New IPC Friction

Smoothed Semi-Implicit Friction [Li et al. 2020]

In IPC, for a contact primitive pair k, the smoothed friction force is defined as
$F_k = -\mu_k \lambda_k T_k \frac{u_k}{||u_k||} f_1(||u_k||)$
where

$$f_1(y) = \begin{cases} -\frac{y^2}{\epsilon_v^2 h^2} + \frac{2y}{\epsilon_v h}, & y \in (0, h\epsilon_v) \\ 1, & y \geq h\epsilon_v, \end{cases}$$

Here

- μ_k is the friction coefficient
- λ_k is the normal force Lagrange multiplier
- T_k is the tangent basis
- $u_k = T_k^T(x - x^t)$ is the tangent space relative displacement
- ϵ_v is the velocity threshold which is seen as static by the friction model

For a semi-implicit temporal discretization, we define
$F_k = -\mu_k \lambda_k^t T_k^t \frac{u_k}{||u_k||} f_1(||u_k||)$
with
$u_k = (T_k^t)^T(x - x^t)$,
i.e. fixing λ_k and T_k to the value at the beginning of the time step, so that F_k is integrable and we have the friction potential
$E_{f,k} = \mu_k \lambda_k^t f_0(||u_k||)$
where
$f_0' = f_1$

By keep updating λ_k and T_k to the converged Newton solution in the same time step and repeat Newton loops, there could be a possibility (no guarantee) that the final solution converge to the fully implicit friction. It would be great to also provide this refinement option in the code while by default turning off (only 1 refinement iteration).

Normal Force Lagrange Multiplier

For the new IPC model, we have the barrier augmented incremental potential
$E(x) + h^2 \sum_k A_k \kappa b(d_k(x))$
where A_k is the area weighting.
At stationary, we have
$\bigtriangledown E(x) + h^2 \sum_k A_k \kappa \frac{\partial b}{\partial d}(d_k(x)) \bigtriangledown d_k(x)$

If we rewrite above as
$\bigtriangledown E(x) - h^2 \sum_k \lambda_k \bigtriangledown d_k(x)$
with
$\lambda_k = -A_k \kappa \frac{\partial b}{\partial d}(d_k(x))$
since $\bigtriangledown d$ is unitless, λ_k is in the unit of force to be used in the friction model.

Note that this is different from original IPC's $\lambda_k = -\frac{\kappa}{h^2} \frac{\partial b}{\partial d}(d_k(x))$ because of our constitutive remodeling for IPC.

Then for implementation we in fact used $E(x) + h^2 \sum_k A_k \kappa b(d_k^2(x))$, so at stationary we have $\bigtriangledown E(x) + h^2 \sum_k A_k \kappa \frac{\partial b}{\partial d}(d_k^2(x)) 2 d_k(x) \bigtriangledown d_k(x)$ instead. Therefore, in code we compute
$\lambda_k = -A_k \kappa \frac{\partial b}{\partial d}(d_k^2(x)) 2 d_k(x)$
and together we have
$E(x) + h^2 \sum_k A_k \kappa b(d_k(x)) + h^2 \sum_l E_{f,l}(x, x^t)$
as the incremental potential with contact and friction, note also here h^2 is not canceled in friction computation as in Li et al. [2020].

$u_k(x) = \frac{\gamma}{2\alpha\beta} x + v^n h + h^2(1-\gamma)a^n - \frac{\gamma}{2\alpha\beta}\tilde{x} = \frac{\gamma}{2\alpha\beta} x - \hat{x}^n$
For Newmark, the weight is 2.

Tangent Basis

The new IPC does not alter the friction basis of original IPC.

TODO: derive 2D PP and PE friction basis following the 3D derivations in IPC technical supplement.

PE (p, e0, e1)
closestPoint $\frac{(e_1-e_0)\cdot(p-e_0)}{(e_1-e_0)\cdot(e_1-e_0)}$
tanBasis $(e_1 - e_0)$.norm()

For more implementation details like Hessian computation, can check the appendix of C-IPC