In 1923, Arthur Compton discovered Compton scattering
- This is where x-rays incident on an atom reflect at a different wavelength
- The classical model, with $p_\gamma = 0$, predicted no shift:
- But in reality, there was a shift:
- This showed that light does carry momentum, and therefore supported the photon model
- Using conservation of momentum and conservation of energy, you can arrive at the following equation:

$$\lambda - \lambda' = \frac{h}{m_0 \cdot c}[1 - \cos(\theta)] \tag{4.1}$$

4.3 Milikan's Oil Drop

5/2/25

- In 1897, Robert Milikan devised an experiment to measure the charge of the electron:
- The 2 plates are connected to a power source, producing an emf
- A nozzle containing oil is small to be sprayed
- There is a camera or microscope to observe the drops
- The drops become charged due to friction with the nozzle
- Oil drops that move up are large charge or low mass, and vice versa
- There is also the bouyant and viscous forces
- $\vec{f_b} = \frac{4}{3}\pi g r^3 \rho_{air} \hat{e}_z$ since the oil is displacing the air
- from Stoke's Law, $\vec{f_v} = -6\pi\eta_{air} \cdot rv$
- $\vec{f_v} = -6\pi\eta_{air}$
- Since it is hard to measure the properties of the drops explicitly, we can instead formulate expressions for them:

$$r = \sqrt{\frac{6\eta_{air} v_{off}}{2(\rho_{oil} - \rho_{air})g}} \tag{4.2}$$

$$q = \frac{g\sqrt{2}(v_{on} + v_{off})d}{V_0} \cdot \sqrt{\frac{\eta_{air}^3 v_{off}}{(\rho_{oil} - \rho_{air})g}} \tag{4.3}$$

- You then repeat this multiple times and record the charge of each drop 6/2/25
- There will be a random distribution of charges, however the smallest difference in charge will be $e = 1.60 \cdot 10^{-19}$

4.4 Quantum Behaviour

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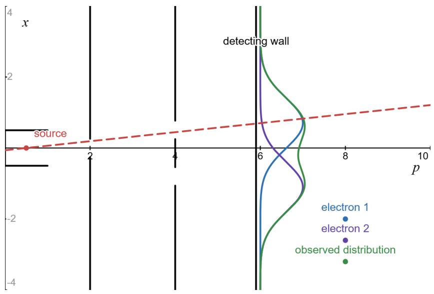
Young's Double Slit

- If particles are sent through the slits, you expect the following distributions:



- And if waves are sent through:
- When Young's double slit experiment was performed with electrons, the distribution found was that of a wave
- However when you put a light on the other side of the slits, the distribution turns out like a particle

5 Mathematical Methods II

5.1 Non Cartesian Co-Ordinates

Polar Co-Ordinates

31/1/25

- for $x\hat{\imath} + y\hat{\jmath}$, the equivalent polar basis vectors are:
  $\hat{r} = (\cos(\theta), \sin(\theta))$, $\hat{\theta} = (-\sin(\theta), \cos(\theta))$ where $\tan(\theta) = \frac{y}{x}$
- Then, $\cos(\theta)\hat{r} - \sin(\theta)\hat{\theta} = \hat{\imath}$ and $\sin(\theta)\hat{r} + \cos(\theta)\hat{\theta} = \hat{\jmath}$

3D Polar Co-Ordinates

- There are 2 main 3D Polar Coordinate Bases:
- Spherical $(\hat{r}, \hat{\theta}, \hat{\varphi})$
- And Cylindrical $(\hat{\rho}, \hat{\theta}, \hat{z})$

6 Derivations

6.1 Analysis II

6.2 Waves and Fields

eqn 2.1 Underdamping Motion Form

- $m\ddot{x} = -kx - b\dot{x}$
- $\Rightarrow \ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0$
- This has a characteristic equation $\lambda^2 + \frac{b}{m}\lambda + \frac{k}{m}$
- With soloutions $\lambda = -\frac{b}{2m} \pm \frac{1}{2}\sqrt{(\frac{b}{m}^2) - 4\frac{k}{m}}$
- For case $b^2 < 4mk, \lambda = -\frac{b}{2m} \pm i\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$

$$\Rightarrow x(t) = Re[Ae^{-\frac{b}{2m}t}e^{\pm i\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}}]$$

- $\Rightarrow \boxed{x(t) = Ae^{-\frac{b}{2m}t}\cos(\omega t + \varphi)}$

eqn 2.2 Resonance Frequency

- For $\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = \frac{1}{m}f_d, f_d = d\cos(\omega_d t + \varphi)$ DE
- We know that for FHM, $f = f_d$ so:
- Assume $x(t) = Re[\mathcal{A}e^{i\omega_d t}], F_d = Re[\mathcal{D}e^{i\omega_d t}]$ where $\mathcal{D} = De^{i\varphi_d}$
- Into DE: $-\omega_d^2\mathcal{A}e^{i\omega_d t} + \frac{b}{m}i\omega_d\mathcal{A}e^{i\omega_d t} + \frac{k}{m}\mathcal{A}e^{i\omega_d t} = \frac{1}{m}\mathcal{D}e^{i\omega_d t}$
- $\Rightarrow \mathcal{A}(\omega_d^2 - \frac{b}{m}i\omega_d - \frac{k}{m}) = -\frac{1}{m}\mathcal{D}$
- $\Rightarrow \mathcal{A} = \frac{\mathcal{D}/m}{\omega_d^2 - \frac{b}{m}i\omega_d - \omega_0^2}$ where $\omega_0 = \sqrt{\frac{k}{m}}$
- Then, amplitude $A = |\mathcal{A}| = \frac{D/m}{\sqrt{(\omega_d^2 - \omega_0^2)^2 + \frac{b^2}{m^2}\omega_d^2}}$
- It is obvious that A_{max} occurs at the minimum of the denominator
- $\frac{d\mathcal{W}}{d\omega_d} = \frac{d}{d\omega_0}[\sqrt{(\omega_d^2 - \omega_0^2)^2 + \frac{b^2}{m^2}\omega_d^2}] = \frac{\frac{1}{2}[4(\omega_d^3 - \omega_d\omega_0^2) + 2\frac{b}{m}^2\omega_d]}{\mathcal{W}}$
- $\frac{d\mathcal{W}}{d\omega_d}|_{\omega_r} \Rightarrow 2\omega_r[2(\omega_r^2 - \omega_0^2) + \frac{b}{m}^2] = 0$
- $\omega_d \neq 0 \Rightarrow \omega_r^2 = \omega_0^2 - \frac{b^2}{2m^2}$
- $\Rightarrow \boxed{\omega_r = \sqrt{\frac{k}{m} - \frac{b^2}{2m^2}}}$

eqn 2.3 Coupled Harmonic Oscillator

- $m\ddot{x}_1 = -kx_1 - k^{prime}(x_1 - x_2)$
- $m\ddot{x}_2 = -kx_2 - k^{prime}(x_2 - x_1)$

6.3 Modern Physics

eqn 3.1 Rayleigh-Jeans Law

- Firstly, we begin by noting that $R(\lambda) = \rho(f) \cdot \frac{c}{4\pi}$ where $\rho(f)$ is the energy density inside a Jeans cube
- Then, $\rho(f) = N(f) \cdot E_{avg}$
- $N(f) = \frac{8\pi f^2}{c^3}$, and $E_{avg} = kT$
- Then $\rho(f) = \frac{8\pi f^2 kT}{c^3}$
- $= \frac{8\pi kT}{\lambda^2}$
- $\Rightarrow \boxed{\dfrac{2ckT}{\lambda^4}}$

eqn 3.2 Planck's Law

eqn 3.3 Bohr Model

- Energies in the atom:
  - $E_k = \frac{1}{2}m_e v^2 + \frac{1}{2}m_n \cdot 0^2$
  - $E_p = \frac{Ze^2}{4\pi\epsilon_0 r}$
- Forces in the atom:
  - $f_c = \frac{m_e v^2}{r}$
  - $f_e = \frac{-Ze^2}{4\pi\epsilon_0 r^2}$
- Where Z=atomic number
- Total energy $E_n = E_k + E_p = \frac{1}{2}m_e v^2 - \frac{Ze^2}{4\pi\epsilon_0 r}$
- Hypothesize stationary orbit: $|f_c| = |f_e|$
- $\frac{m_e v^2}{r} = \frac{Ze^2}{4\pi\epsilon_0 r}$ (Eq1)
- $\Rightarrow m_e v^2 = \frac{Ze^2}{4\pi\epsilon_0 r}$
- $\Rightarrow E_n = \frac{1}{2}\frac{Ze^2}{4\pi\epsilon_0 r} - \frac{Ze^2}{4\pi\epsilon_0 r} = -\frac{1}{2}\frac{Ze^2}{4\pi\epsilon_0 r}$ (Eq2)

- Hypothesize quantized angular momentum: $L_n = m_e v r_n = n\hbar$
- $\Rightarrow v = \frac{n\hbar}{m_e r_n}$
- From Eq1, $m_e(\frac{n\hbar}{m_e r_n})^2 = \frac{Ze^2}{4\pi\epsilon_0 r_n} \Rightarrow r_n = \frac{n^2}{Z} \cdot \frac{4\pi\epsilon_0\hbar^2}{m_e e^2}$
- Then, setting $n = 1, Z = 1$, we find the ground state in a hydrogen atom has $r_b = 5.29 \cdot 10^{-11}$
- Then, from r_n, Eq2,
- $2(4\pi\epsilon_0\hbar)^2 \cdot \frac{Z}{n}^2$
- Then, for $Z = 1, n = 1$, or the case of the ground state of the hydrogen atom:
- $E_0 = -13.6\ eV$
- $\boxed{E_\gamma = hf = E_0(\frac{1}{n_i^2} - \frac{1}{n_f^2})}$

6.4 Intro to Quantum Physics

eqn 4.1 Compton Wavlength

Milikan's Experiment

For plates with p.d. V_0, separation d, and droplet of charge q and radius r moving at velocity v:

- $\vec{f_e} = q \cdot \vec{E} = q \cdot \frac{V_0}{q} \cdot \hat{e}_z$
- $\vec{f_g} = mg \cdot \hat{e}_z$ and $m = V \cdot \rho_{oil} = \frac{4}{3}\pi r^3 \rho_{oil} \Rightarrow \vec{f_g} = \frac{4}{3}\pi r^3 \rho_{oil} g \hat{e}_z$
- oil displacing air $\Rightarrow \vec{f_b} = w_{displaced} = v_{air}\rho_{air}$
- $v_{air} = v_{oil} \Rightarrow \vec{f_b} = \frac{4}{3}\pi g r^3 \rho_{air} \hat{e}_z$
- from Stoke's Law, $\vec{f_v} = -6\pi\eta_{air} \cdot rv$

eqn 4.2 Milikan's Oil Drop Radius

If we set $V_0 = 0$ and consider the terminal velocity of the drop v_{off}

- $\vec{f_g} + \vec{f_b} + \vec{f_v} = 0$
- Note, $\vec{f_g} + \vec{f_b} + \vec{f_v} = 0 \Rightarrow f_v = f_g - f_b$
- $\frac{4}{3}\pi\rho_{oil} g r^3 \hat{e}_z - \frac{4}{3}\pi\rho_{air} g r^3 \hat{e}_z - 6\pi\eta_{air} v_{off} \hat{e}_z = 0$
- $\frac{4}{3}\pi(\rho_{oil} - \rho_{air} g) r^3 = 6\eta_{air} v_{off} r$

- $\boxed{r = \sqrt{\frac{6\eta_{air}v_{off}}{2(\rho_{oil} - \rho_{air})g}}}$

eqn 4.3 Milikan's Oil Drop Charge

apply voltage V_0 such that drop moves upwards to a terminal velocity v_{on}

- $\vec{f_e} + \vec{f_v'} + \vec{f_b} + \vec{f_e} = 0$
- $f_g\hat{e}_z + f_v'\hat{e}_z - f_b\hat{e}_z - f_e\hat{e}_z = 0$
- $f_e = f_v' + (f_g - f_b) = f_v' + f_v$, where f_v = viscous force from off case
- $\frac{qV_0}{d} = 6\pi\eta_{air}rV_{on} + 6\pi\eta_{air}v_{off} = 6\pi\eta_{air}r(V_{on} + V_{off})$
- Leading to:
- $\boxed{q = \frac{g\sqrt{2}(V_{on} + V_{off}d)}{V_0} \cdot \sqrt{\frac{\eta_{air}^3 V_{off}}{(\rho_{oil} - \rho_{air})g}}}$

6.5 Maths Methods II