1 Answer Distribution Calculations

Basic Requirements:

$$\begin{aligned}
f(t) &= \lambda e^{-\lambda t} \\
F(T) &= \int_0^T f(t)dt \\
F(\infty) &= 1 \\
E[f(t)] &= \mu
\end{aligned}$$

Determine $F(T)$:

$$\begin{aligned}
F(T) &= \int_0^T f(t)dt \\
&= \int_0^T \lambda e^{-\lambda t}dt \\
&= \left[-e^{-\lambda t}\right]_0^T \\
&= -e^{-\lambda T} + e^{-\lambda 0} \\
F(T) &= 1 - e^{-\lambda T}
\end{aligned}$$

Determine λ:

$$\begin{aligned}
E[f(t)] &= \int_0^\infty tf(t)dt \\
&= \int_0^\infty \lambda te^{-\lambda t}dt \\
u = t & \qquad dv = \lambda e^{-\lambda t}dt \\
du = dt & \qquad v = -e^{-\lambda t} \\
&= \left[-te^{-\lambda t}\right]_0^\infty - \int_0^\infty -e^{-\lambda t}dt \\
&= \left[-te^{-\lambda t} - \frac{e^{-\lambda t}}{\lambda}\right]_0^\infty \\
&= -\infty e^{-\lambda\infty} - \frac{e^{-\lambda\infty}}{\lambda} + 0e^{-\lambda 0} + \frac{e^{-\lambda 0}}{\lambda} \\
&= 0 - 0 + 0 + \frac{1}{\lambda} \\
\mu &= \frac{1}{\lambda} \\
\lambda &= \frac{1}{\mu}
\end{aligned}$$

Forcing time below a maximum:

$$
\begin{aligned}
f(t) &= A\lambda e^{-\lambda t} \\
F(T) &= \int_0^T f(t)dt \\
F(t_{max}) &= 1
\end{aligned}
$$

$$
\begin{aligned}
F(T) &= \int_0^T f(t)dt \\
&= \int_0^T A\lambda e^{-\lambda t}dt \\
&= \left[-Ae^{-\lambda t}\right]_0^T \\
&= A\left(-e^{-\lambda T} + e^{-\lambda 0}\right) \\
F(T) &= A\left(1 - e^{-\lambda T}\right)
\end{aligned}
$$

$$
\begin{aligned}
F(t_{max}) &= A\left(1 - e^{-\lambda t_{max}}\right) \\
1 &= A\left(1 - e^{-\lambda t_{max}}\right) \\
A &= \frac{1}{1 - e^{-\lambda t_{max}}}
\end{aligned}
$$

Determine $T(F)$:

$$
\begin{aligned}
F(T) &= A\left(1 - e^{-\lambda T}\right) \\
\frac{F}{A} &= 1 - e^{-\lambda T} \\
e^{-\lambda T} &= 1 - \frac{F}{A} \\
\ln\left(e^{-\lambda T}\right) &= \ln\left(1 - \frac{F}{A}\right) \\
-\lambda T &= \ln\left(1 - \frac{F}{A}\right) \\
T(F) &= \frac{-1}{\lambda}\ln\left(1 - \frac{F}{A}\right)
\end{aligned}
$$

Final Equations:

$$\mu = 12$$
$$t_{max} = 25$$
$$\lambda = \frac{1}{12}$$
$$A = \frac{1}{1 - e^{-25/12}}$$

$$f(t) = A\lambda e^{-\lambda t}$$
$$f(t) = \frac{1}{12(1 - e^{-25/12})} e^{-t/12}$$

$$F(T) = A\left(1 - e^{-\lambda T}\right)$$
$$F(T) = \frac{1}{1 - e^{-25/12}}\left(1 - e^{\frac{-T}{12}}\right)$$

$$T(F) = \frac{-1}{\lambda}\ln\left(1 - \frac{F}{A}\right)$$
$$T(F) = -12\ln\left(1 - F(1 - e^{-25/12})\right)$$