Optimal Voltage Control Using Online Exponential Barrier Method

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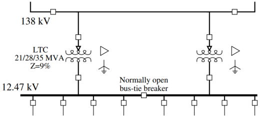
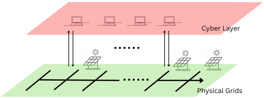
NAPS 2025 – Hartford, CT, Oct 27, 2025

Reference: P. Zhang and B. Zhang, "Optimal voltage control using online exponential barrier method," arXiv preprint arXiv:2506.10247, 2025.

Voltage Control in Distribution Systems



Traditional: Tap-changing transformers [1], capacitor banks



Modern: Inverter-based resources (fast-time-scale)

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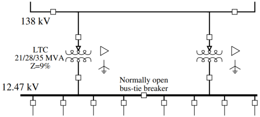

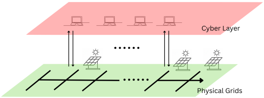

Traditional: Tap-changing transformers [1], capacitor banks

Modern: Inverter-based resources (fast-time-scale)

Excessive power injection from inverters $\implies$ **Overvoltage problem**

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Literature-Optimal Power Flow (OPF)

OPF Problem

$$
\begin{aligned}
\min_{\mathbf{p},\mathbf{q}} \quad & \text{(control + system cost)} \\
\text{s.t.} \quad & \text{power flow model} \\
& \underline{\mathbf{v}} \leq \mathbf{v} \leq \bar{\mathbf{v}} \quad \text{(voltage constraint)} \\
& \underline{\mathbf{p}} \leq \mathbf{p} \leq \bar{\mathbf{p}} \\
& \underline{\mathbf{q}} \leq \mathbf{q} \leq \bar{\mathbf{q}} \quad \text{(physical capacity)}
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$$

- Approximation (Linear DistFlow), convex relaxation (SDP, SOCP)
- Centralized [Low'14, Dall'Anese'14], decentralized [Zhang'14, Molzahn'17]

Challenge 1: Require the exact distribution system model

Literature-Optimal Power Flow (OPF)

- Model learning + model-based optimization:
  Two-stage: ICNN + convex optimization [Chen'20]
  Simultaneous: convex body chasing + robust control [Yeh'22]

Challenge 2: Result in conservative actions

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- Feedback controller: $p(q)(t+1) = p(q)(t) + f(v(t))$
  Online primal-dual [Dall'Anese'16], discrete projected gradient flow [Häberle'20], safe gradient flow [Colot'23], stability-constrained RL-based controller [Feng'24]

Challenge 3: Intermediate voltage violation under inaccurate model

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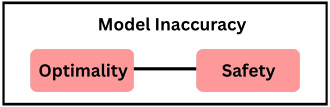

Look for a better trade-off between optimality and safety under model inaccuracies

Online Exponential Barrier Method

Focus on overvoltage problem and use linearized AC power flow

Problem Formulation

$$\min_{\mathbf{u}\in\mathbb{R}^{2n}} \quad c(\mathbf{u}) := \frac{1}{2}\mathbf{u}^\top \mathbf{Q}\mathbf{u} \tag{1a}$$

$$\text{s.t.} \quad \mathbf{B}\mathbf{u} + \mathbf{e} \leq \bar{\mathbf{x}} \tag{1b}$$

$$\underline{\mathbf{u}} \leq \mathbf{u} \leq \bar{\mathbf{u}} \tag{1c}$$

$\mathbf{u} = [\mathbf{u}^{\mathbf{p}}; \mathbf{u}^{\mathbf{q}}] \in \mathbb{R}^{2n}$ adjustment of inverter power

$\mathbf{B} = [\mathbf{R} \quad \mathbf{X}] \in \mathbb{R}^{n\times 2n}$ linearized AC power flow

$\mathbf{e} = \mathbf{R}(\mathbf{p}^o_{pv} - \mathbf{p}_{\mathbf{e}}) + \mathbf{X}(\mathbf{q}^o_{pv} - \mathbf{q}_{\mathbf{e}})$ voltage drop caused by original pv and loads

$\bar{\mathbf{u}} = [\bar{\mathbf{p}}_{pv} - \mathbf{p}^o_{pv}; \bar{\mathbf{q}}_{pv} - \mathbf{q}^o_{pv}] \geq 0$ and $\underline{\mathbf{u}} = [\underline{\mathbf{p}}_{pv} - \mathbf{p}^o_{pv}; \underline{\mathbf{q}}_{pv} - \mathbf{q}^o_{pv}] \leq 0$

- We denote **actual** pv injections by $\mathbf{p}_{pv} = \mathbf{p}^o_{pv} + \mathbf{u}^{\mathbf{p}}$, $\mathbf{q}_{pv} = \mathbf{q}^o_{pv} + \mathbf{u}^{\mathbf{q}}$
- We assume $\mathbf{u} = [-\mathbf{p}^o_{pv}, -\mathbf{q}^o_{pv}]$ is a **feasible** solution

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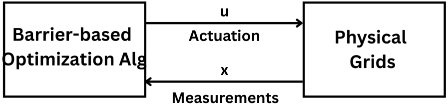
Assumption 1: Cost function is strongly convex, i.e.,$\mathbf{Q} \succ 0$

- **Main idea:** Synthesize barrier method and online feedback optimization



$$\tilde{c}(\mathbf{u}|\alpha, \mathbf{B}) = c(\mathbf{u}) + \sum_i^n \frac{\alpha_i}{\beta_i} e^{\beta_i(\mathbf{b}_i^\top \mathbf{u} + \mathbf{e}_i - \bar{\mathbf{x}}_i)}. \tag{2}$$

Theorem 1. (Optimality condition)

Under Assumption 1 and model $\mathbf{B}$, there exists the unique barrier parameter $\alpha \in \mathbb{R}_+^n$ with fixed $\beta \in \mathbb{R}_+^n$, such that the augmented cost function $\tilde{c}(\mathbf{u}|\alpha, \mathbf{B})$ is strongly convex and its global minimum $\mathbf{u}^*$ satisfies the optimality condition of (1a)-(1b). And α_i, $i \in A$ is the solution to

$$\begin{bmatrix} \mathbf{Q} & \mathbf{B}_A^\top \\ \mathbf{B}_A & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \alpha_A \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \bar{\mathbf{x}}_A - \mathbf{e}_A, \end{bmatrix}, \tag{3}$$

and $\alpha_i = 0$, $i \in N \setminus A$, with A as the set of active constraints at $\mathbf{u}^*$.

$$\tilde{c}(\mathbf{u}|\alpha,\mathbf{B}) = c(\mathbf{u}) + \sum_{i}^{n} \frac{\alpha_i}{\beta_i} e^{\beta_i(\mathbf{b}_i^\top \mathbf{u} + \mathbf{e}_i - \bar{\mathbf{x}}_i)}. \tag{2}$$

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Proof sketch:

$$\nabla_{\mathbf{u}} \tilde{c}(\mathbf{u}|\alpha,\mathbf{B}) = \mathbf{Q}\mathbf{u} + \sum_{i=1}^{n} \alpha_i e^{\beta_i(\mathbf{b}_i^\top \mathbf{u} + e_i - \bar{x}_i)} \mathbf{b}_i, \tag{4}$$

with $e^{\beta_i(\mathbf{b}_i^\top \mathbf{u}^* + \mathbf{e}_i - \bar{\mathbf{x}}_i)} = e^0 = 1$, $i \in A$ and $\alpha_i = 0$, $i \notin A$.

$$\implies \nabla_{\mathbf{u}} c(\mathbf{u}^*) + \sum_{i \in A} \alpha_i \mathbf{b}_i = 0. \tag{5}$$

Optimality Analysis

$$\tilde{c}(\mathbf{u}|\alpha, \mathbf{B}) = c(\mathbf{u}) + \sum_{i}^{n} \frac{\alpha_i}{\beta_i} e^{\beta_i(\mathbf{b}_i^\top \mathbf{u} + \mathbf{e}_i - \bar{\mathbf{x}}_i)}. \tag{2}$$

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Why not log barrier?

1. Classic barrier method features double-loop
2. Log may explode around the boundaries
3. Facing trouble when outside the safety limit

Algorithm

Although A is unknown, we may choose the node with largest voltage deviation (attention node) for overvoltage problem

Algorithm 1 Online Exponential Barrier Method for Optimal Voltage Control

Input 1: Safety limit $\bar{\mathbf{x}}$, Cost coefficient $\mathbf{Q}$, Barrier curvature β, Solar efficiency bound κ, Stopping criterion K
Input 2: Estimated dynamics $\hat{\mathbf{B}}$, Voltage deviation $\mathbf{x}$ under maximum solar energy $\mathbf{p_{av}}$, step-size η
if $\max_{i\in N} \mathbf{x}_i \geq \bar{\mathbf{x}}_i$ **then**
  (s1) Initialize the safe action $\mathbf{u}(k) = [(\kappa-1)\mathbf{p_{av}} \quad \mathbf{0}]^T$ and observe new voltage deviation $\mathbf{x}(k)$ with $k=0$
  (s2) Compute barrier weights $\hat{\alpha}^s$ for augmented cost $\tilde{c}$
  for $k=0$ to K **do**
    (s3) Compute and implement adjusted power setpoint $\mathbf{u}(k+1) = \mathbf{u}(k) - \eta F(\mathbf{u}(k)|\alpha^s)$
    (s4) Measure voltage $\mathbf{x}(k+1)$ and control input $\mathbf{u}(k+1)$
    if $\min_{i\in N} \mathbf{u}_i(k+1) = \underline{\mathbf{u}}_i$ or $\max_i \mathbf{u}_i(k+1) = \bar{\mathbf{u}}_i$ **then**
      (s5) Activate saturation by updating weights $\hat{\alpha}^s$
    end if
    if $\arg\max_{i\in N} \mathbf{x}_i(k+1) \neq \arg\max_{i\in N} \mathbf{x}_i(k)$ **then**
      (s6) Switch attention node by updating weights $\hat{\alpha}^s$
    end if
  end for
end if

- Inaccurate model $\hat{\mathbf{B}}$
- Measurements:
  $\mathbf{x}_i = \mathbf{b}_i^\top \mathbf{u} + \mathbf{e}_i,\ i \in N$
- calculate barrier weights α^s
- Update rule:

$$\mathbf{u}(k+1) = \mathbf{u}(k) - \eta F(\mathbf{u}(k)|\alpha^s)$$

$$F(\mathbf{u}|\alpha^s) := \nabla_{\mathbf{u}} c(\mathbf{u}) + \sum_i^n \alpha_i^s e^{\beta_i(\mathbf{x}_i - \bar{\mathbf{x}}_i)} \hat{\mathbf{b}}_i$$

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- **Closed-form** selection of barrier weights $\alpha^s \in \mathbb{R}^n$
- **Safety guarantee** for converged point (power systems structure matters!)

Safety Guarantee

Safety barrier selection

$\alpha_i^s = \hat{\alpha}_i + \gamma^s$ at $i = \arg\max_j \mathbf{x}_j$ with some $\gamma^s \geq 0$, and $\alpha_j^s = 0, j \in N \setminus \{i\}$, with $\hat{\alpha}_i$ as the solution to (3) under $\hat{\mathbf{B}}$.

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$\hat{\mathbf{B}}$ affects both calculation of α^s and update rule of $\mathbf{u}$

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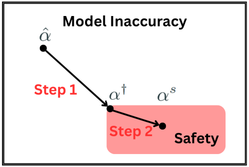

Road map for proof of safety guarantee, with optimality pursued by calculation of "α"

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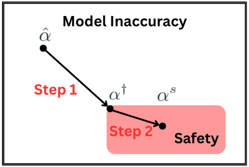

Road map for proof of safety guarantee, with optimality pursued by calculation of "α"

Assumption 2: $\mathbf{Q} \succ 0$ is diagonal

- Step 1: **Theorem 2:** existence the proper barrier weights $\alpha^\dagger$
- Step 2: **Theorem 3:** specify γ^s to achieve $\alpha^s \geq \alpha^\dagger$, which guarantees the safety ($\mathbf{B}$ and $\hat{\mathbf{B}}$ are element-wise positive)

Theorem 3. (Safety guarantee)

Under Assumptions 1-2 and $\hat{\mathbf{B}}$, if $\bar{\mathbf{u}} = 0$, when **Algorithm** with $\alpha^s \in \mathbb{R}^n$ and fixed $\beta \in \mathbb{R}^n_+$ achieves the convergence, denoted by $\mathbf{u}^s$, we have $\alpha_i^s \geq 0$, at $i = \arg\max_j \mathbf{b}_j^\top \mathbf{u}^\mathbf{s} + \mathbf{e}_j$ and

$$\mathbf{b}_i^\top \mathbf{u}^\mathbf{s} + \mathbf{e}_i - \bar{\mathbf{x}}_i \leq 0, \tag{4}$$

with $\|\mathbf{B} - \hat{\mathbf{B}}\| \leq \epsilon_\mathbf{B}$ and

$$\gamma^s = \frac{\epsilon_\mathbf{B}}{|\hat{\mathbf{b}}_{i,Au}^\top \mathbf{Q}_{Au}^{-1} \hat{\mathbf{b}}_{i,Au}|} (\|\underline{\mathbf{u}}\| + \|\mathbf{Q}_{Au}^{-1} \hat{\mathbf{b}}_{i,Au}\| |\hat{\alpha}_i|). \tag{5}$$

Proof

- Discuss different node i at the convergence, $i = \arg\max_j \mathbf{b}_j^\top \mathbf{u}^\mathbf{s} + \mathbf{e}_j$.

  1. If $\mathbf{e}_i < \bar{\mathbf{x}}_i$, i.e., the node without initial violation, we have (4) holds.
  2. If $\mathbf{e}_i \geq \bar{\mathbf{x}}_i$, as $\bar{\mathbf{u}} = 0$, $\alpha_i^\dagger$ is non-negative. Based on **Theorem 2**, there exists $\alpha^\dagger$ drives the convergence on the boundary. Through the first-order perturbation analysis on two set of linear equations, we obtain γ^s in (5) and $\alpha_i^s \geq \alpha_i^\dagger \geq 0$.

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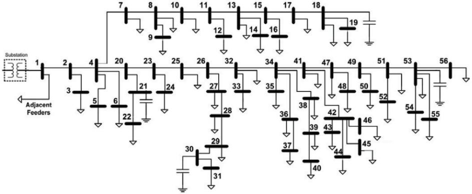
   By contradiction, assume $\mathbf{b}_i^\top \mathbf{u}^s + \mathbf{e}_i - \bar{\mathbf{x}}_i > 0$.
   For $\mathbf{u}_j^s \in (\underline{\mathbf{u}}_j, \bar{\mathbf{u}}_j)$, we have $F(\mathbf{u}^s|\alpha^s)_j = F(\mathbf{u}^\dagger|\alpha^\dagger)_j = 0$, i.e.,

$$
\begin{aligned}
&(\mathbf{Q}\mathbf{u}^s + \alpha_i^s e^{\beta_i(\mathbf{b}_i^\top \mathbf{u}^s + \mathbf{e}_i - \bar{\mathbf{x}}_i)} \hat{\mathbf{b}}_i)_j \\
&= (\mathbf{Q}\mathbf{u}^\dagger + \alpha_i^\dagger e^{\beta_i(\mathbf{b}_i^\top \mathbf{u}^\dagger + \mathbf{e}_i - \bar{\mathbf{x}}_i)} \hat{\mathbf{b}}_i)_j = 0,
\end{aligned}
\tag{6}
$$

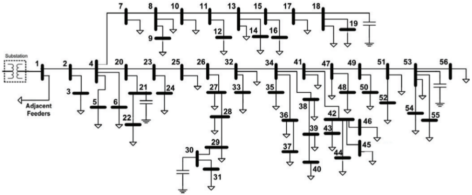
   With the assumptions 1-2 and $\hat{\mathbf{b}}_i \geq 0$ (power system characteristics), we have $\mathbf{u}_j^s \leq \mathbf{u}_j^\dagger$. For $\mathbf{u}_j^s = \underline{\mathbf{u}}_j$, we have $\mathbf{u}_j^s = \mathbf{u}_j^\dagger$.
   Thus, we get $\mathbf{b}_i^\top \mathbf{u}^s + \mathbf{e}_i - \bar{\mathbf{x}}_i \leq \mathbf{b}_i^\top \mathbf{u}^\dagger + \mathbf{e}_i - \bar{\mathbf{x}}_i = 0$, which contradicts the assumption. Above all, we conclude that $\mathbf{b}_i^\top \mathbf{u^s} + \mathbf{e}_i - \bar{\mathbf{x}}_i \leq 0$.

Simulation



SCE 56-bus distribution grids

- Arbitrarily installed inverters: Buses $\{2, 4, 7, 8, 11, 13, 14, 17, 22, 27, 30, 31, 33, 34, 36, 37, 39, 40, 41, 42, 43, 45, 47, 48, 51, 55, 56\}$
- Inverter capacity: $\mathbf{u^p} \in [-\bar{\mathbf{p}}_{pv}, \mathbf{0}]$ and $\mathbf{u^q} \in [-0.4\bar{\mathbf{p}}_{pv}, 0.4\bar{\mathbf{p}}_{pv}]$
- Model inaccuracies: $\epsilon_{\mathbf{B}_1}/\|\mathbf{B}\| = 44.1\%$ and $\epsilon_{\mathbf{B}_2}/\|\mathbf{B}\| = 52.8\%$



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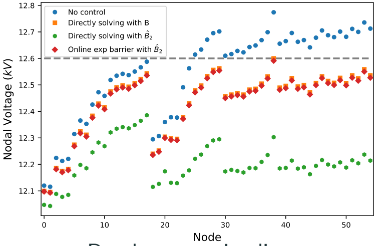
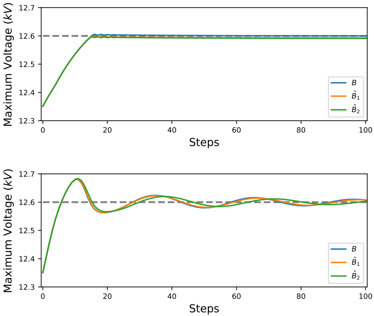
Inaccurate Models	$\hat{\mathbf{B}}_1$	$\hat{\mathbf{B}}_2$
Safety Factor γ^s	2.7×10^{-3}	4.5×10^{-3}
Barrier Weight α^s	3.5×10^{-3}	5.4×10^{-3}

Initial parameters with $\eta_i = 200,\ i \in N$



Results on optimality



Results on safety ($\eta_{\mathbf{u}} = 0.01$)

Control groups:

- No control
- Directly solving the LCQP in (1a)-(1c) with $\mathbf{B}$ and $\hat{\mathbf{B}}$
- Regularized online primal-dual method [Dall'Anese'16] with $\eta_{\mathbf{u}} = 0.05$, $\eta_{\lambda} = 0.02$

Conclusion

Contributions of this work

- Proposed online exponential barrier method that unifies online feedback optimization (**Optimality**) and barrier method (**Safety**)
- Provided closed-form selection of barrier weights
- Proved safety guarantee under model inaccuracies at the convergence
- Improved intermediate voltage constraint satisfaction

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Ongoing & Future Work

- Global convergence analysis
- Theoretical analysis on optimality
- Incorporate various constraints and generalize to time-varying loads and solar energy with tracking properties

Acknowledgements

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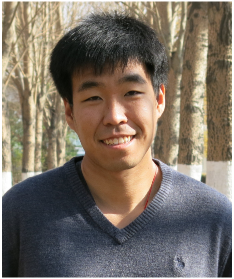

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Reference: P. Zhang and B. Zhang, "Optimal voltage control using online exponential barrier method," arXiv preprint arXiv:2506.10247, 2025.

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The authors would like to thank Manish. K. Singh for valuable discussions that helped improve this work.

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