Introduction to structural equation modelling - advanced modelling techniques

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Overview

- Advanced SEM topics covered today:

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    - Latent variables

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    - Composite variables

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    - Interactions

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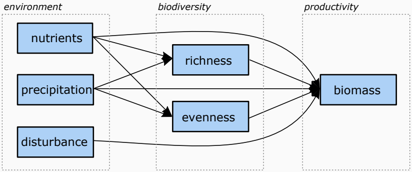
    - Composite variables
    - Interactions
    - Complex survey designs

Revisiting the meta-model



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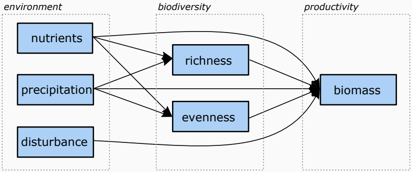

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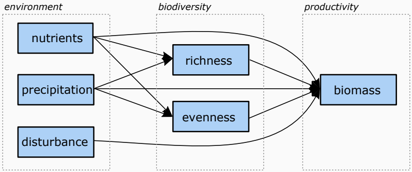

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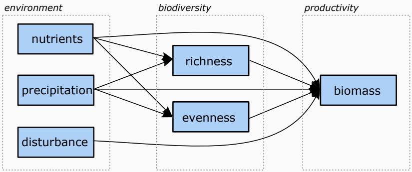

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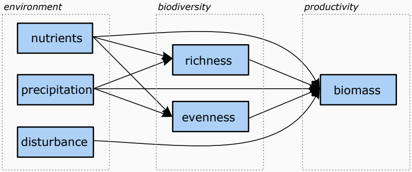

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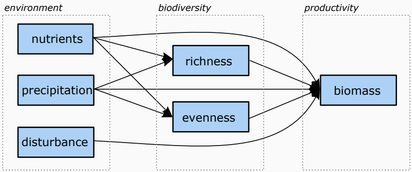

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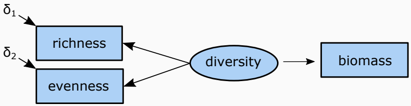
- Or, because we measure variables with error.
- This is where latent and composite variables are needed.

Latent variables

Latent variables



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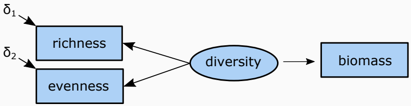

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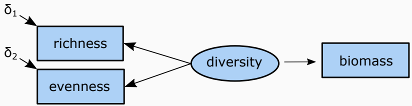

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- Graphically, latent variables are often represented by an oval node shapes (ellipses).

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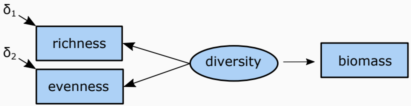

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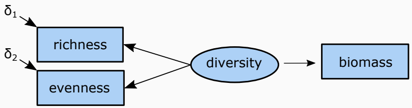

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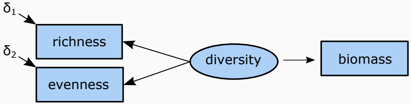

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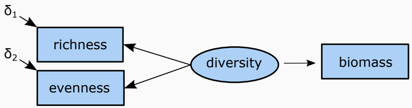

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- In contrast to indicators, latent variables are free of random or systematic measurement errors.

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- *Structural model* is one with directed paths between latent variables.

Latent variables

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- To estimate effect of latent variable on other variables.

Composite variables

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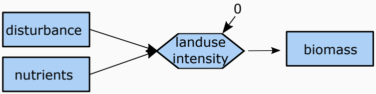

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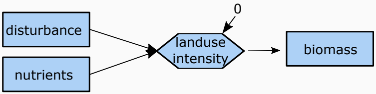

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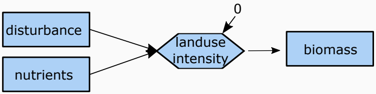

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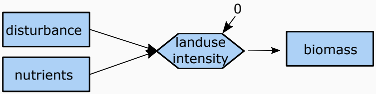

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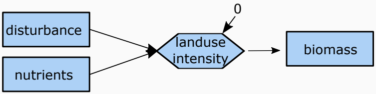

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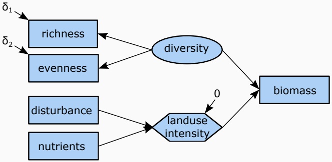
- Composites are often shown as hexangular shapes or ellipses (latent composites).

Key differences between latent and composite variables



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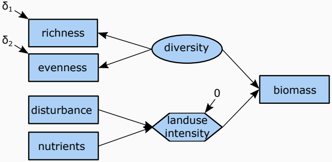

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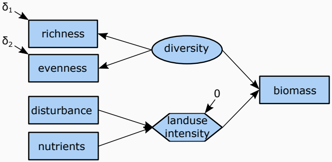

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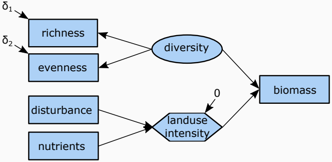

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- In the case of a composite variable, there is no assumption about the relation between the indicators.

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- Rule of thumb 2: If the meaning of the construct changes after dropping one of the indicators, the construct likely is a composite.

Interactions

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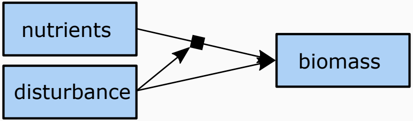

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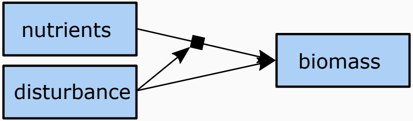

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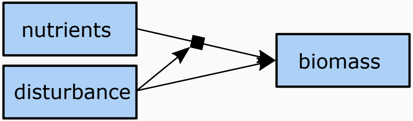

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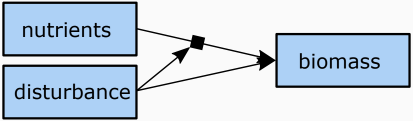

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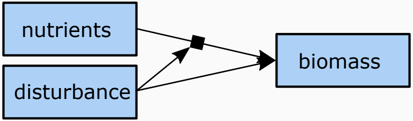

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- In regression, the interaction is represented by a coefficient that estimates the effect of the product of the two predictors.

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- Lavaan offers the “group” argument to specify for which groups coefficients should be estimated.
- Importantly, groups have to be categorical (e.g., sex, age class).

Interactions (multi group)

Lavaan allows to introduce equality constraints on various aspects via the `group.equal` argument:

```
mod <- sem(model, group = "age_class", group.equal =
c("regressions"), data = dd)
```

Additional constraints could be:

```
group.equal=c(
"intercepts",
"means",
"regressions",
"residuals",
"residual.covariances")
```

- Even more control by having the same name for different parameters:

```
model <- '
y ~ c("b1", "b1") * x1 + c("b2", "b2") * x3
x2 ~ c("b3", "b4") * x1
x3 ~ c("b5", "b5") * x2
'
```

Same coefficients for all but the effect of x1 on x2.

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    - within groups such as families, nests
- Nesting violates the principle of being independent and identically distributed.
- Necessary to account for this structure in the data in the model.

Complex sampling structure

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- The add-on package lavaan.survey allows the analysis of stratified, clustered or weighted data.

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- The add-on package lavaan.survey allows the analysis of stratified, clustered or weighted data.
- lavaan objects can be processed further with a specific data structure:
    1) initialize the design
    2) post-process the lavaan object and compute the adjusted results.
    3) Result is a corrected lavaan object.

Complex sampling structure

```
design <- svydesign(ids = ~ plot, strata = ~ field, nest = TRUE,
summary(design)

fit.nested <- lavaan.survey(lavaan.fit = model, survey.design =
```

- Needed to specify the study design

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- Needed to specify the study design
- Here we have plots (`ids`) nested in fields (`strata`)
- Next, we can refit the `simple` model from before with `lavaan.survey` using the specified study design as an argument.

Questions?

Live coding session

Your turn: working with the Seabloom dataset

Exercise 1

Start with the following model:

```
library("lavaan")

simple <-
"mass.above ~ nadd + disk + rich + even + precip.mm
rich ~ nadd + precip.mm
even ~ nadd + precip.mm

rich ~~ even"

fit.simple <- sem(simple, data = seabloom, estimator = "MLM")
summary(fit.simple)
```

Exercise 2

- Construct latent variable diversity based on richness, evenness and ens.pie

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Exercise 4

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    - What do you conclude?
    - Build composite manually.

Exercise 5

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    - Incorporate composite into full model

Exercise 6

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Exercise 7

- Account for nested experimental design with the `lavaan.survey` package:

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- Account for nested experimental design with the `lavaan.survey` package:
    - Add individual plots nested in fields.