

Introduction to Numerical Analysis

Assignment 4 - Linear Algebra

Exercise 1

$$\|x - y\| \geq |\|x\| - \|y\|| \quad \forall x, y \in R^n$$

Exercise 2

$$\|AB\| \leq \|A\| \|B\| \quad \forall A, B \in R^n \times R^n$$

Exercise 3

For the linear system $Ax = b$ given by:

$$\begin{pmatrix} 9.7 & 6.6 \\ 4.1 & 2.8 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 9.7 \\ 4.1 \end{pmatrix}$$

estimate the **cond(A)**

Exercise 4

Solve the linear system:
$$\begin{cases} 4x_1 - x_2 + x_3 = 7 \\ 4x_1 - 8x_2 + x_3 = -21 \\ -2x_1 + x_2 + 5x_3 = 15 \end{cases}$$

using:

1. Jacobi
2. Gauss-Seidel

Starting in both from the initial guess: $x^0 = (1, 2, 2)$

Exercise 5

The values $x_1 = x_2 = 1.000$ are the solutions to:
$$\begin{cases} 1.133x_1 + 5.281x_2 = 6.414 \\ 24.14x_1 - 1.210x_2 = 22.93 \end{cases}$$

1. Use four-digit arithmetic and **Gaussian Elimination** without pivoting to find a computed approximate solution to the system.
2. Same as above but use partial pivoting.

Exercise 6

Use the power method with 9 iterations to locate an eigenvalue and eigenvector

for the matrix: $\begin{pmatrix} 5 & -1 & 7 \\ -1 & -1 & 1 \\ 7 & 1 & 5 \end{pmatrix}$ Check with Python/MATLAB.