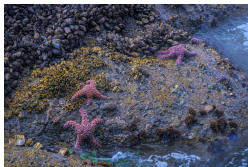


# Introduction to structural equation modelling - basic modelling

Frank Pennekamp

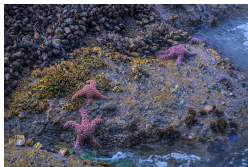
11/11/2020

# Research questions



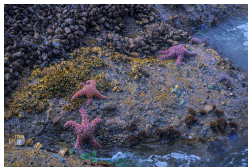
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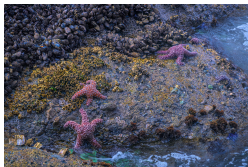
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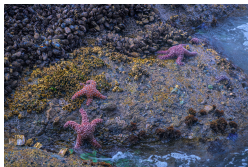
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- ▶ Research questions are often about understanding cause and effect.

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- ▶ Causality is central: SEM implicitly assumes that the relationships among variables represent causal links.

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- ▶ The arrow above indicates a causal relationship.

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- ▶ Include interaction terms can test main effects and interaction effects.

## Two goals of SEM:

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- 1) Understand the patterns of correlation/covariance among a set of variables.
- 2) Explain as much of their variance as possible with the model specified.

# Thinking about the model

A SEM is usually specified based on theory to determine and validate a proposed causal process and/or model.

Which variables to include?

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Good practice: Make a table / graph of putative causal relationships before analysis.

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- ▶ Garbage in - garbage out (both data quality and ecologically meaningful).

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# SEM modelling philosophy

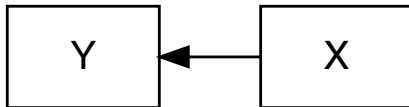
- ▶ Continuum between theory (hypothesis-driven) to exploratory (data-driven) modelling.



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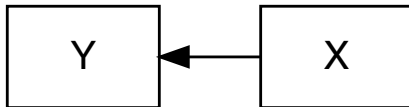
- ▶ Continuum between theory (hypothesis-driven) to exploratory (data-driven) modelling.
- ▶ Important to be explicit about the approach taken.

## Graphical models:



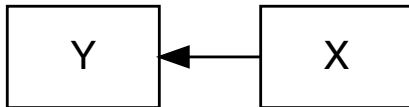
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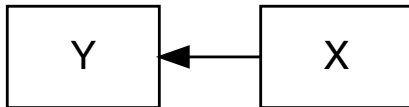
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- ▶ Edges (two-headed arrows) are non-causal relationships such as X and Y are correlated.

# Introduction to the dataset



We will use an experimental dataset collected at the Cedar Creek Ecosystem Science Reserve to examine long-term consequences of human-driven environmental changes ecosystem responses to:

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# Introduction to the dataset

Understanding the recovery of a grassland for two decades following an intensive agricultural disturbance under ambient and elevated nutrient conditions.

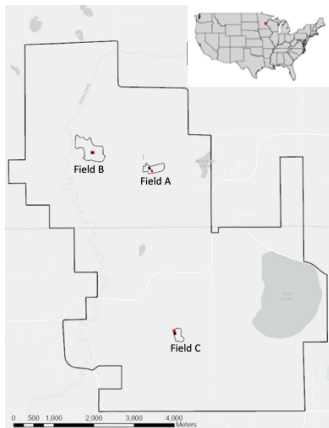
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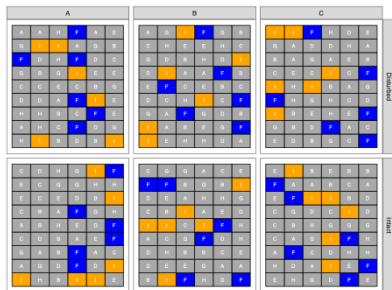
- 1) How has aboveground biomass changed as a function of disturbance (disking) and nutrient addition?
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# Introduction to the dataset



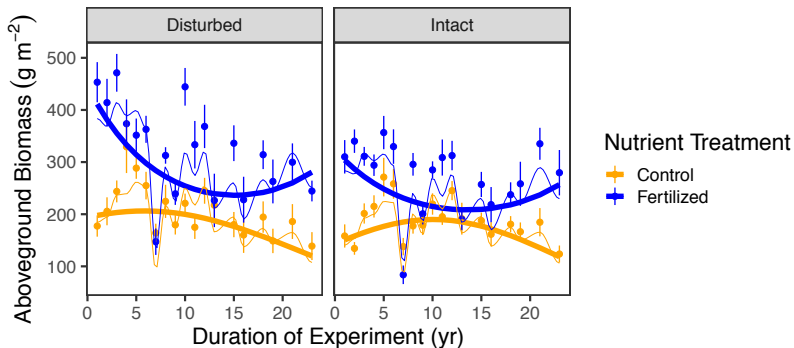
Map showing location of the study site (Cedar Creek Ecosystem Science Reserve), the location of the three study fields within the reserve, and location of the 35 x 55 m intact (black) and disturbed (red) plots within each field.

# Introduction to the dataset



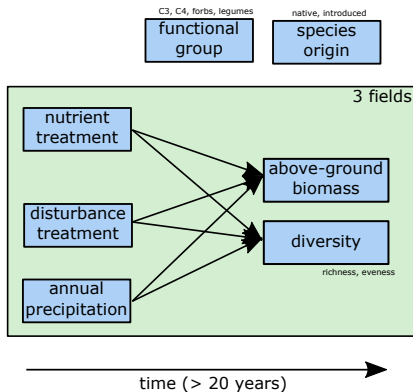
Location of the 4 x 4 m nutrient treatment plots within each 35 x 55 m Intact or Disturbed plot within each of three fields (A, B, and C). Letters indicate the nutrient treatments, and the colored plots are treatments that are the focus of the analyses presented here: Control (orange) and 9.5 g N m<sup>-2</sup> yr<sup>-1</sup> (blue).

# Introduction to the dataset



Effect of soil disturbance (disking) and nutrient enrichment on live, aboveground plant biomass. Colors indicate nutrient addition treatment: Control and NPK+ (all nutrients plus  $9.5 \text{ g N m}^{-2} \text{ yr}^{-1}$ ).

# Question of interest

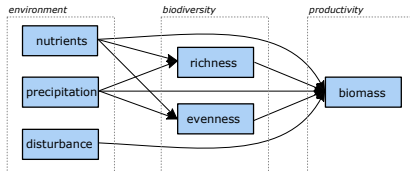


- What is the effect of richness on biomass?

# Meta-model

A metamodel summarizes the concept behind a model and links it to theory.

*Productivity (biomass) is directly influenced on the one hand by the environment (nutrients, disturbance and precipitation) and on the other hand by biodiversity (richness and evenness). Also some elements of the environment influence biodiversity and thus, have an additional indirect effect on productivity via biodiversity.*



## A simple bivariate model.



```
lm(biomass ~ precipitation)
```

- Linear regression



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- ▶ Linear regression
- ▶ Regression coefficient quantifies the strength of relationship

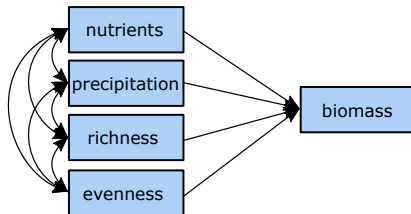
## A simple bivariate model.



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- ▶ Linear regression
- ▶ Regression coefficient quantifies the strength of relationship
- ▶ Change in Y for one unit change in X

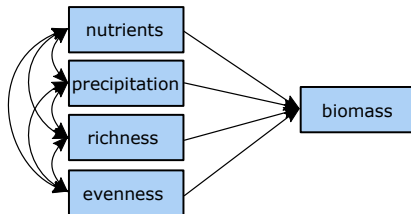
## Multiple independent variables



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- Often more than one independent variable important = multiple regression

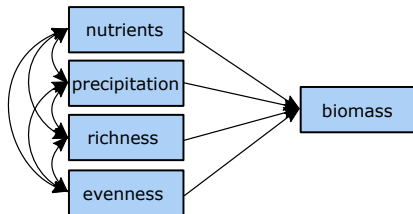
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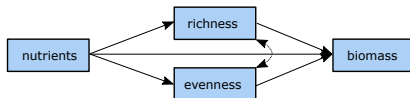
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- ▶ Estimates partial regression coefficients (i.e. effect of  $x_1$  on  $y$  when  $x_2$  is fixed)
- ▶ All effects are direct.

## Multiple independent variables



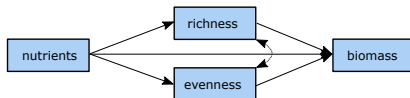
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lm(richness ~ nutrients)
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- Indirect effect is the effect of an independent variable on a dependent variable through one or more intervening or mediating variables.

## Multiple independent variables



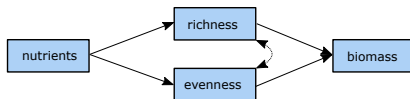
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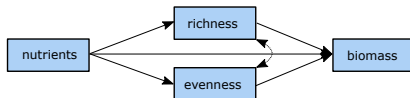
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- ▶ Indirect effect is the effect of an independent variable on a dependent variable through one or more intervening or mediating variables.
- ▶ Indirect effects can be quantified by the product of the compound path.

# Mediation

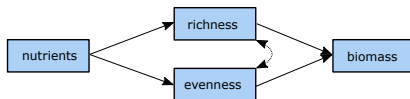


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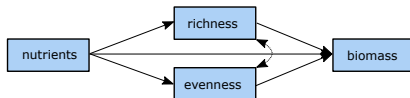




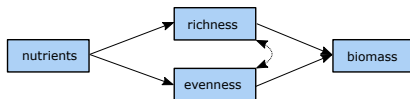
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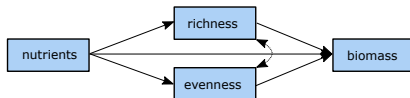
- ▶ Tests whether a particular variable has a mediating effect.
- ▶ Often used to test underlying mechanisms.



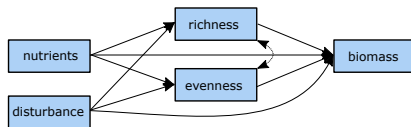
# Mediation



- ▶ Tests whether a particular variable has a mediating effect.
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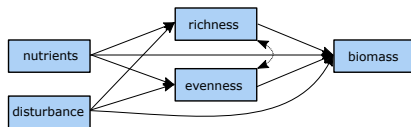


## System level approach



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- 8) The total effect (including undirected paths) is equivalent to the total correlation.

## Goodness-of-fit

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- ▶ The larger the sample size, the more precise (unbiased) the estimates will be.

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- 1) Review the relevant theory and research literature to support model specification
- 2) Specify a model (e.g., diagram, equations)
- 3) Determine model identification
- 4) Select measures for the variables represented in the model
- 5) Collect data
- 6) Conduct preliminary descriptive statistical analysis (e.g., scaling, missing data, collinearity issues, outlier detection)
- 7) Estimate parameters in the model
- 8) Assess model fit
- 9) Re-specify the model if meaningful
- 10) Interpret and present results visually