1 Answer Distribution Calculations

Basic Requirements:

$$
\begin{aligned}
f(t) &= \lambda e^{-\lambda t} \\
F(T) &= \int_0^T f(t)dt \\
F(\infty) &= 1 \\
E[f(t)] &= \mu
\end{aligned}
$$

Determine $F(T)$:

$$
\begin{aligned}
F(T) &= \int_0^T f(t)dt \\
&= \int_0^T \lambda e^{-\lambda t}dt \\
&= \left[-e^{-\lambda t}\right]_0^T \\
&= -e^{-\lambda T} + e^{-\lambda 0} \\
F(T) &= 1 - e^{-\lambda T}
\end{aligned}
$$

Determine λ:

$$
\begin{aligned}
E[f(t)] &= \int_0^\infty t f(t)dt \\
&= \int_0^\infty \lambda t e^{-\lambda t}dt
\end{aligned}
$$

$$
\begin{aligned}
u &= t & dv &= \lambda e^{-\lambda t}dt \\
du &= dt & v &= -e^{-\lambda t}
\end{aligned}
$$

$$
\begin{aligned}
&= \left[-te^{-\lambda t}\right]_0^\infty - \int_0^\infty -e^{-\lambda t}dt \\
&= \left[-te^{-\lambda t} - \frac{e^{-\lambda t}}{\lambda}\right]_0^\infty \\
&= -\infty e^{-\lambda\infty} - \frac{e^{-\lambda\infty}}{\lambda} + 0e^{-\lambda 0} + \frac{e^{-\lambda 0}}{\lambda} \\
&= 0 - 0 + 0 + \frac{1}{\lambda} \\
\mu &= \frac{1}{\lambda}
\end{aligned}
$$

Determine $T(F)$:

$$\begin{aligned}
F(T) &= 1 - e^{-\lambda T} \\
1 - F &= e^{-\lambda T} \\
\ln(1 - F) &= \ln(e^{-\lambda T}) \\
-\lambda T &= \ln(1 - F) \\
T(F) &= \frac{-\ln(1 - F)}{\lambda}
\end{aligned}$$

Final Equations:

$$\begin{aligned}
\mu &= 12 \\
12 &= \frac{1}{\lambda} \\
\lambda &= \frac{1}{12}
\end{aligned}$$

$$\begin{aligned}
f(t) &= \frac{1}{12} e^{-t/12} \\
F(T) &= 1 - e^{-T/12} \\
T(F) &= -12 \ln(1 - F)
\end{aligned}$$