

Introduction to structural equation modelling - advanced modelling techniques

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- Advanced SEM topics covered today:

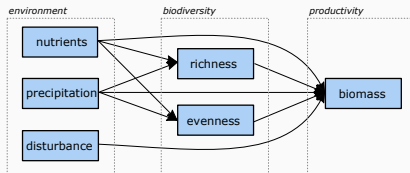
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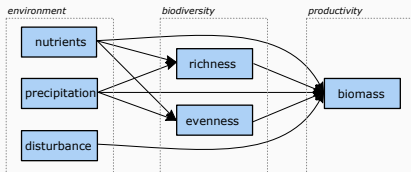
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 - Latent variables
 - Composite variables
 - Interactions
 - Complex survey designs

Revisiting the meta-model



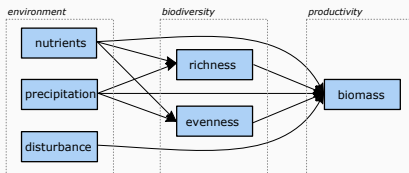
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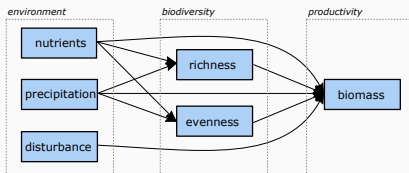
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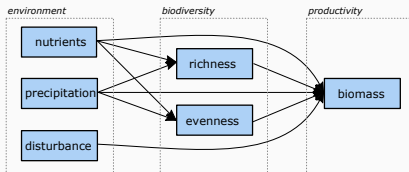
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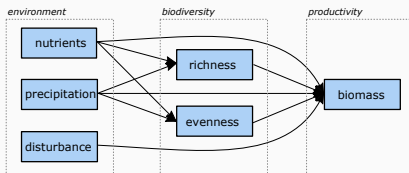
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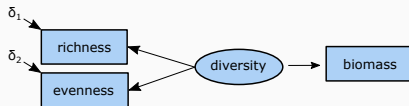
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- Or, because we measure variables with error.
- This is where latent and composite variables are needed.

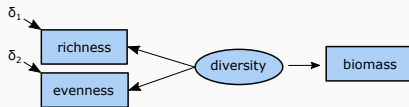
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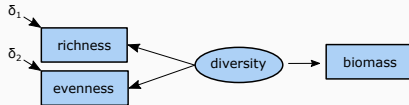
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Latent variables



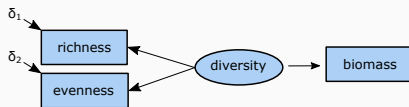
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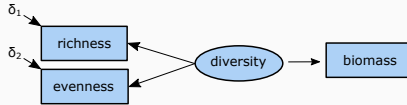
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- Graphically, latent variables are often represented by an oval node shapes (ellipses).

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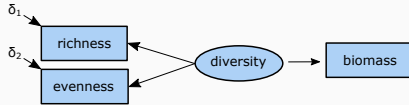
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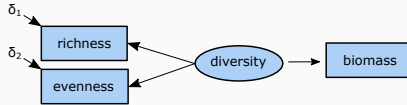
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- *Structural model* is one with directed paths between latent variables.

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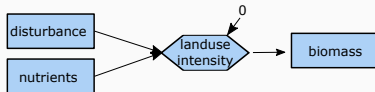
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2) Full model including latent variable

- To estimate effect of latent variable on other variables.

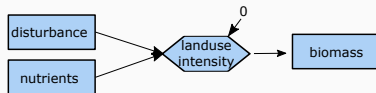
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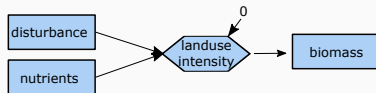
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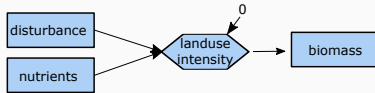
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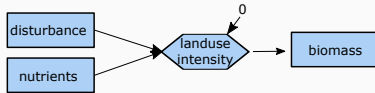
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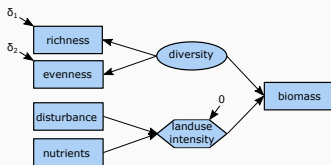
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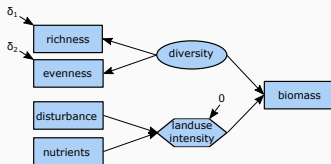
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- Composites are often shown as hexangular shapes or ellipses (latent composites).

Key differences between latent and composite variables



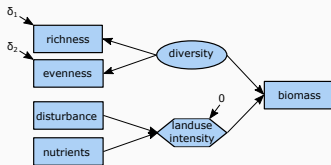
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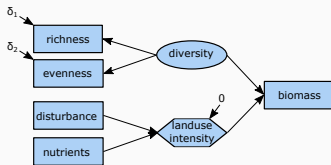
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- For composite variables, the flow of causation is reversed and the indicators are independent entities.
- In the case of a composite variable, there is no assumption about the relation between the indicators.

Key differences between latent and composite variables

How distinguish latent and composites:

- Rule of thumb 1: if the indicators are redundant, they likely belong to a latent variable.

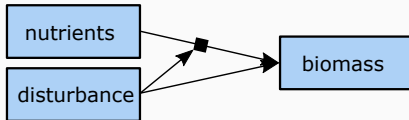
Key differences between latent and composite variables

How distinguish latent and composites:

- Rule of thumb 1: if the indicators are redundant, they likely belong to a latent variable.
- Rule of thumb 2: If the meaning of the construct changes after dropping one of the indicators, the construct likely is a composite.

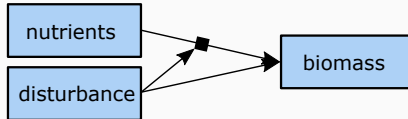
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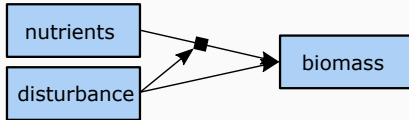
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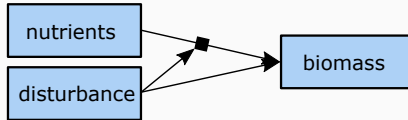
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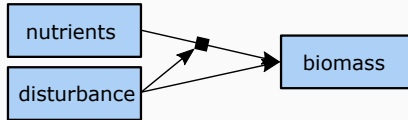
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- Such a behaviour is called an interaction, which indicates that the effect of the two main effects are different when combined.
- Both positive and negative interactions are possible.
- In regression, the interaction is represented by a coefficient that estimates the effect of the product of the two predictors.

Interactions can be modelled in different ways in lavaan:

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- Importantly, groups have to be categorical (e.g., sex, age class).

Interactions (multi group)

Lavaan allows to introduce equality constraints on various aspects via the `group.equal` argument:

```
mod <- sem(model, group = "age_class", group.equal =  
c("regressions"), data = dd)
```

Additional constraints could be:

```
group.equal=c(  
  "intercepts",  
  "means",  
  "regressions",  
  "residuals",  
  "residual.covariances")
```

Interactions (multi group)

- Even more control by having the same name for different parameters:

```
model <- '  
y ~ c("b1", "b1") * x1 + c("b2", "b2") * x3  
x2 ~ c("b3", "b4") * x1  
x3 ~ c("b5", "b5") * x2  
,
```

Same coefficients for all but the effect of x1 on x2.

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 - 4) general misspecification

Complex sampling structure

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 - within groups such as families, nests
- Nesting violates the principle of being independent and identically distributed.
- Necessary to account for this structure in the data in the model.

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- `lavaan` objects can be processed further with a specific data structure:
 - 1) initialize the design
 - 2) post-process the `lavaan` object and compute the adjusted results.
 - 3) Result is a corrected `lavaan` object.

```
design <- svydesign(ids = ~ plot, strata = ~ field, nest = TRUE,  
summary(design)
```

```
fit.nested <- lavaan.survey(lavaan.fit = model, survey.design =
```

- Needed to specify the study design

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- Needed to specify the study design
- Here we have plots (**ids**) nested in fields (**strata**)
- Next, we can refit the **simple** model from before with **lavaan.survey** using the specified study design as an argument.

Questions?

Live coding session

Your turn: working with the
Seabloom dataset

Exercise 1

Start with the following model:

```
library("lavaan")

simple <-
"mass.above ~ nadd + disk + rich + even + precip.mm
rich ~ nadd + precip.mm
even ~ nadd + precip.mm

rich ~~ even"

fit.simple <- sem(simple, data = seabloom, estimator = "MLM")
summary(fit.simple)
```


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Exercise 3

- Construct latent variable diversity only based on richness and evenness:

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 - Build composite manually.

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- Account for nested experimental design with the `lavaan.survey` package:

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 - Add individual plots nested in fields.