Logic : conditional statement (Syllogism, Aristotelian)

Statement :	If p, then q	OR	$p \rightarrow q$
Contrapositive :	If $\neg q$, then $\neg p$	OR	$\neg q \rightarrow \neg p$
Inverse :	If $\neg p$, then $\neg q$	OR	$\neg p \rightarrow \neg q$
Converse :	If q, then p	OR	$q \rightarrow p$

Logic : conditional statement (Syllogism, Aristotelian)

Statement :	If Wings,	then	Bird
Contrapositive :	If NOT Bird,	then	NOT Wings
Inverse :	If NOT Wings,	then	NOT Bird
Converse :	If Bird,	then	Wings

Logic : conditional statement (Syllogism, Aristotelian)

Statement :	If Fish,	then	Swim
Contrapositive :	If NOT Swim,	then	NOT Fish

Inverse :	If NOT Fish,	then	NOT Swim
Converse :	If Swim,	then	Fish

Logic : conditional statement (Python aka English way)

Statement :	Swim if Fish	(If Fish,	then Swim)
Contrapositive :	Not Fish if Not Swim	(If NOT Swim,	then NOT Fish)
Inverse :	Not Swim if Not Fish	(If NOT Fish,	then NOT Swim)
Converse :	Fish if Swim	(If Swim,	then Fish)

Other Logic Rules

And : $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ short-hand as $p \wedge q \wedge r$ (intersection)

Or : $(p \vee q) \vee r \equiv p \vee (q \vee r)$ short-hand as $p \vee q \vee r$ (union)

Negation :

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Distributive law:

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

Boolean Algebra

And : $\wedge$ like multiply $\otimes$ more restrictive, relates to intersection

Or : $\vee$ like adding $\oplus$ more inclusive, relates to union

False is typically treated as 0

True is typically treated as 1

Does logic rules coincide with the algebraic rules

on numbers that we expect?

Let's check…

Boolean Algebra

Let's check…

False $\leftrightarrow$ 0 | True $\leftrightarrow$ 1 | And $\leftrightarrow \wedge \leftrightarrow \otimes$ | Or $\leftrightarrow \vee \leftrightarrow \oplus$

$F \vee F \equiv F$ | $0 \oplus 0 = 0$ ✓

$F \vee T \equiv T$ | $0 \oplus 1 = 1$ ✓

$T \vee T \equiv T$ | $1 \oplus 1 = 2$?? It's okay. Anything > 0 usually considered True

$F \wedge F \equiv F$ | $0 \otimes 0 = 0$ ✓

$F \wedge T \equiv F$ | $0 \otimes 1 = 0$ ✓

$T \wedge T \equiv T$ | $1 \otimes 1 = 1$ ✓

Boolean Algebra

continue…

$\mathrm{F} \vee \mathrm{F} \equiv \mathrm{F} \quad | \quad 0 \oplus 0 = 0$

, Or : $(p \vee q) \vee r \equiv p \vee (q \vee r)$ short-hand as $p \vee q \vee r$ (union)

Negation : $\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

Distributive law:

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

Boolean Algebra

Let's check…

Or	F	T
F	F	T
T	T	T

$\oplus$	0	1
0	0	1
1	1	2 (ok)

And	F	T
F	F	F
T	F	T

$\otimes$	0	1
0	0	0
1	0	1

Boolean Algebra

Let's check distributive laws…

(1) $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$

(2) $p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$

(1) $p \otimes (q \oplus r) \equiv (p \otimes q) \oplus (p \otimes r)$ ✓

(2) $p \oplus (q \otimes r) \equiv (p \oplus q) \otimes (p \oplus r)$?????????

The second distributive law is not familiar to us in regular algebra.

It is only true for Boolean algebra.