Online Voltage Regulation of Distribution Systems with Disturbance-Action Controllers

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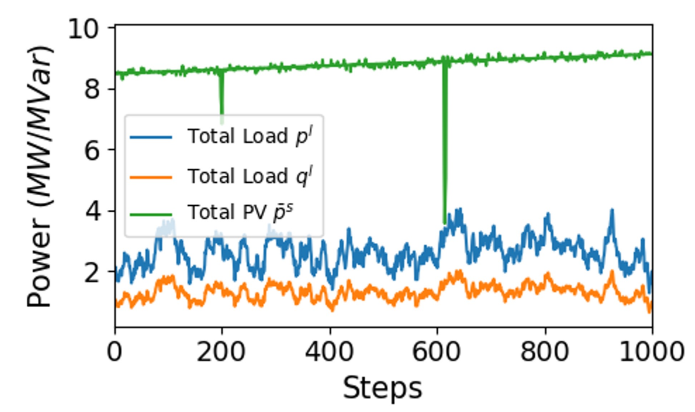
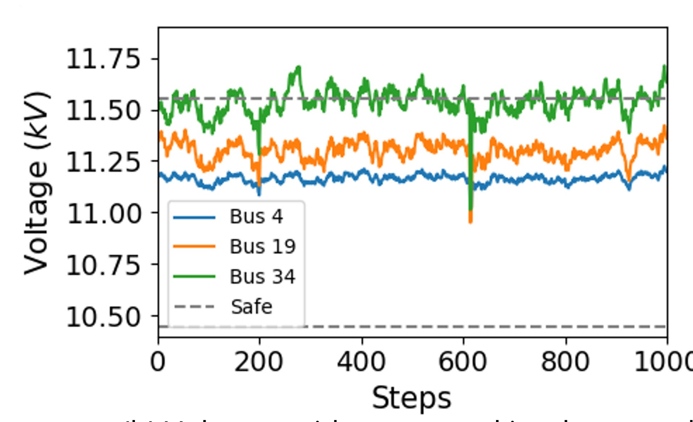
Overvoltage? Control Installed Inverters



(a) Total load and PV generation profiles



(b) Voltages with no control implemented

Optimal Power Flow

$$\min_{u\in\mathbb{R}^{2n}} \quad c(x,u) \qquad (1a)$$
$$\text{s.t.} \quad x = \mathcal{P}(u;u^l) \qquad (1b)$$
$$\underline{u} \le u \le \bar{u} \qquad (1c)$$
$$\underline{x} \le x \le \bar{x} \qquad (1d)$$

x : Voltage deviation, u : Control input,
u^l : Uncontrollable loads,
c : Loss function, $\mathcal{P}$: Power flow model
$\underline{[.]}$ and $\overline{[.]}$: Lower and upper bounds

Challenges

System-level

- Inaccurate estimation on the line parameters and topology
- Fast-changing operation conditions on loads and solar

Inverter-level

- Communication delay
- Inaccessibility to load information

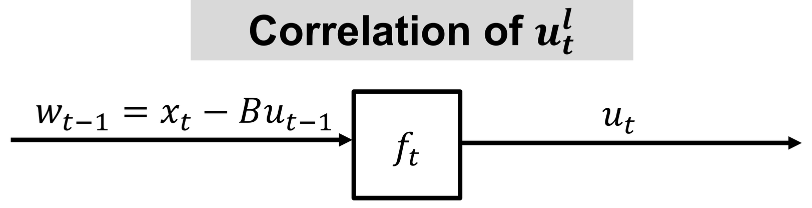
Disturbance-Action Controller (DAC)

Reformulate as linear system

$$x_{t+1} = B(u_t - u_t^l) = Bu_t + w_t \qquad (2)$$

B : Linearized power flow model, ω_t: Voltage drop of uncontrollable loads

Correlation of u_t^l



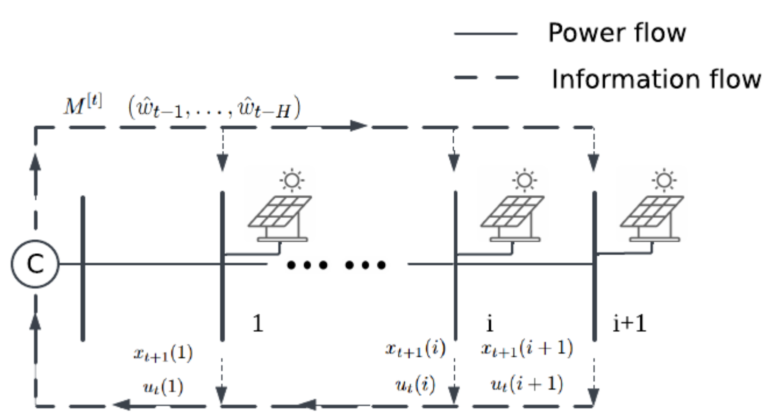
- **Linear policy:** DAC (System-level Approach [1])

$$u_t = \left[\tilde{u}_t + \sum_{i=1}^{H} M_i^{[t]}\hat{w}_{t-i}\right]_{\underline{u}_t}^{\bar{u}_t} \qquad (3)$$

Online Feedback Optimization

- **Receive measurement** $\hat{w}_t = x_{t+1} - \hat{B}u_t$ (4a)
- **Conduct policy gradient** $M^{[t+1]} = M^{[t]} - \eta\nabla_M c_t(x_{t+1}(M;\hat{w}), u_t(M;\hat{w}))$ (4b)

Main Results



Framework of DAC-based online voltage control

Data requirements

- Voltage deviation
- Controllable power injection

Stability Condition

Control theory

- Input-to-state stability is satisfied, under saturated action and memoryless dynamics
- Our target is to drive the controller to avoid fluctuation

Definition 1. The linear system (2) with saturated control input $u_t \in [\underline{u}, \bar{u}]$ is said to avoid the fluctuation if

$$\Pi_\tau^{\tau+1} I(u_\tau) \ge 0,\ \forall \tau \in \{1,\dots,T\}, \quad I(u) = \begin{cases} 1, & u \in [\bar{u},\infty) \\ 0, & u \in (\underline{u},\bar{u}) \\ -1, & u \in (-\infty,\underline{u}]. \end{cases} \qquad (5)$$

Theorem 7. Under *Assumptions 1-4*, it is sufficient to achieve stability on the state and input variables with model estimation error as $\|B - \hat{B}\| \le \epsilon_B$, by choosing initialization of controller as $\|M^{[0]}\| \le \frac{2\tilde{U}}{\epsilon_B\tilde{U}+W}$ and learning rate as

$$\eta \le \frac{2\tilde{U}}{LDd(1+\kappa_B)(\epsilon_B\tilde{U}+W)^2}. \qquad (6)$$

Optimization theory

- Convergence under static case

With descent lemma, the condition on learning rate is

$$\eta \le \frac{2}{LDd(1+\kappa_B)(\epsilon_B\tilde{U}+W)}. \qquad (7)$$

Performance Degradation

With the formulation of linear system, we can quantify the performance variation of DAC under model inaccuracies

Theorem 8. Suppose that the disturbance-action controller is implemented with the stability condition on η and $\|M^{[0]}\|$, and the estimation error is bounded by $\epsilon_B \le \frac{W}{\tilde{U}}$. Then, it holds true that $\|\Delta u_t\| \le \bar{Y}_t$ and $\|\Delta x_{t+1}\| \le \bar{X}_{t+1}$, where $\bar{Y}_t$ and $\bar{X}_{t+1}$ are defined as follows,

$$\bar{Y}_t \triangleq \begin{cases} (\bar{M}(\kappa_B+\epsilon_B))^{t-1}\|\Delta u_1\| & \text{if } \bar{M}(\kappa_B+\epsilon_B) \le 1, \\ \min\{(\bar{M}(\kappa_B+\epsilon_B))^{t-1}\|\Delta u_1\|, \tilde{U}\} & \text{if } \bar{M}(\kappa_B+\epsilon_B) > 1, \end{cases} \qquad (8)$$

$$\bar{X}_{t+1} \triangleq \kappa_B \bar{Y}_t, \qquad (9)$$

with the base case calculated as $\|\Delta\hat{w}_0\| \le \tilde{U}\epsilon_B$ and $\|\Delta u_1\| \le \bar{M}\tilde{U}\epsilon_B$.

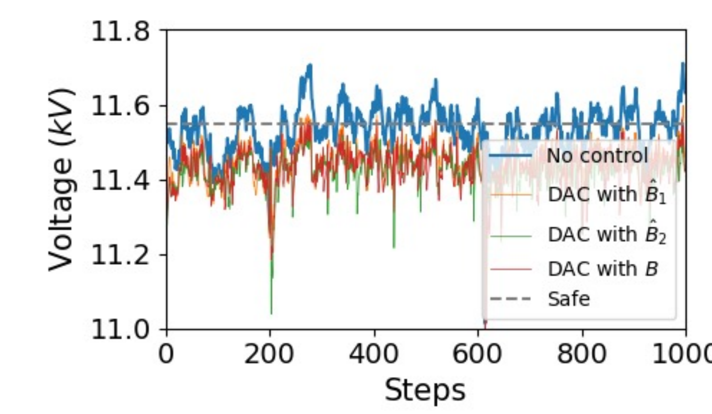
Simulation Results

Design of loss function

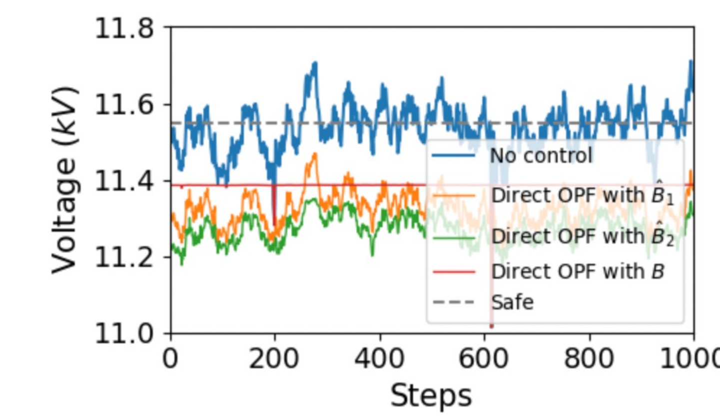
$$c_t = \eta_1\|u_t - \tilde{u}_t\|_2^2 + \eta_2\|\max(x_t - \bar{x}, 0)\|_2^2 \qquad (10)$$

(a) Implementation: $\bar{x} = 0.55 * (1 - \epsilon_B)$; (b) Evaluation: $\bar{x} = 0.55$

Robustness against model inaccuracy

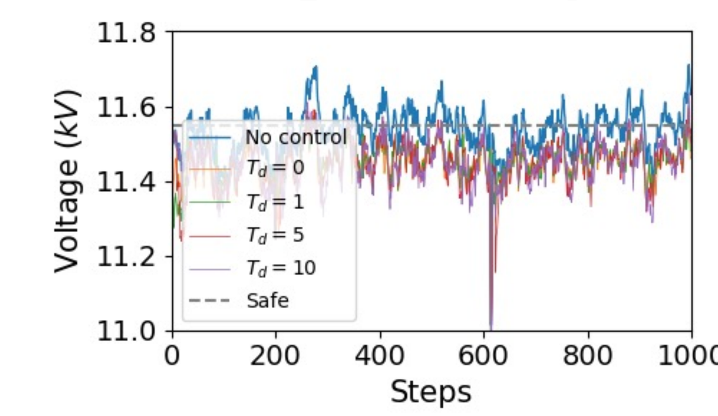


(a) Disturbance-action controller

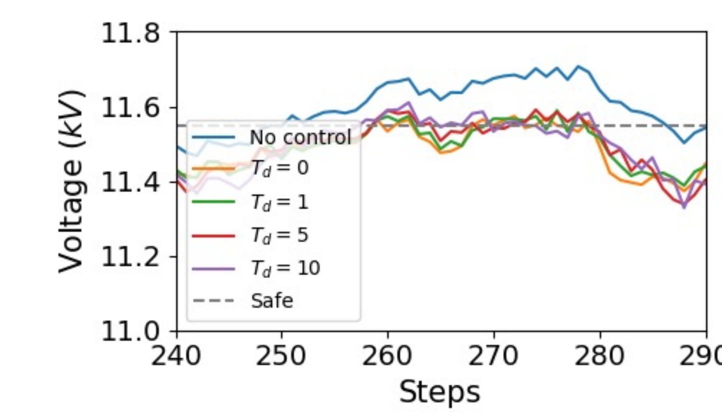


(b) Direct optimization method

Robustness against latency

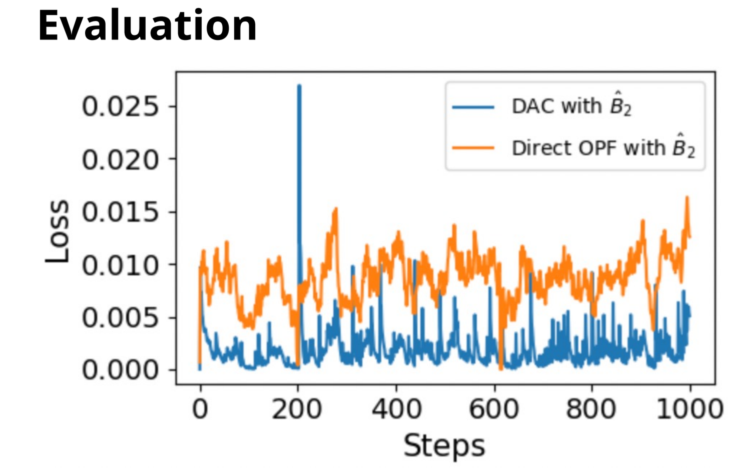


(a) Overall voltage profile at Node 34

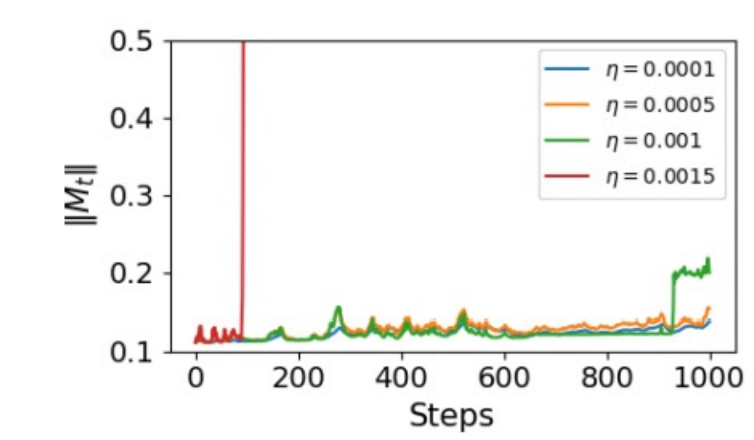


(b) Local voltage profile at Node 34

Evaluation



(a) Comparison of overall cost



(b) Controller parameters

Conclusion & Future Work

Conclusion

- Policy-based feedback control law for online voltage control
- Robustness to model inaccuracy and latency
- Stability condition on controller design

Future work

- Nonlinear policy: Neural network
- Decentralized controller design
- Safety guarantee: Online exponential barrier method [2]

$$\tilde{c}_t = c_t + \sum_i^n \frac{\hat{\alpha}_i}{\hat{\beta}_i} e^{\hat{\beta}_i(x_{t,i} - \bar{x}_i)} \qquad (11)$$

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References
[1] Wang, Yuh-Shyang, Nikolai Matni, and John C. Doyle. "A system-level approach to controller synthesis." IEEE Transactions on Automatic Control 64.10 (2019): 4079-4093.
[2] Zhang, Peng, and Baosen Zhang. "Optimal Voltage Control Using Online Exponential Barrier Method." arXiv preprint arXiv:2506.10247 (2025)
[3] Zhang, Peng, and Baosen Zhang. "Online Voltage Regulation of Distribution Systems with Disturbance-Action Controllers." arXiv preprint arXiv:2412.00629 (2024).