

# Introduction to structural equation modelling - advanced modelling techniques

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Advanced SEM topics:

- Latent variables

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- Composite variables

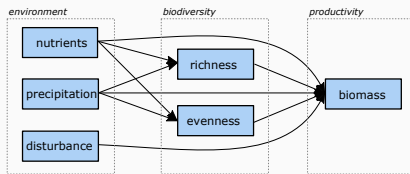
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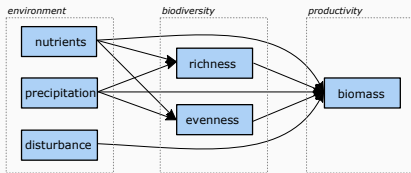
- Latent variables
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- Interactions
- Complex survey designs

# Revisiting the meta-model



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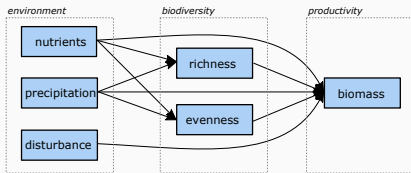
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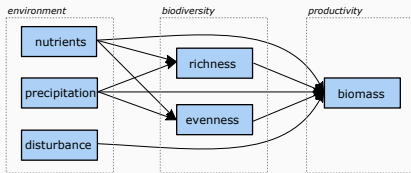


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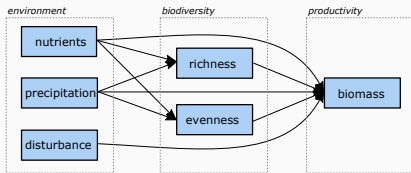
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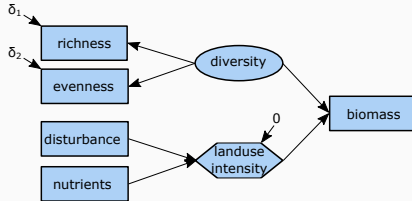
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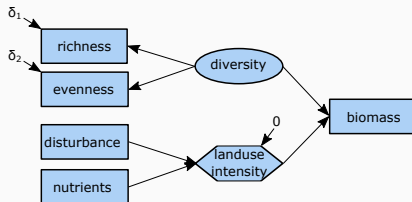
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- However, some of the parts of the model may be difficult to quantify and measure directly.
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- Or, because we measure variables with error.
- This is where latent and composite variables are needed.

# Latent variables



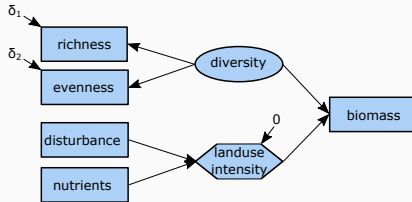
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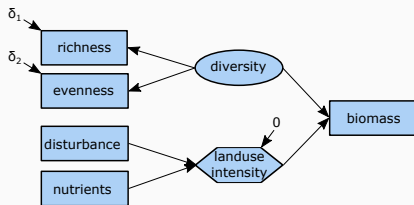
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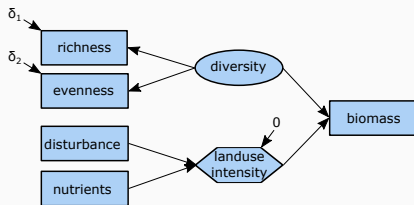
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- Graphically, latent variables are often represented by an oval node shapes (ellipses).

# Latent variables



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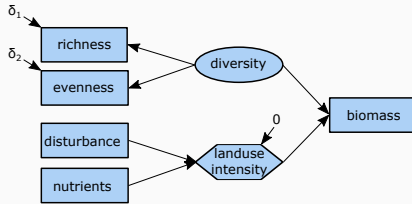
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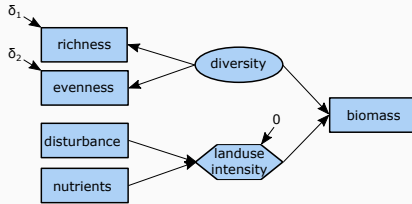


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- *Structural model* is one with directed paths between latent variables.

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## Confirmatory factor analysis

- Multi-indicator latent variables test hypothesis that multiple indicator variables are generated by the same underlying process.
- Test that latent variable has given rise to emergent properties that, by virtue of a common cause, are correlated.
- Precursor to evaluation of any structural models in which the latent variables appear.



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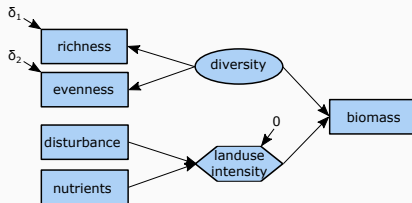
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  - 4) general misspecification

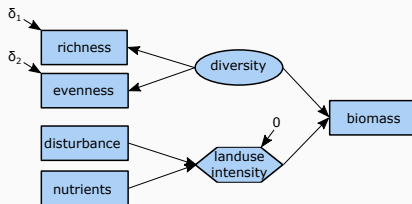


# Composite variables



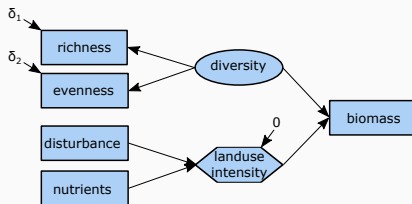
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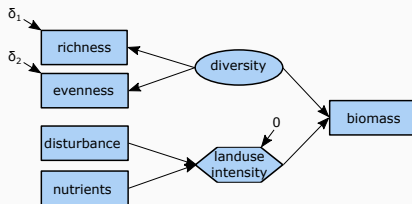
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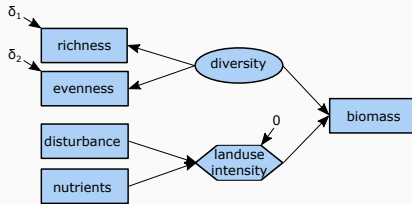
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- they are visualized by arrows pointing from the indicators to the composite (formative or causal indicators).
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- Composites are often shown as hexangular shapes or ellipses (latent composites).

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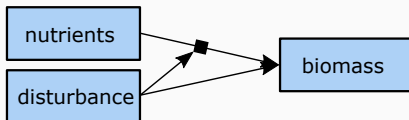
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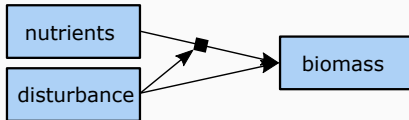
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- Rule of thumb 1: if the indicators are redundant, they likely belong to a latent variable.
- Rule of thumb 2: If the meaning of the construct changes after dropping one of the indicators, the construct likely is a composite.
- As a latent variable represents an underlying, data-generating process, its indicators need to be correlated to each other.
- In the case of a composite variable, there is no assumption about the relation between the indicators. Thus, correlation among indicators are uninformative about the direction of the causal flow, but a lack of such contraindicates that the construct is a latent

## Interactions (Moderation)



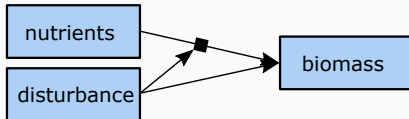
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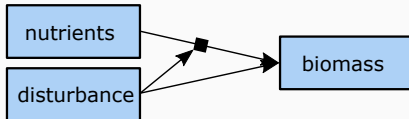
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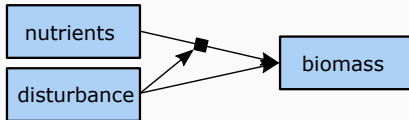
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- Such a behaviour is called an interaction, which indicates that the effect of the two main effects are different when combined.
- Both positive and negative interactions are possible.
- In regression, the interaction is represented by a coefficient that estimates the effect of the product of the two predictors.



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- 2) Composites

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- Importantly, groups have to be of categorical nature.

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## Interactions (Moderation)

Lavaan allows to introduce equality constraints on various aspects via the `group.equal` argument:

```
mod <- sem(model, group = "age_class", group.equal =  
c("regressions"), data = dd)
```

Additional constraints could be:

```
group.equal=c(  
  "intercepts",  
  "means",  
  "regressions",  
  "residuals",  
  "residual.covariances")
```

- Even more control by having the same name for different parameters:

```
model <- '  
y ~ c("b1", "b1") * x1 + c("b2", "b2") * x3  
x2 ~ c("b3", "b4") * x1  
x3 ~ c("b5", "b5") * x2  
,
```

Same coefficients for all but the effect of x1 on x2.



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  - 2) post-process the `lavaan` object and compute the adjusted results.
  - 3) Result is a corrected `lavaan` object.





Questions?