Lecture 5.1
Camera calibration

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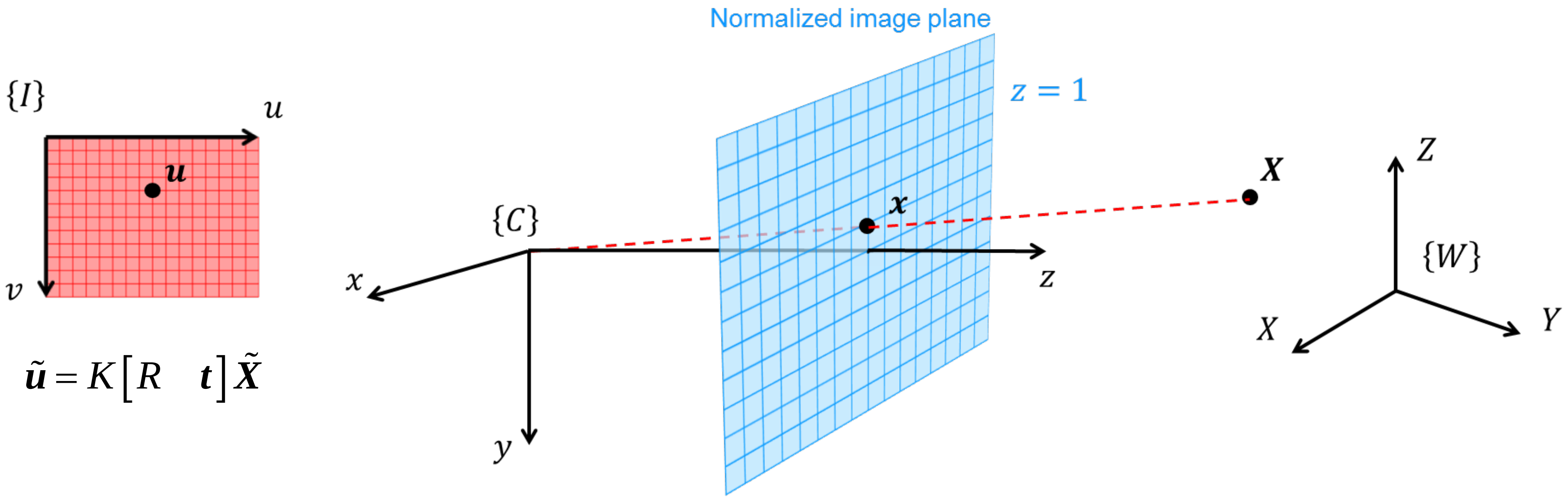
Introduction



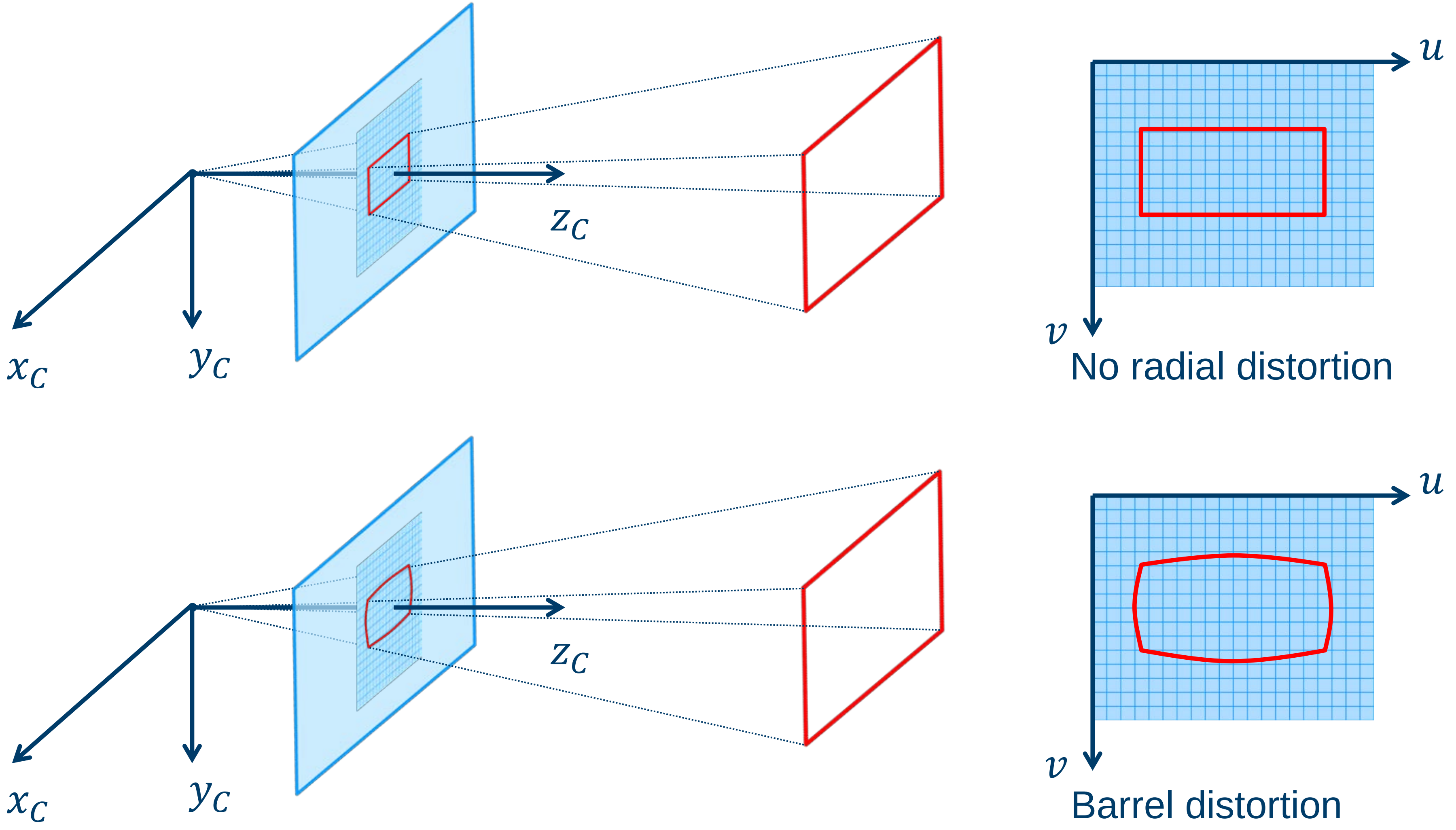
$$\tilde{\boldsymbol{u}} = K\begin{bmatrix} R & \boldsymbol{t} \end{bmatrix}\tilde{\boldsymbol{X}}$$

- The perspective camera model describes a 3D to 2D transformation that is consistent with the pinhole geometry

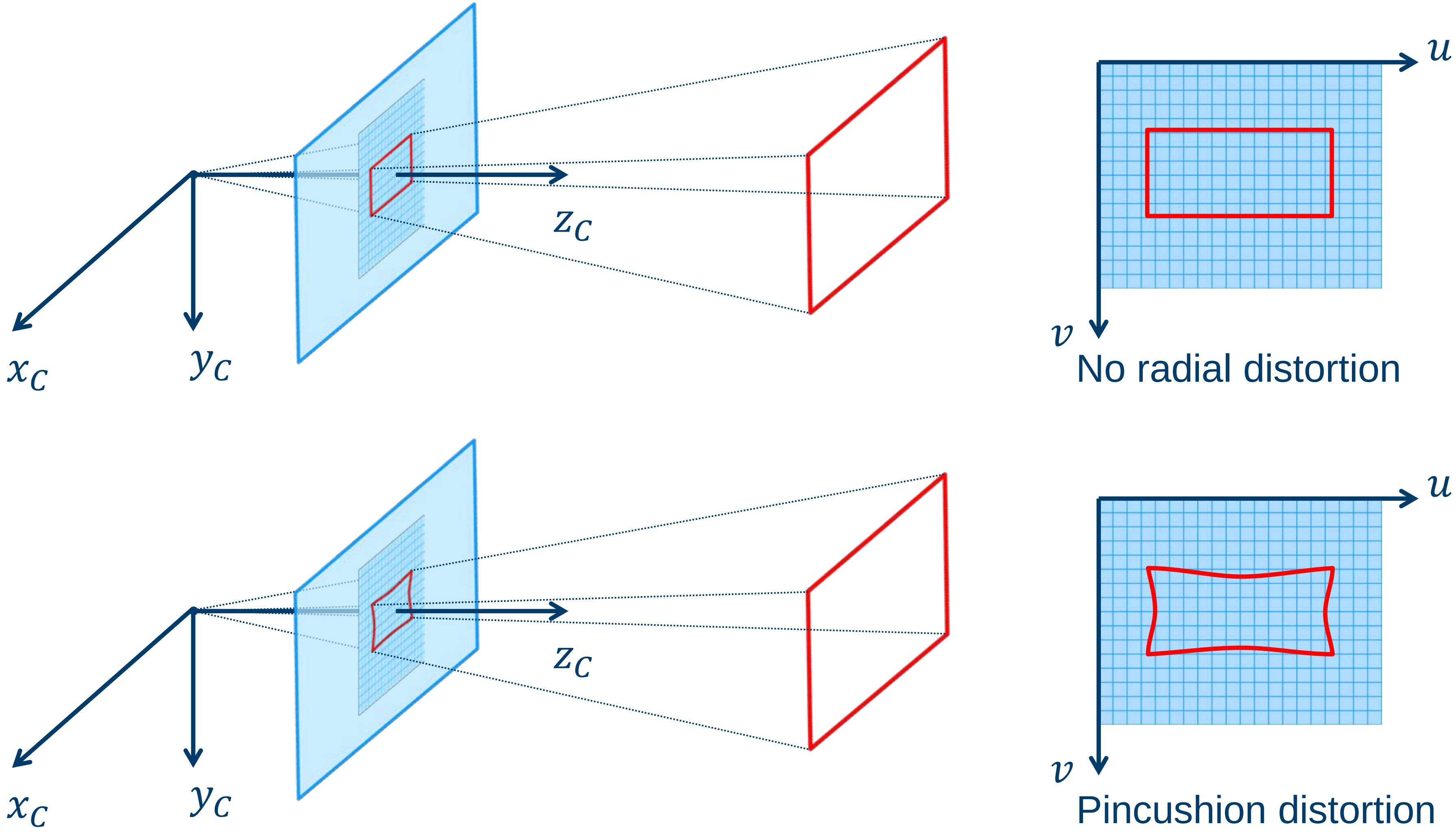
- No cameras fit this model perfectly – They all suffer from some kind of distortion

- If we want to use images for geometrical computations we need to take this distortion into account

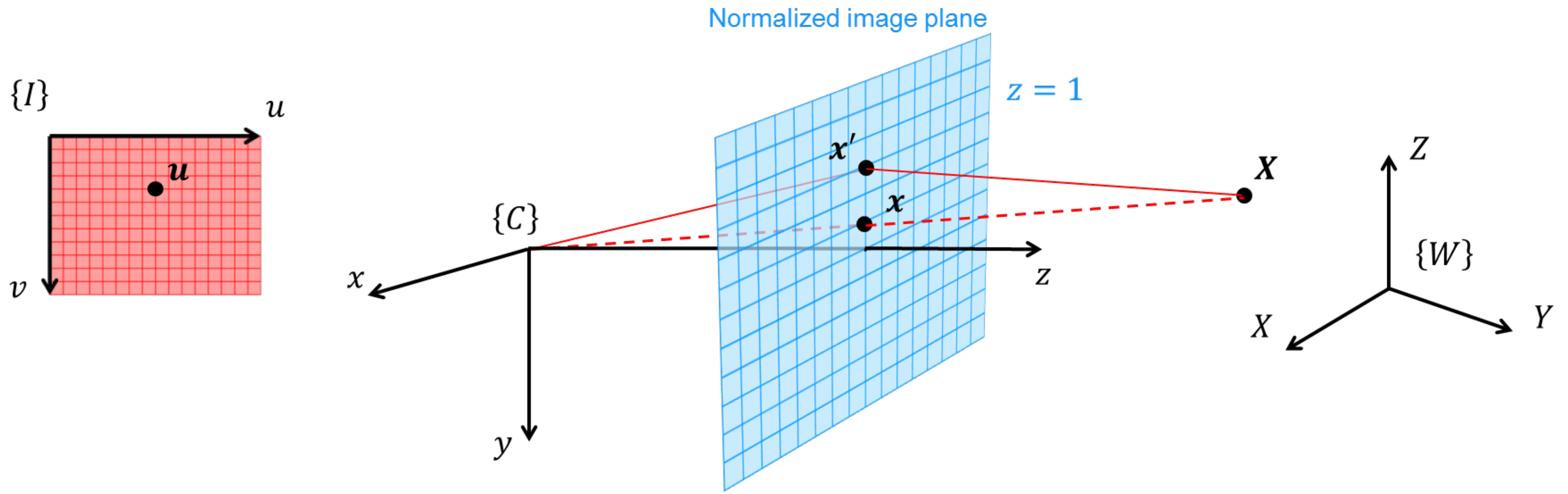
Introduction



Introduction



Undistortion



- A distortion model describes the relationship between undistorted coordinates $\boldsymbol{x}$ and distorted coordinates $\boldsymbol{x}'$ of the normalized image plane

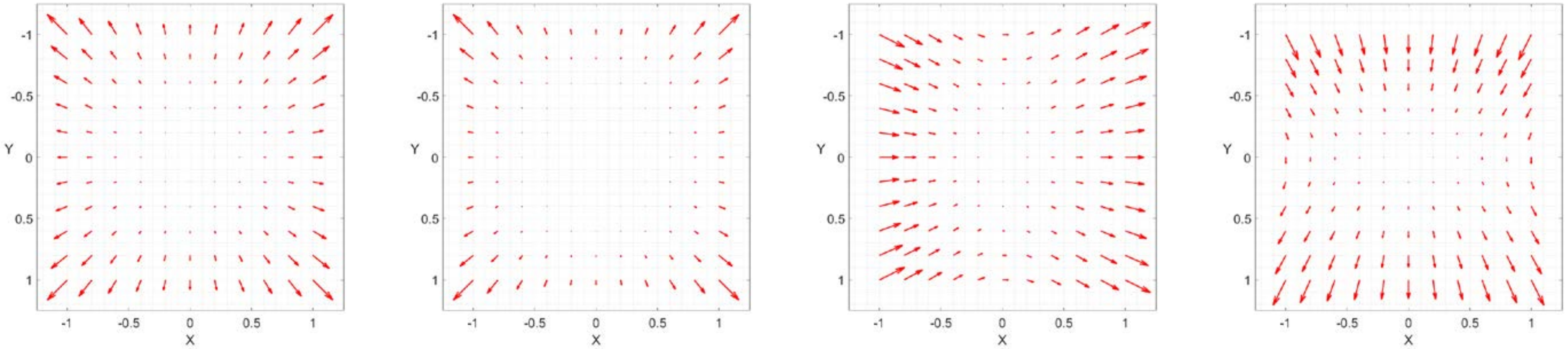
- This example model describes both radial distortion and tangential distortion

$$x = x'\left(1 + k_1 r^2 + k_2 r^4\right) + 2 p_1 x' y' + p_2\left(r^2 + 2x'^2\right)$$
$$y = y'\left(1 + k_1 r^2 + k_2 r^4\right) + p_1\left(r^2 + 2y'^2\right) + 2 p_2 x' y'$$

where $r^2 = x'^2 + y'^2$

Undistortion

$k_1 = 0.05$ $\quad$ $k_2 = 0.05$ $\quad$ $p_1 = 0.05$ $\quad$ $p_2 = 0.05$



$k_{1,2} =$ radial distortion parameters $\quad$ $p_{1,2} =$ tangential distortion parameters

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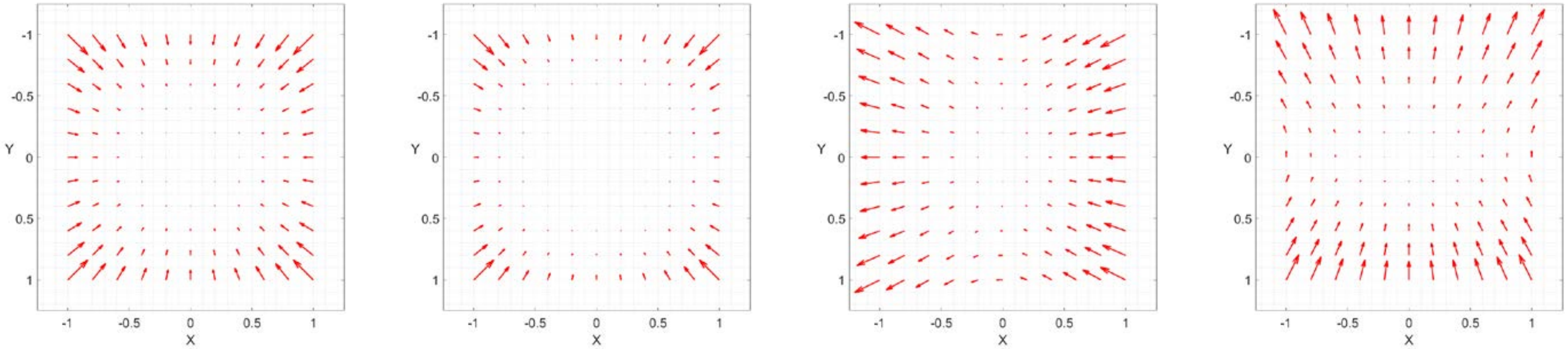
Undistortion

$k_1 = -0.05$ $\quad$ $k_2 = -0.05$ $\quad$ $p_1 = -0.05$ $\quad$ $p_2 = -0.05$



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Undistortion

Original image

$$\tilde{\boldsymbol{u}} \neq K\begin{bmatrix} R & \boldsymbol{t} \end{bmatrix}\tilde{\boldsymbol{X}}$$

undistortion

$\longrightarrow$

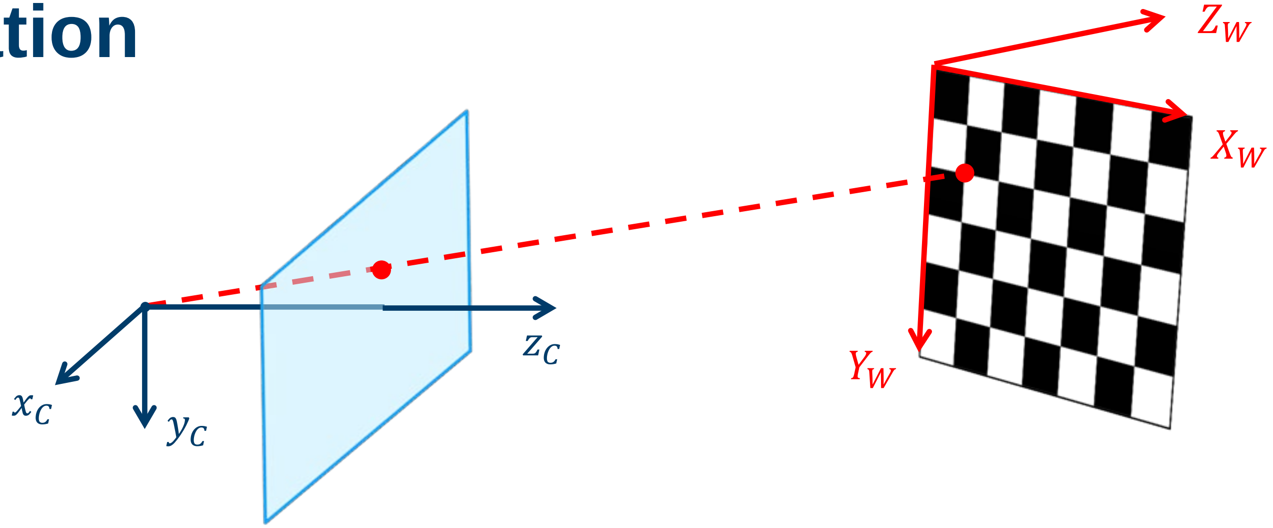
Undistorted image

$$\tilde{\boldsymbol{u}} = K\begin{bmatrix} R & \boldsymbol{t} \end{bmatrix}\tilde{\boldsymbol{X}}$$

- We can use the distortion model to warp the original image into the so called undistorted image
- The undistorted image satisfy the perspective camera model and are thus well suited for geometrical computations
- Since the distortion model depends on K for the undistorted camera, it is custom to estimate both in a common calibration process

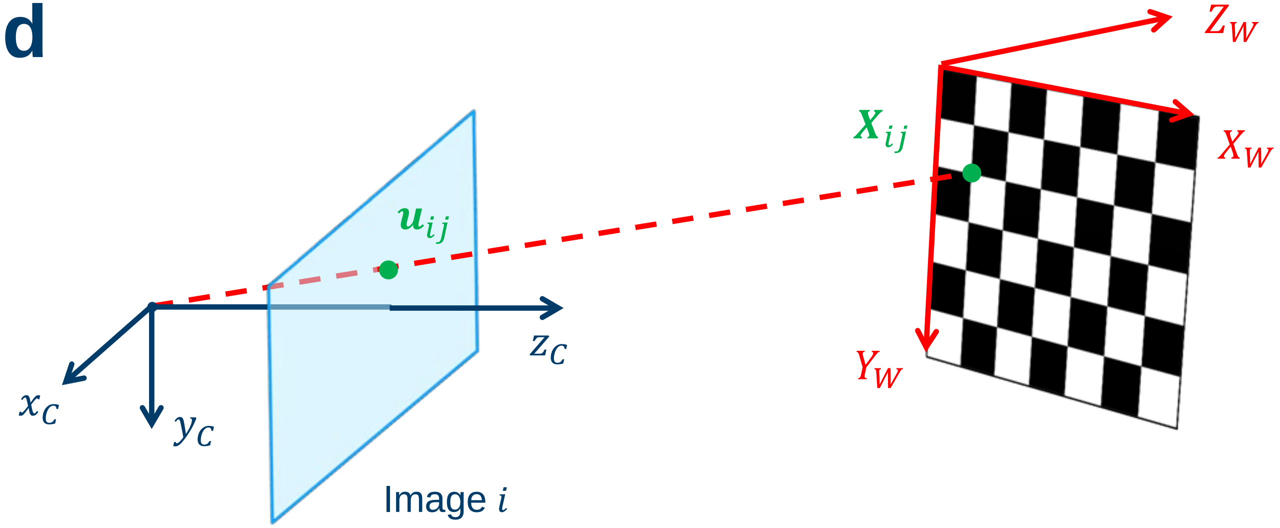
Images: http://www.robots.ox.ac.uk/~vgg/hzbook/

Camera calibration



- Camera calibration is a process where we estimate the intrinsic parameters f_u, f_v, s, c_u, c_v and distortion parameters for a camera
- One of the most commonly used calibration algorithms was described by Zhengyou Zhang in the paper ''*A Flexible New Technique for Camera Calibration*'' in 2000
- Zhang's method is based on using a planar calibration object, e.g. a chessboard where we know the size of the tiles

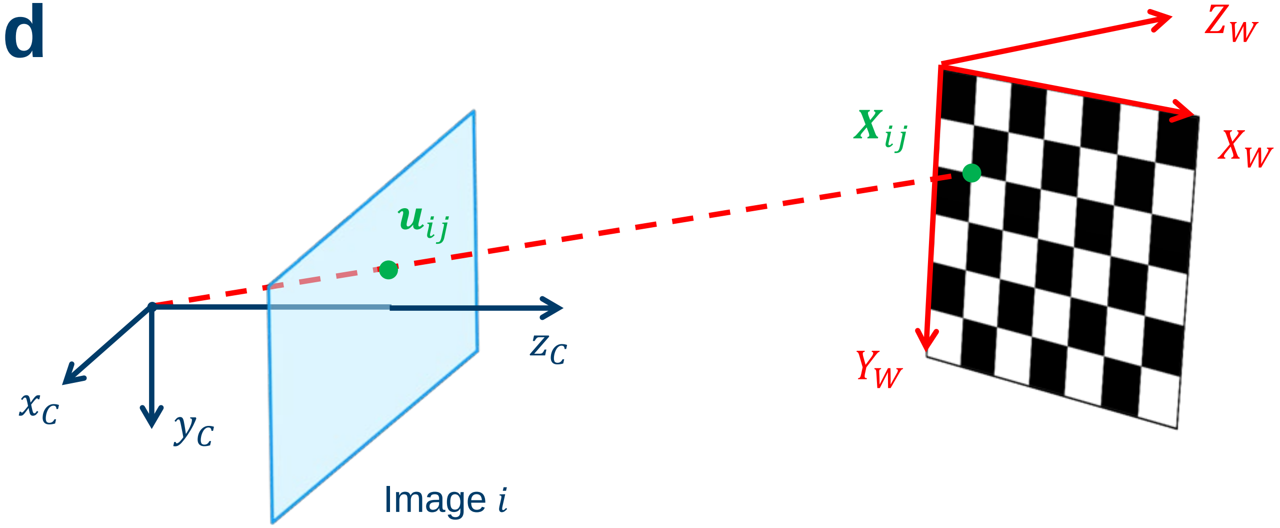
Zhang's method



1. Capture multiple images (at least 3) of the planar calibration object
2. For each image, estimate the homography H between the 2D surface of the calibration object and the image
3. Based on these homographies, estimate K (DLT) and from this all R_i's and $\boldsymbol{t}_i$'s
4. Use the estimated parameters as the starting point of an iterative non-linear optimization of the full set of intrinsic parameters (including distortion parameters $\boldsymbol{\kappa}$)

$$\min_{K,\boldsymbol{\kappa},R_i,\boldsymbol{t}_i} \sum_i \sum_j \left\| \boldsymbol{u}_{ij} - \hat{\mathbf{u}}\left(K,\boldsymbol{\kappa},R_i,\boldsymbol{t}_i;\boldsymbol{X}_{ij}\right) \right\|^2$$

Zhang’s method

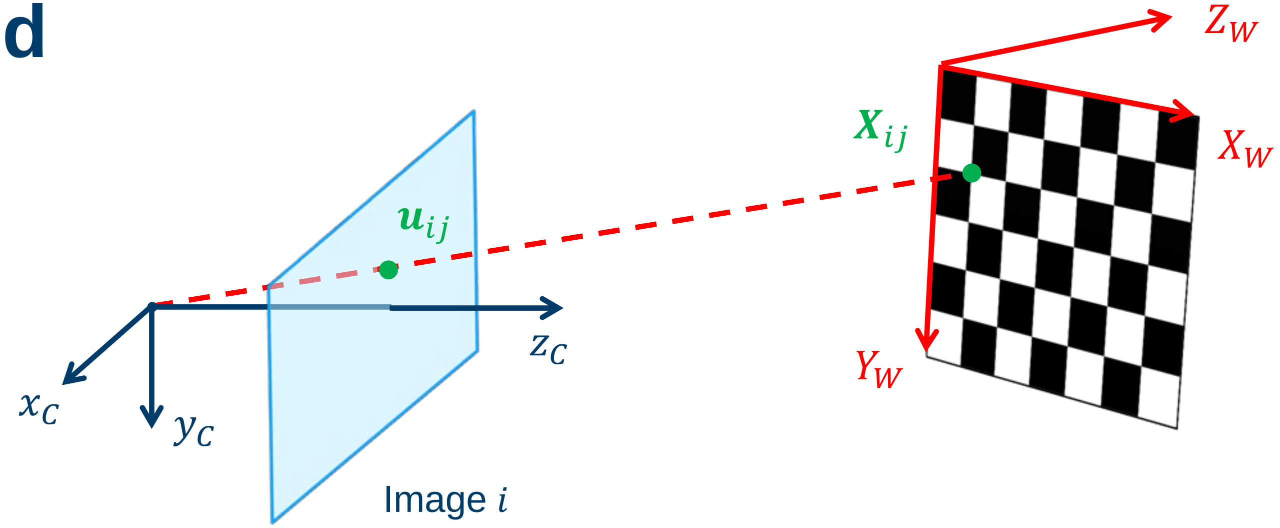


- Given that the calibration object is planar, the 3D-2D relationship between points on the calibration object and points in the image is described by a homography

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = K \underbrace{\begin{bmatrix} \boldsymbol{r}_1 & \boldsymbol{r}_2 & \boldsymbol{r}_3 & \boldsymbol{t} \end{bmatrix}}_{[R \;\; \boldsymbol{t}]} \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = K \begin{bmatrix} \boldsymbol{r}_1 & \boldsymbol{r}_2 & \boldsymbol{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = H \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

- We can estimate the homography H from a minimum of 4 point correspondences between the image and the calibration object

Zhang's method



- Since $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are columns of a rotation matrix, they impose 2 constraints on the elements of H

$$\left.\begin{aligned} \boldsymbol{r}_1^T\boldsymbol{r}_2 &= 0 \\ \boldsymbol{r}_1^T\boldsymbol{r}_1 &= \boldsymbol{r}_2^T\boldsymbol{r}_2 \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} \boldsymbol{h}_1^T K^{-T} K^{-1}\boldsymbol{h}_2 &= 0 \\ \boldsymbol{h}_1^T K^{-T} K^{-1}\boldsymbol{h}_1 &= \boldsymbol{h}_2^T K^{-T} K^{-1}\boldsymbol{h}_2 \end{aligned}\right.$$

- From N images we get N homographies, giving rise to $2N$ constraints allowing us to solve for the different elements of K using linear methods (DLT)

Summary

- Undistortion
  - For geometrical computations we work on undistorted images/feature points

- Calibration
  - Estimate K + distortion model parameters from images
  - Zhang's method
  - OpenCV, Matlab, Kalibr (https://github.com/ethz-asl/kalibr)

- Additional reading
  - Szeliski: 6.3

- Optional reading
  - *A flexible new technique for camera calibration*, by Z. Zhang