Fifth Assignment in "Introduction to Numerical Analysis", Fall Sem. 2021

Numerical Integration

Given: $I = \int_0^1 \ln(1 + x^2)$

(a) Find an approximation I^{num} for I, by composite Trapezoidal rule, with **m** sub-intervals of [0,1]. (Your answer should be depend on the parameter **m**). Simplify your answer as possible.

(b) Find the number of sub-intervals **m** (as small as possible)**,** such that an absolute value of error $\left|E^{total}\right| \le 10^{-3}$. Write code in MATLAB or Python, which calculate I^{num} for the **m** value you found. Add the code and print the value of I^{num}.

(c) What can you say about the sign of the error $E^{total} = I - I^{num}$ for any $\mathbf{m} \geq 1$?

Explain your answer.

Finite Difference Exercise

Solve the Model Problem $\frac{dy}{dx} = x + y$; $y(0) = 1$ using Euler's Method (EM) with h=0.1 and the 2nd order Runge-Kutta (RK2) with λ=2/3, h=0.1.

(a) Compare the solution with the exact solution $y = 2e^x - x - 1$ at x values between 0 and 1, with a step-size of 0.1 (i.e., x=0; x=0.1; x=0.2; … ;x=0.8; x=0.9; x=1.0).

(b) Compare between RK2 and EM.

(c) Count the number of times f(x,y) was evaluated in both methods.

(d) How many time f(x,y) is called with Euler's method at h=0.05.

(e) Compare both methods for accuracy and efficiency: to be fair to both methods, adjust the value of h in EM so that both EM and RK2 use the same number of function evaluations. Now, compare the accuracy between the two methods.

Finite Element Exercise

Solve the Model Problem $\frac{d^2y}{dx^2} - (1 - \frac{x}{5})\mathrm{y} = \mathrm{x}$; $y(1) = 2$; $y(3) = -1$ using the Rayleigh-Ritz method. Approximate $y(x)$ with $u(x) = \sum_{i=1}^{5} c_i N_i(x)$ where $N_i(x)$ are the triangular basis functions defined in class. Using the variational method find the coefficients C_1, C_2, C_3, C_4, C_5 (the first and last are given as the boundary conditions).

Good luck.