We will cover the following topics:

- 1. What is deep learning?
- 2. Why deep learning? Why now?
- 3. Perceptron: forward propagation.
- 4. activation function: introduce non-linearities.
- 5. Deep neural network:
    - 5.1. example
    - 5.2 loss function
        - cross entropy
        - MSE
    - 5.3 Loss optimization
        - gradient descent
    - 5.4 Loss function can be hard to optimize
        - learning rate: adaptive learning rate
        - Stochastic Gradient Descent:

easy to compute but very noisy

- Mini-batches:

- 5.5 Overfitting:
  - Regularization:
    - what is it: constrains optimization problem to discourage comlex models
    - why do we need it:

improve generalization of model on unseen data

- techniques:
  - dropout: during training, randomly set some activations to 0.

- force network to not rely on any 1 node.

- Early stopping:
  - Stop

training before we have a chance to overfit.

- 6. Summary.

What is Deep Learning?

Artificial Intelligence

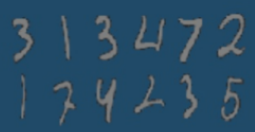
Any technique that enables computers to mimic human behavior

Machine Learning

Ability to learn without explicitly being programmed

Deep Learning

Extract patterns from data using neural networks



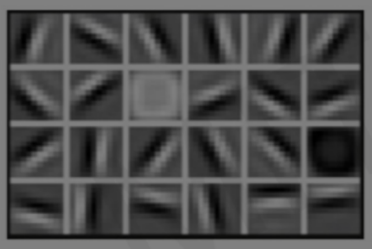
Why Deep Learning and Why Now?

Why Deep Learning?

Hand engineered features are time consuming, brittle, and not scalable in practice

Can we learn the **underlying features** directly from data?

Low Level Features



Lines & Edges

Mid Level Features

Eyes & Nose & Ears

High Level Features

Facial Structure

Why Now?

Neural Networks date back decades, so why the resurgence?

1952 Stochastic Gradient Descent

1958 Perceptron
- Learnable Weights

1986 Backpropagation
- Multi-Layer Perceptron

1995 Deep Convolutional NN
- Digit Recognition

1. Big Data
- Larger Datasets
- Easier Collection & Storage

IMAGENET

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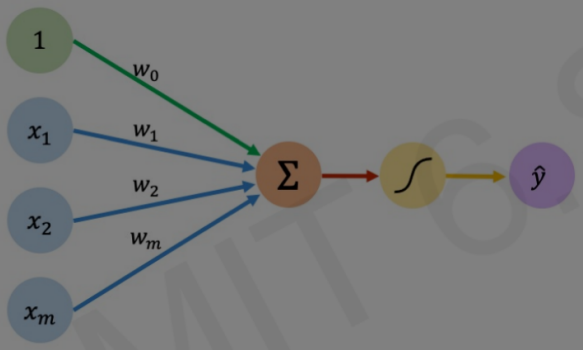
2. Hardware
- Graphics Processing Units (GPUs)
- Massively Parallelizable

3. Software
- Improved Techniques
- New Models
- Toolboxes

TensorFlow

The Perceptron: Forward Propagation



Inputs Weights Sum Non-Linearity Output

Output

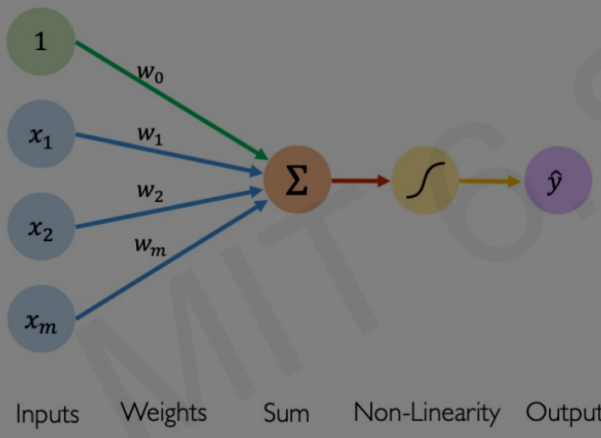
Linear combination of inputs

$$\hat{y} = g\left(w_0 + \sum_{i=1}^{m} x_i\, w_i\right)$$

Non-linear activation function

Bias

The Perceptron: Forward Propagation

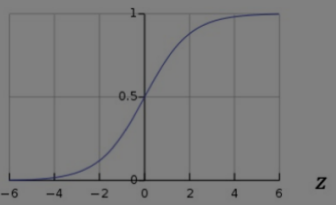


Activation Functions

$$\hat{y} = g(w_0 + \boldsymbol{X}^T \boldsymbol{W})$$

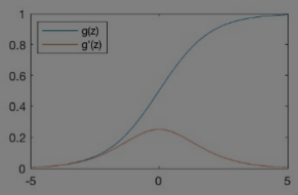
- Example: sigmoid function

$$g(z) = \sigma(z) = \frac{1}{1 + e^{-z}}$$



Common Activation Functions

Sigmoid Function

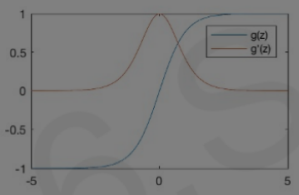


$$g(z) = \frac{1}{1 + e^{-z}}$$

$$g'(z) = g(z)(1 - g(z))$$

`tf.math.sigmoid(z)`

Hyperbolic Tangent

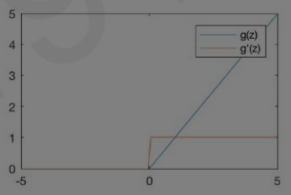


$$g(z) = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$$

$$g'(z) = 1 - g(z)^2$$

`tf.math.tanh(z)`

Rectified Linear Unit (ReLU)



$$g(z) = \max(0, z)$$

$$g'(z) = \begin{cases} 1, & z > 0 \\ 0, & \text{otherwise} \end{cases}$$

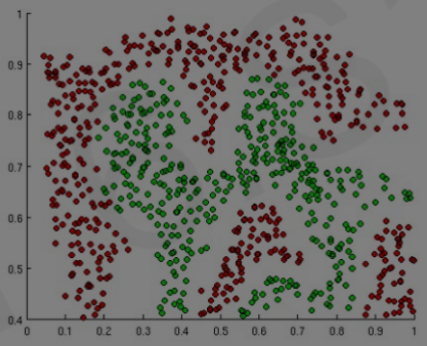
`tf.nn.relu(z)`

TensorFlow code blocks

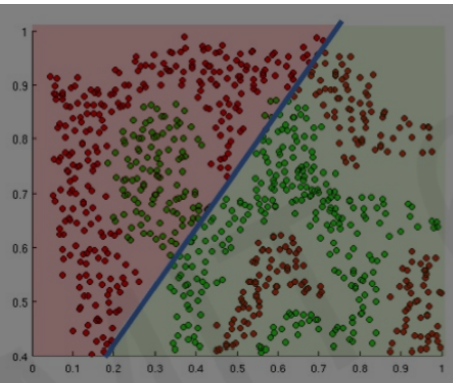
NOTE: All activation functions are non-linear

Importance of Activation Functions

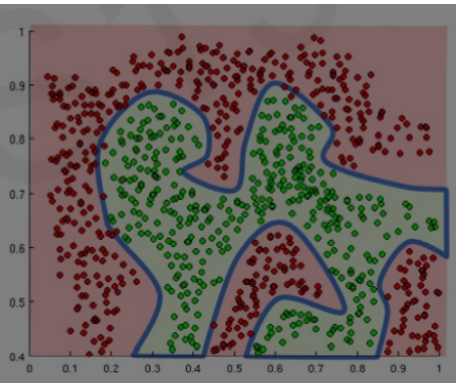
*The purpose of activation functions is to **introduce non-linearities** into the network*



What if we wanted to build a neural network to distinguish green vs red points?

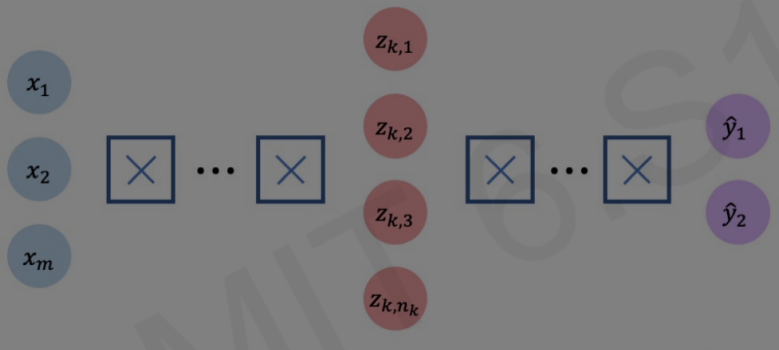


Linear activation functions produce linear decisions no matter the network size



Non-linearities allow us to approximate arbitrarily complex functions

Deep Neural Network




```
import tensorflow as tf

model = tf.keras.Sequential([
  tf.keras.layers.Dense(n1),
  tf.keras.layers.Dense(n2),
  ⋮
  tf.keras.layers.Dense(2)
])
```

$$z_{k,i} = w_{0,i}^{(k)} + \sum_{j=1}^{n_{k-1}} g(z_{k-1,j})\, w_{j,i}^{(k)}$$

Example Problem

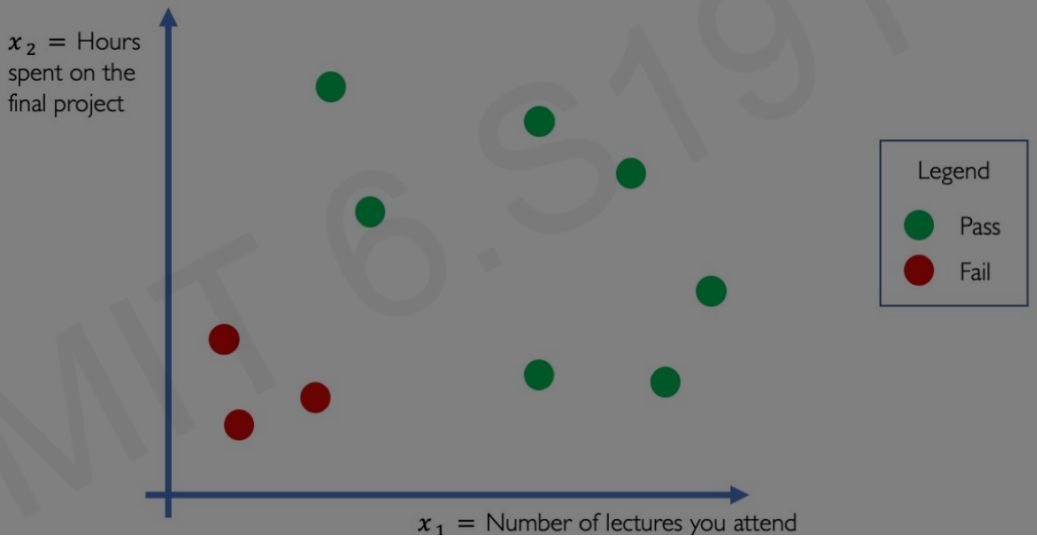
Will I pass this class?

Let's start with a simple two feature model

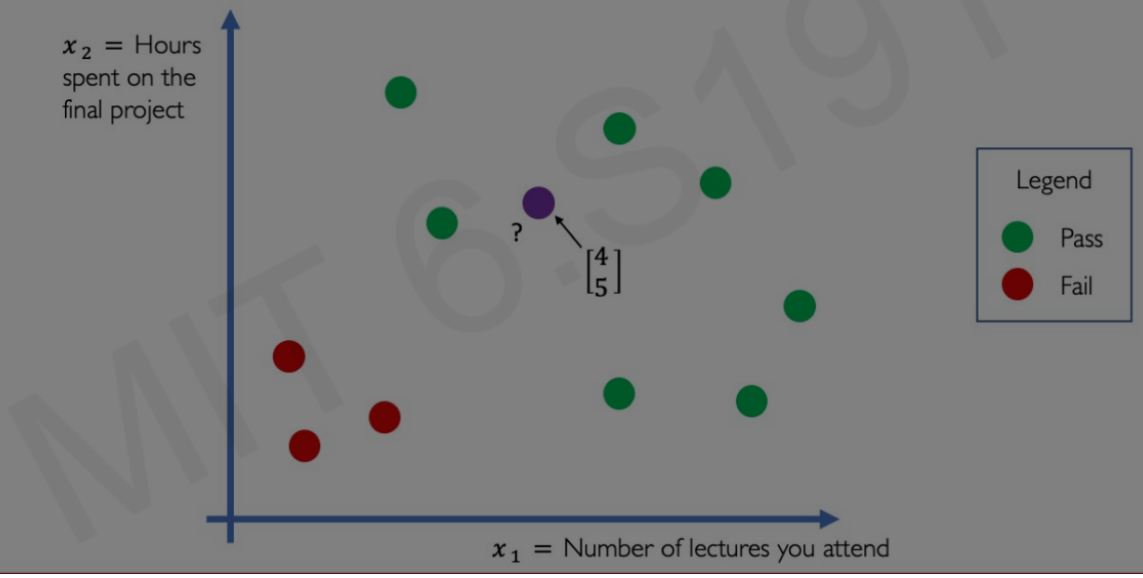
x_1 = Number of lectures you attend

x_2 = Hours spent on the final project

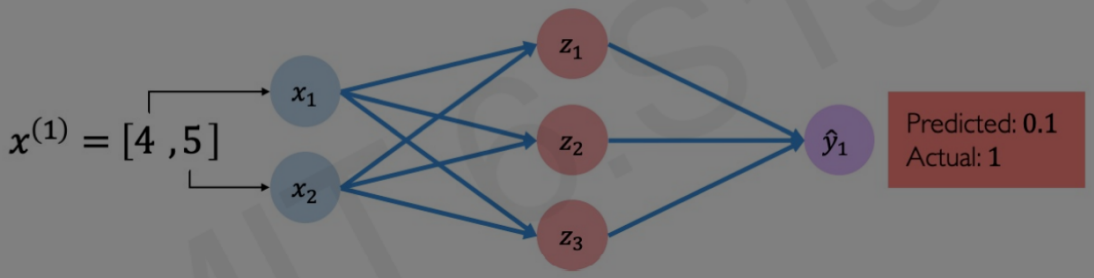
Example Problem: Will I pass this class?



Example Problem: Will I pass this class?

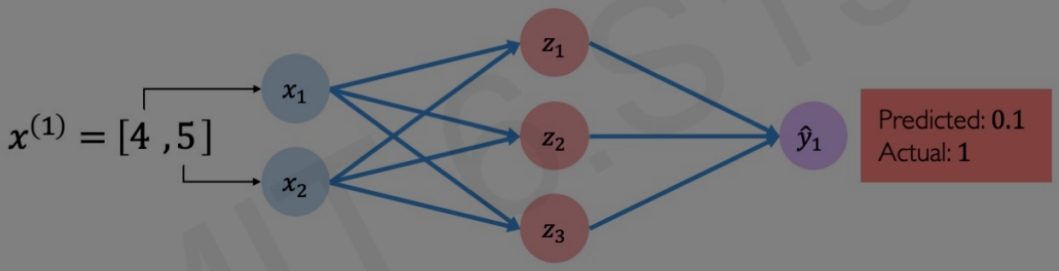


Example Problem: Will I pass this class?



Quantifying Loss

*The **loss** of our network measures the cost incurred from incorrect predictions*

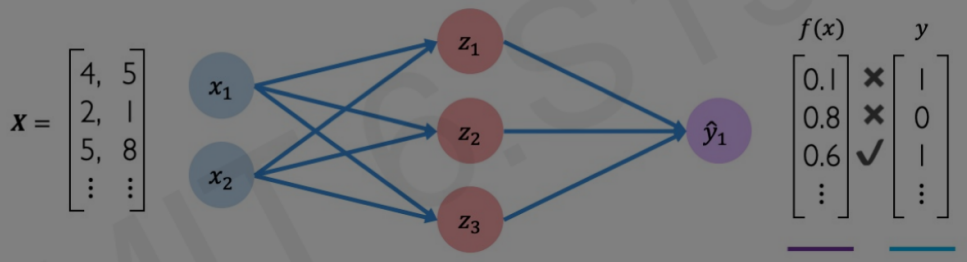


$$\mathcal{L}\left(f\left(x^{(i)}; \boldsymbol{W}\right), y^{(i)}\right)$$

Predicted Actual

Empirical Loss

*The **empirical loss** measures the total loss over our entire dataset*



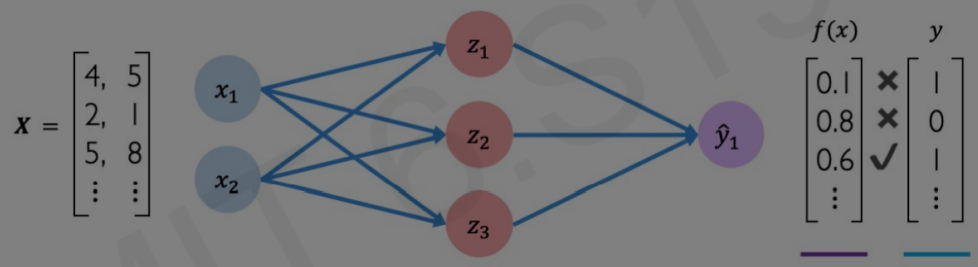
$$\boldsymbol{J}(\boldsymbol{W}) = \frac{1}{n}\sum_{i=1}^{n} \mathcal{L}\left(f\left(x^{(i)}; \boldsymbol{W}\right), y^{(i)}\right)$$

Predicted Actual

Also known as:
- Objective function
- Cost function
- Empirical Risk

Binary Cross Entropy Loss

Cross entropy loss *can be used with models that output a probability between 0 and 1*



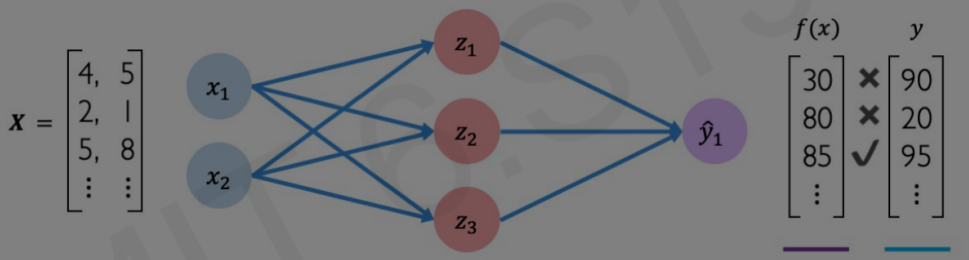
$$J(W) = -\frac{1}{n}\sum_{i=1}^{n} y^{(i)} \log\left(f(x^{(i)}; W)\right) + (1 - y^{(i)}) \log\left(1 - f(x^{(i)}; W)\right)$$

Actual Predicted Actual Predicted

```
loss = tf.reduce_mean( tf.nn.softmax_cross_entropy_with_logits(y, predicted) )
```

Mean Squared Error Loss

Mean squared error loss *can be used with regression models that output continuous real numbers*



$$J(W) = \frac{1}{n}\sum_{i=1}^{n} \left(y^{(i)} - f(x^{(i)}; W)\right)^2$$

Actual Predicted

```
loss = tf.reduce_mean( tf.square(tf.subtract(y, predicted)) )
loss = tf.keras.losses.MSE( y, predicted )
```

Loss Optimization

We want to find the network weights that ***achieve the lowest loss***

$$\boldsymbol{W}^* = \underset{\boldsymbol{W}}{\text{argmin}} \frac{1}{n} \sum_{i=1}^{n} \mathcal{L}\left(f\left(x^{(i)}; \boldsymbol{W}\right), y^{(i)}\right)$$

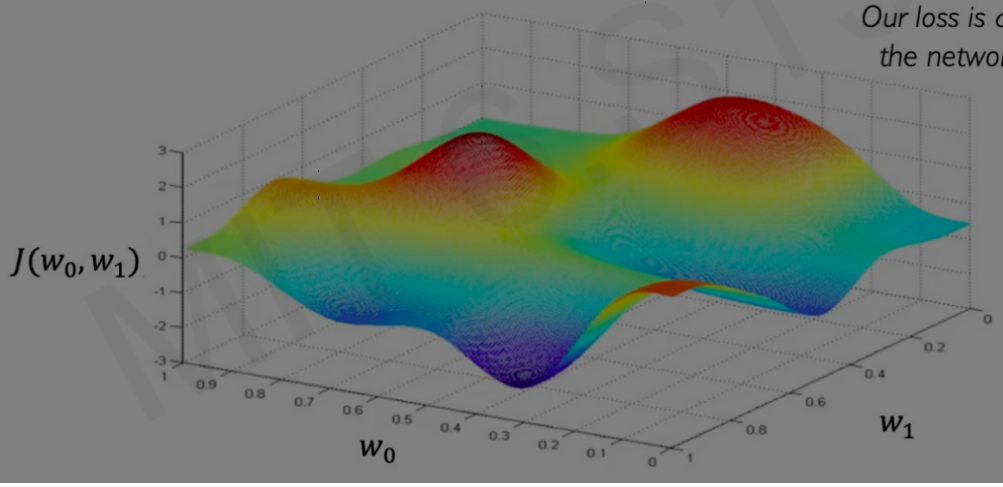
$$\boldsymbol{W}^* = \underset{\boldsymbol{W}}{\text{argmin}}\, J(\boldsymbol{W})$$

Remember:
$\boldsymbol{W} = \{\boldsymbol{W}^{(0)}, \boldsymbol{W}^{(1)}, \dots\}$

Loss Optimization

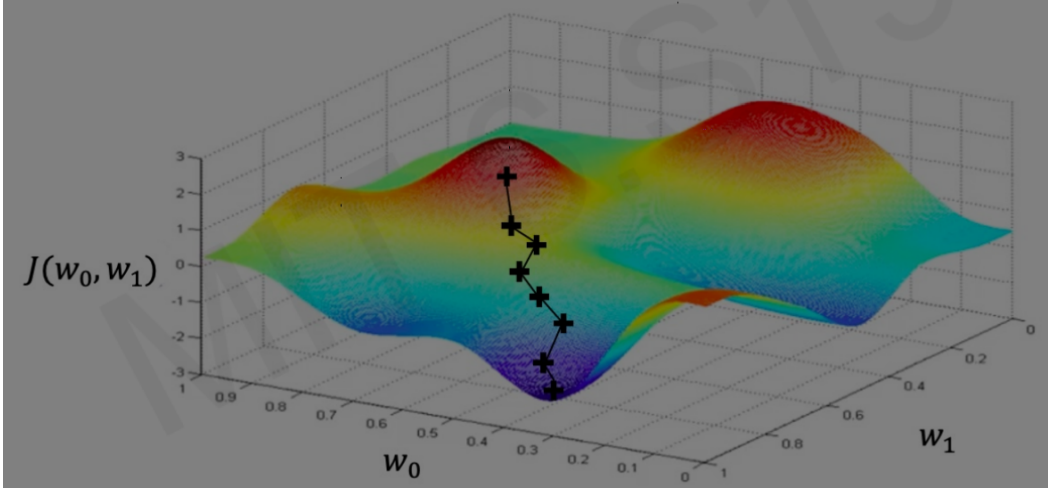
$$\boldsymbol{W}^* = \underset{\boldsymbol{W}}{\text{argmin}}\, J(\boldsymbol{W})$$

Remember:
Our loss is a function of the network weights!



Gradient Descent

Repeat until convergence

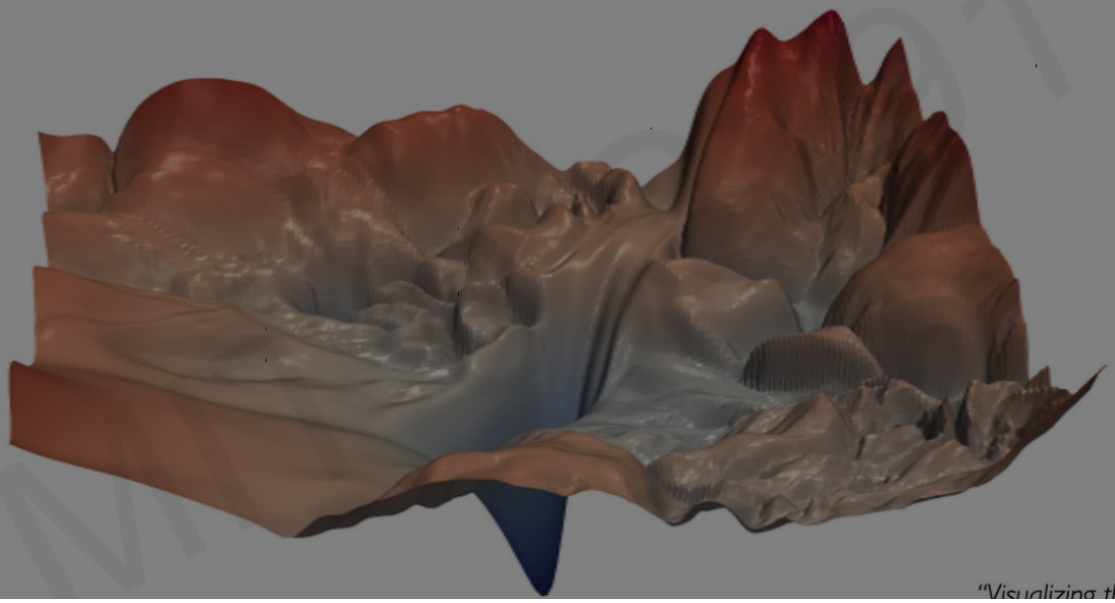


Gradient Descent

Algorithm

1. Initialize weights randomly $\sim \mathcal{N}(0, \sigma^2)$
2. Loop until convergence:
3. Compute gradient, $\frac{\partial J(\boldsymbol{W})}{\partial \boldsymbol{W}}$
4. Update weights, $\boldsymbol{W} \leftarrow \boldsymbol{W} - \eta \frac{\partial J(\boldsymbol{W})}{\partial \boldsymbol{W}}$
5. Return weights

Training Neural Networks is Difficult



"Visualizing the loss la

Loss Functions Can Be Difficult to Optimize

Remember:
Optimization through gradient descent

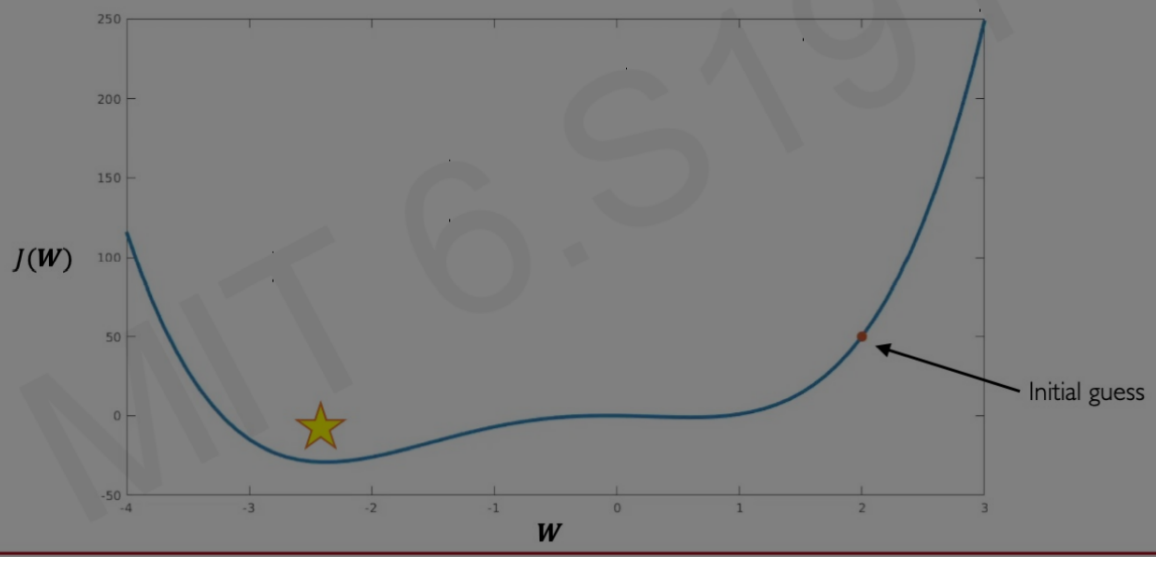
$$\boldsymbol{W} \leftarrow \boldsymbol{W} - \eta \frac{\partial J(\boldsymbol{W})}{\partial \boldsymbol{W}}$$

$$\boldsymbol{W} \leftarrow \boldsymbol{W} - \eta \frac{\partial J(\boldsymbol{W})}{\partial \boldsymbol{W}}$$

How can we set the learning rate?

Setting the Learning Rate

Small learning rate *converges slowly and gets stuck in false local minima*



Large learning rates *overshoot, become unstable and diverge*

Stable learning rates *converge smoothly and avoid local minima*

Try lots of different learning rates and see what works "just right"

Idea 2:

Do something smarter!
Design an adaptive learning rate that "adapts" to the landscape

Adaptive Learning Rates

- Learning rates are no longer fixed
- Can be made larger or smaller depending on:
  - how large gradient is
  - how fast learning is happening
  - size of particular weights
  - etc...

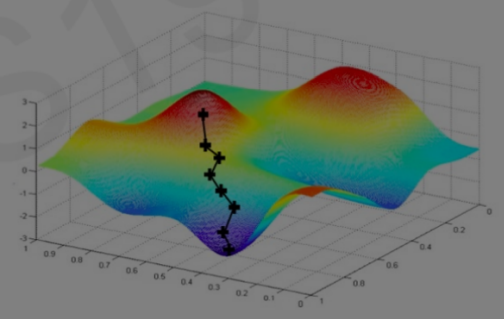
Gradient Descent Algorithms

Algorithm	TF Implementation	Reference
• SGD	`tf.keras.optimizers.SGD`	Kiefer & Wolfowitz. "Stochastic Estimation of the Maximum of a Regression Function." 1952.
• Adam	`tf.keras.optimizers.Adam`	Kingma et al. "Adam: A Method for Stochastic Optimization." 2014.
• Adadelta	`tf.keras.optimizers.Adadelta`	Zeiler et al. "ADADELTA: An Adaptive Learning Rate Method." 2012.
• Adagrad	`tf.keras.optimizers.Adagrad`	Duchi et al. "Adaptive Subgradient Methods for Online Learning and Stochastic Optimization." 2011.
• RMSProp	`tf.keras.optimizers.RMSProp`	

Stochastic Gradient Descent

Algorithm

1. Initialize weights randomly $\sim \mathcal{N}(0, \sigma^2)$
2. Loop until convergence:
3. Pick single data point i
4. Compute gradient, $\frac{\partial J_i(W)}{\partial W}$
5. Update weights, $W \leftarrow W - \eta \frac{\partial J(W)}{\partial W}$
6. Return weights



Easy to compute but **very noisy** (stochastic)!

Mini-batches while training

More accurate estimation of gradient

Smoother convergence

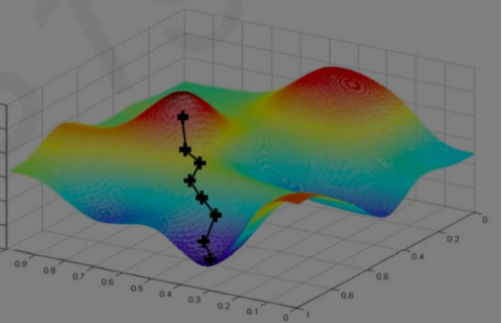
Allows for larger learning rates

Mini-batches lead to fast training!

Can parallelize computation + achieve significant speed increases on GPU's

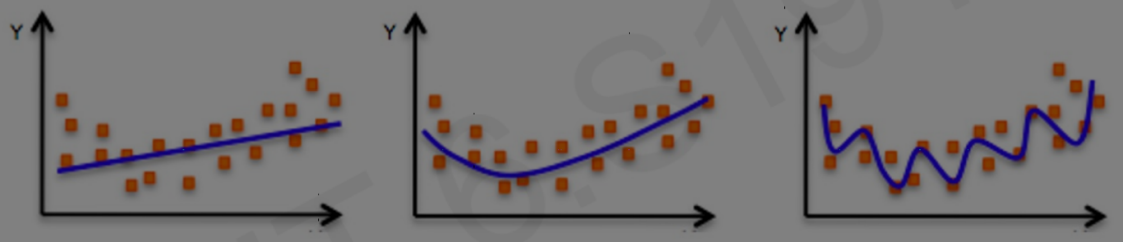
Algorithm

1. Initialize weights randomly $\sim \mathcal{N}(0, \sigma^2)$
2. Loop until convergence:
3. Pick batch of B data points
4. Compute gradient, $\frac{\partial J(\boldsymbol{W})}{\partial \boldsymbol{W}} = \frac{1}{B}\sum_{k=1}^{B}\frac{\partial J_k(\boldsymbol{W})}{\partial \boldsymbol{W}}$
5. Update weights, $\boldsymbol{W} \leftarrow \boldsymbol{W} - \eta\frac{\partial J(\boldsymbol{W})}{\partial \boldsymbol{W}}$
6. Return weights



Fast to compute and a much better estimate of the true gradient!

The Problem of Overfitting



Underfitting
Model does not have capacity to fully learn the data

Ideal fit

Overfitting
Too complex, extra parameters, does not generalize well

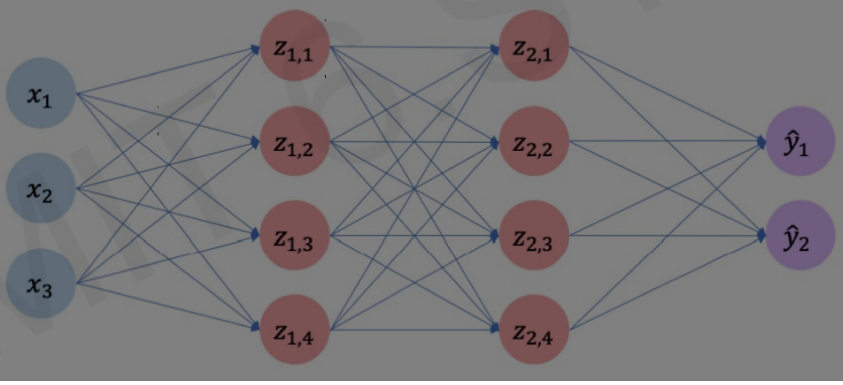
Regularization

What is it?

Technique that constrains our optimization problem to discourage complex models

Regularization 1: Dropout

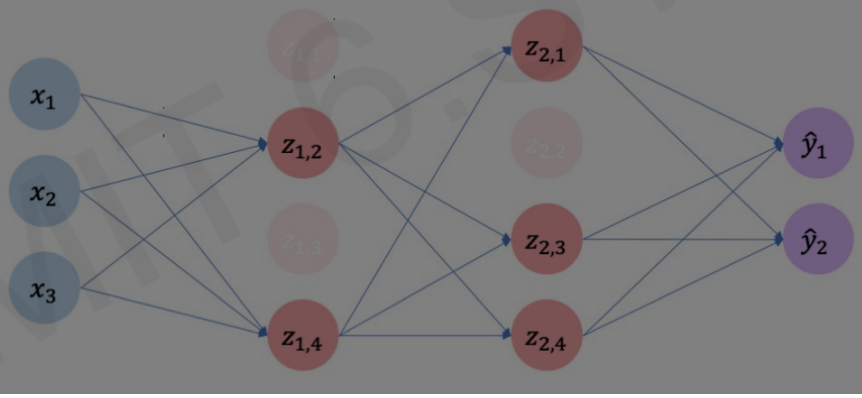
- During training, randomly set some activations to 0



Regularization 1: Dropout

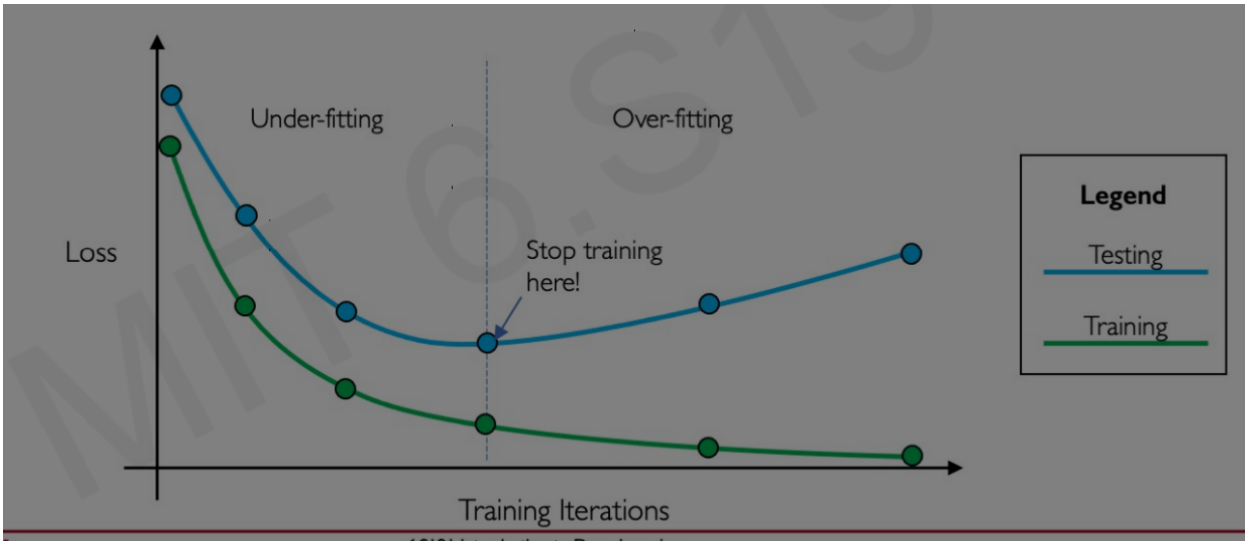
- During training, randomly set some activations to 0
  - Typically 'drop' 50% of activations in layer
  - Forces network to not rely on any 1 node

```
tf.keras.layers.Dropout(p=0.5)
```



Regularization 2: Early Stopping

- Stop training before we have a chance to overfit



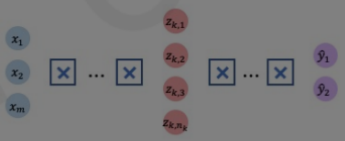
Core Foundation Review

The Perceptron

- Structural building blocks
- Nonlinear activation functions

Neural Networks

- Stacking Perceptrons to form neural networks
- Optimization through backpropagation



Training in Practice

- Adaptive learning
- Batching
- Regularization