Answer Distribution Calculations
Basic Requirements:

$$
\begin{aligned}
f(t) &= A\lambda e^{-\lambda t} \\
F(t_f) &= \int_0^{t_f} f(t)dt \\
F(t_{max}) &= 1 \\
E[f(t)] &= \mu
\end{aligned}
$$

Determine $F(t_f)$:

$$
\begin{aligned}
F(t_f) &= \int_0^{t_f} f(t)dt \\
&= \int_0^{t_f} A\lambda e^{-\lambda t}dt \\
&= \left[-Ae^{-\lambda t}\right]_0^{t_f} \\
&= -Ae^{-\lambda t_f} + Ae^{-\lambda 0} \\
&= A(1 - e^{-\lambda t_f})
\end{aligned}
$$

Determine $F(t_{max})$:

$$
\begin{aligned}
F(t_{max}) &= A(1 - e^{-\lambda t_{max}}) \\
1 &= A(1 - e^{-\lambda t_{max}}) \\
\frac{1}{A} &= 1 - e^{-\lambda t_{max}} \\
\frac{1-A}{A} &= -e^{-\lambda t_{max}} \\
e^{-\lambda t_{max}} &= \frac{A-1}{A} \\
-\lambda t_{max} &= \ln\left(\frac{A-1}{A}\right) \\
\lambda &= \frac{-1}{t_{max}} \ln\left(\frac{A-1}{A}\right)
\end{aligned}
$$

Determine $E(a)$:

$$
\begin{aligned}
E[f(t)] &= \int_0^{t_{max}} t f(t) dt \\
&= \int_0^{t_{max}} A\lambda t e^{-\lambda t} dt \\
u = t \qquad & dv = \lambda e^{-\lambda t} dt \\
du = dt \qquad & v = -e^{-\lambda t} \\
&= A\left[-te^{-\lambda t}\right]_0^{t_{max}} - \int_0^{t_{max}} -Ae^{-\lambda t} dt \\
&= A\left[-te^{-\lambda t} - \frac{e^{-\lambda t}}{\lambda}\right]_0^{t_{max}} \\
&= A\left(-t_{max}e^{-\lambda t_{max}} - \frac{e^{-\lambda t_{max}}}{\lambda} + 0e^{-\lambda 0} + \frac{e^{-\lambda 0}}{\lambda}\right) \\
&= A\left(\frac{1}{\lambda} - t_{max}e^{-\lambda t_{max}} - \frac{e^{-\lambda t_{max}}}{\lambda}\right)
\end{aligned}
$$

$$
\begin{aligned}
\lambda &= \frac{-1}{t_{max}} \ln\left(\frac{A-1}{A}\right) \\
&= A\left(\frac{-t_{max}}{\ln\left(\frac{A-1}{A}\right)} - t_{max}e^{\frac{\ln\left(\frac{A-1}{A}\right)}{t_{max}}t_{max}} + \frac{t_{max}e^{\frac{\ln\left(\frac{A-1}{A}\right)}{t_{max}}t_{max}}}{\ln\left(\frac{A-1}{A}\right)}\right) \\
&= At_{max}\left(\frac{-1}{\ln\left(\frac{A-1}{A}\right)} - e^{\ln\left(\frac{A-1}{A}\right)} + \frac{e^{\ln\left(\frac{A-1}{A}\right)}}{\ln\left(\frac{A-1}{A}\right)}\right) \\
&= At_{max}\left(\frac{\frac{A-1}{A} - 1}{\ln\left(\frac{A-1}{A}\right)} - \frac{A-1}{A}\right) \\
&= At_{max}\left(\frac{\frac{A-1}{A} - 1}{\ln\left(\frac{A-1}{A}\right)} - \frac{\frac{A-1}{A}\ln\left(\frac{A-1}{A}\right)}{\ln\left(\frac{A-1}{A}\right)}\right)
\end{aligned}
$$

$$
\begin{aligned}
\mu &= \frac{At_{max}}{\ln\left(\frac{A-1}{A}\right)}\left(\frac{A-1}{A} - 1 - \frac{A-1}{A}\ln\left(\frac{A-1}{A}\right)\right) \\
\frac{\mu}{t_{max}}\ln\left(\frac{A-1}{A}\right) &= (A-1) - A - (A-1)\ln\left(\frac{A-1}{A}\right) \\
\frac{\mu}{t_{max}}\ln\left(\frac{A-1}{A}\right) &= -1 - A\ln\left(\frac{A-1}{A}\right) + \ln\left(\frac{A-1}{A}\right) \\
\frac{\mu}{t_{max}} &= 1 - \frac{1}{\ln\left(\frac{A-1}{A}\right)} - A \\
\frac{1}{\ln\left(\frac{A-1}{A}\right)} &= 1 - \frac{\mu}{t_{max}} - A \\
\frac{1}{\ln\left(1 - \frac{1}{A}\right)} &= 1 - \frac{\mu}{t_{max}} - A
\end{aligned}
$$

$$
\begin{aligned}
-\lambda t_{max} &= \ln\left(\frac{A-1}{A}\right) \\
t_{max} &= \frac{-1}{\lambda}\ln\left(\frac{A-1}{A}\right)
\end{aligned}
$$

$$
\begin{aligned}
\mu &= A\left(\frac{1}{\lambda} - \frac{-1}{\lambda}\ln\left(\frac{A-1}{A}\right)e^{-\lambda\frac{-1}{\lambda}\ln\left(\frac{A-1}{A}\right)} - \frac{e^{-\lambda\frac{-1}{\lambda}\ln\left(\frac{A-1}{A}\right)}}{\lambda}\right) \\
\mu &= A\left(\frac{1}{\lambda} + \frac{1}{\lambda}\ln\left(\frac{A-1}{A}\right)e^{\ln\left(\frac{A-1}{A}\right)} - \frac{e^{\ln\left(\frac{A-1}{A}\right)}}{\lambda}\right) \\
\mu &= A\left(\frac{1}{\lambda} + \frac{1}{\lambda}\ln\left(\frac{A-1}{A}\right)\left(\frac{A-1}{A}\right) - \frac{A-1}{A\lambda}\right) \\
\mu &= \frac{A}{\lambda} + \frac{A-1}{\lambda}\ln\left(\frac{A-1}{A}\right) - \frac{A-1}{\lambda} \\
\mu &= \frac{1}{\lambda} + \frac{A-1}{\lambda}\ln\left(\frac{A-1}{A}\right) \\
\lambda &= \frac{1}{\mu} + \frac{A-1}{\mu}\ln\left(\frac{A-1}{A}\right)
\end{aligned}
$$

$$
\begin{aligned}
\lambda &= \frac{-1}{t_{max}}\ln\left(\frac{A-1}{A}\right) \\
\frac{-1}{t_{max}}\ln\left(\frac{A-1}{A}\right) &= \frac{1}{\mu} + \frac{A-1}{\mu}\ln\left(\frac{A-1}{A}\right) \\
\frac{-\mu}{t_{max}}\ln\left(\frac{A-1}{A}\right) &= 1 + A\ln\left(\frac{A-1}{A}\right) - \ln\left(\frac{A-1}{A}\right)
\end{aligned}
$$