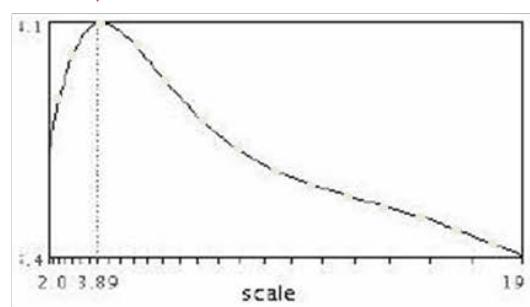


## Lecture 3.2.2 Blob features

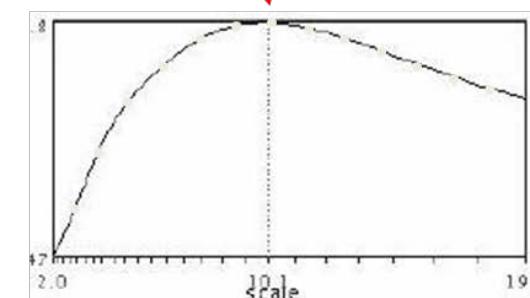
Trym Vegard Haavardsholm

Slides from Svetlana Lazebnik, Grauman&Leibe,  
S. Seitz, James Hays and Noah Snavely

# Automatic scale selection



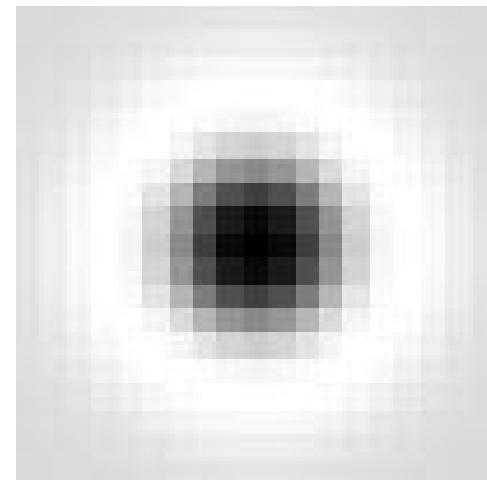
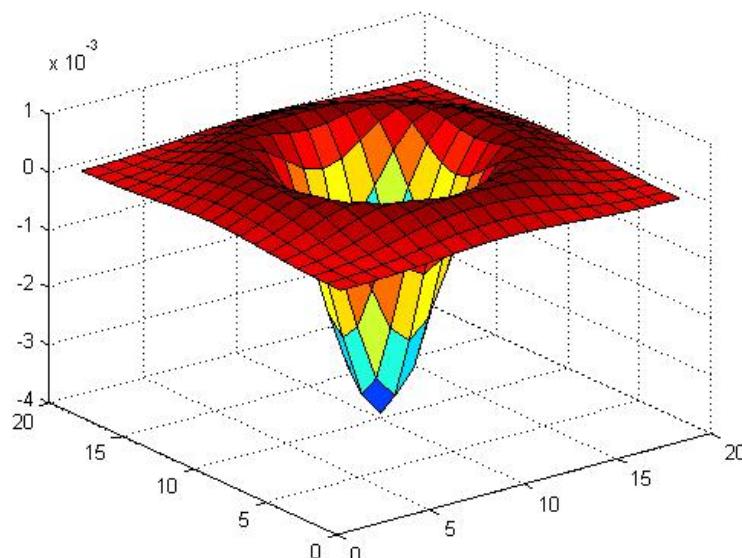
$$f(I_{i_1 \dots i_m}(x, \sigma))$$



$$f(I'_{i_1 \dots i_m}(x', \sigma'))$$

## Another common definition of $f$

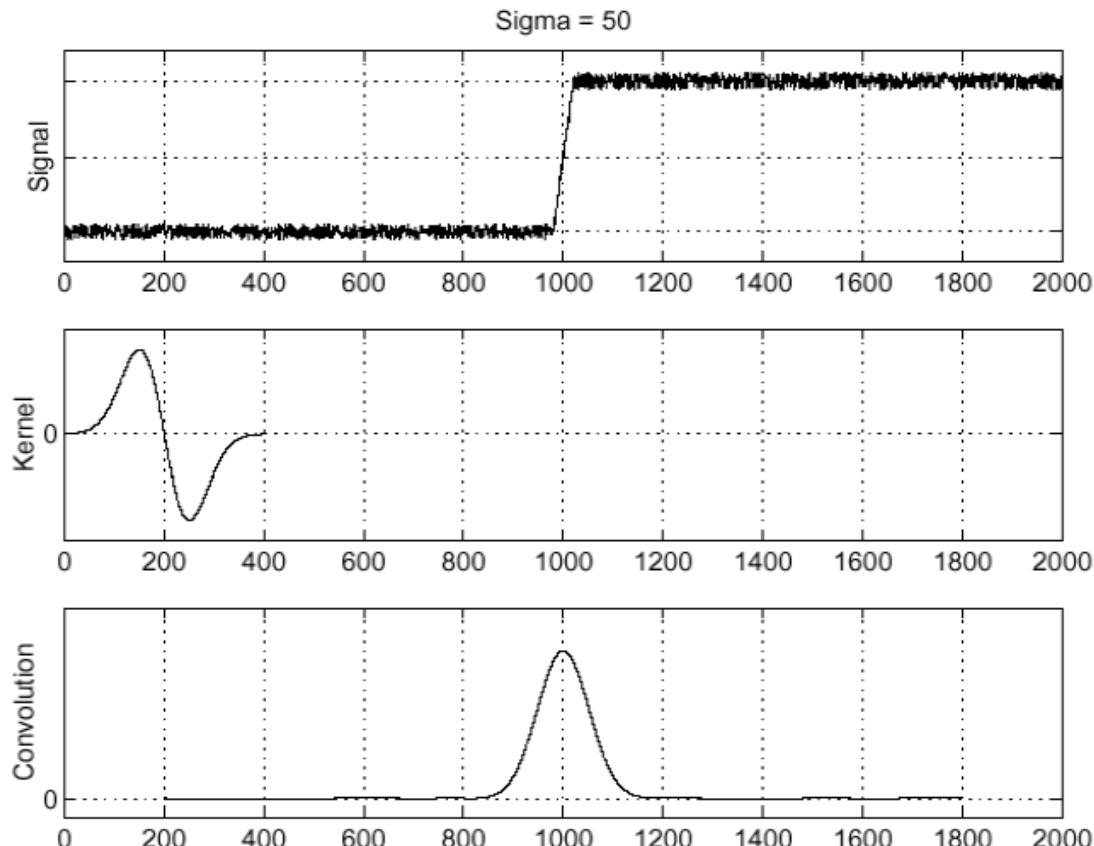
- The *Laplacian of Gaussian (LoG)*



$$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}$$

# Edges and blobs

$$f * \frac{d}{dx} g$$

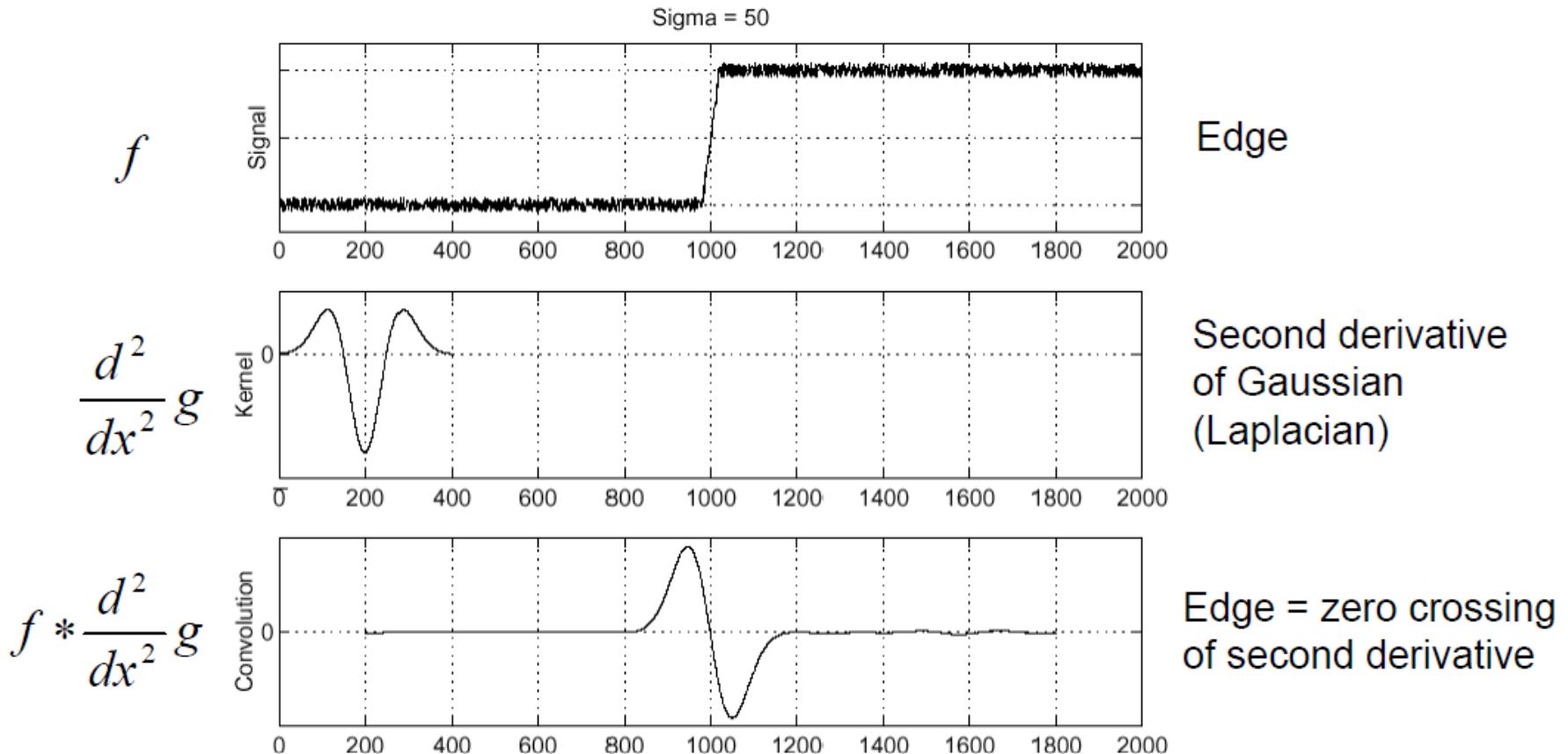


Edge

Derivative  
of Gaussian

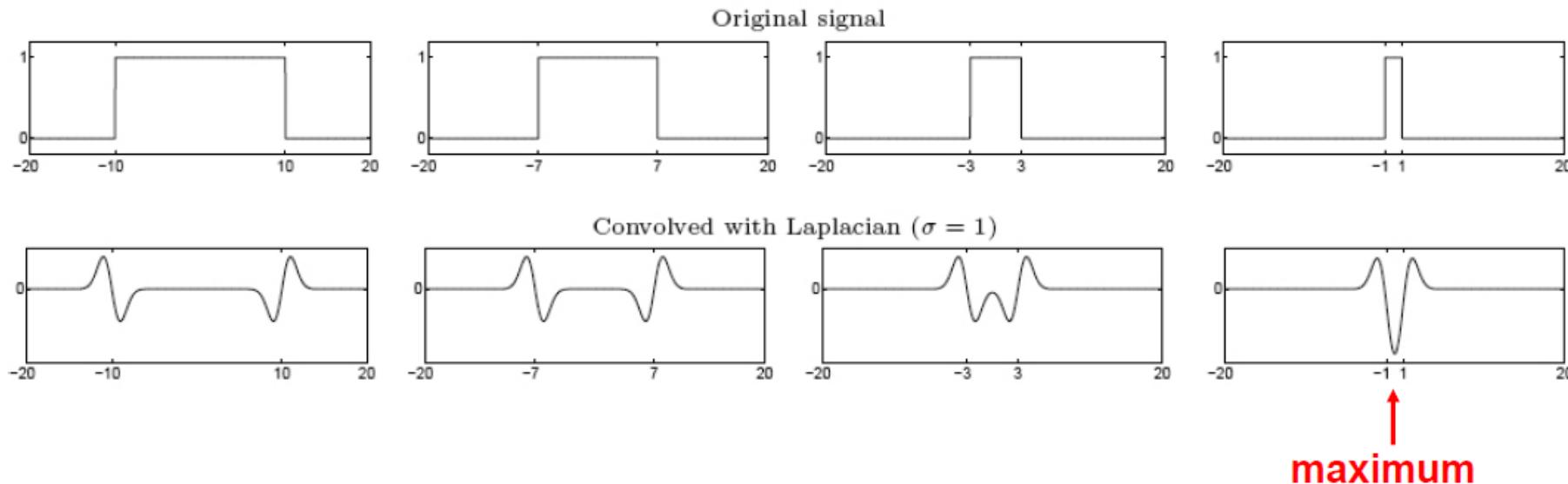
Edge = maximum  
of derivative

# Edges and blobs



# Edges and blobs

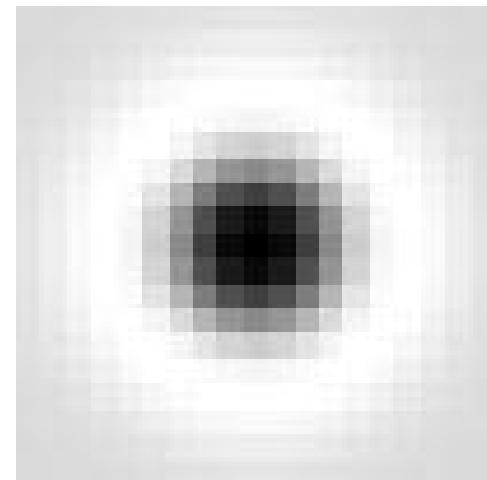
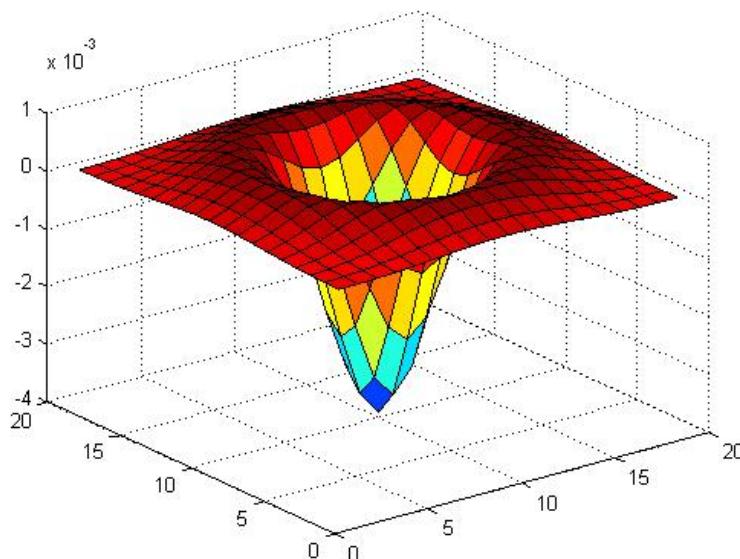
- Edge: Ripple
- Blob: Superposition of two ripples



- The magnitude of the Laplacian response is maximum at the centre of the blob provided the scale of the Laplacian matches the scale of the blob

# Laplacian of Gaussian

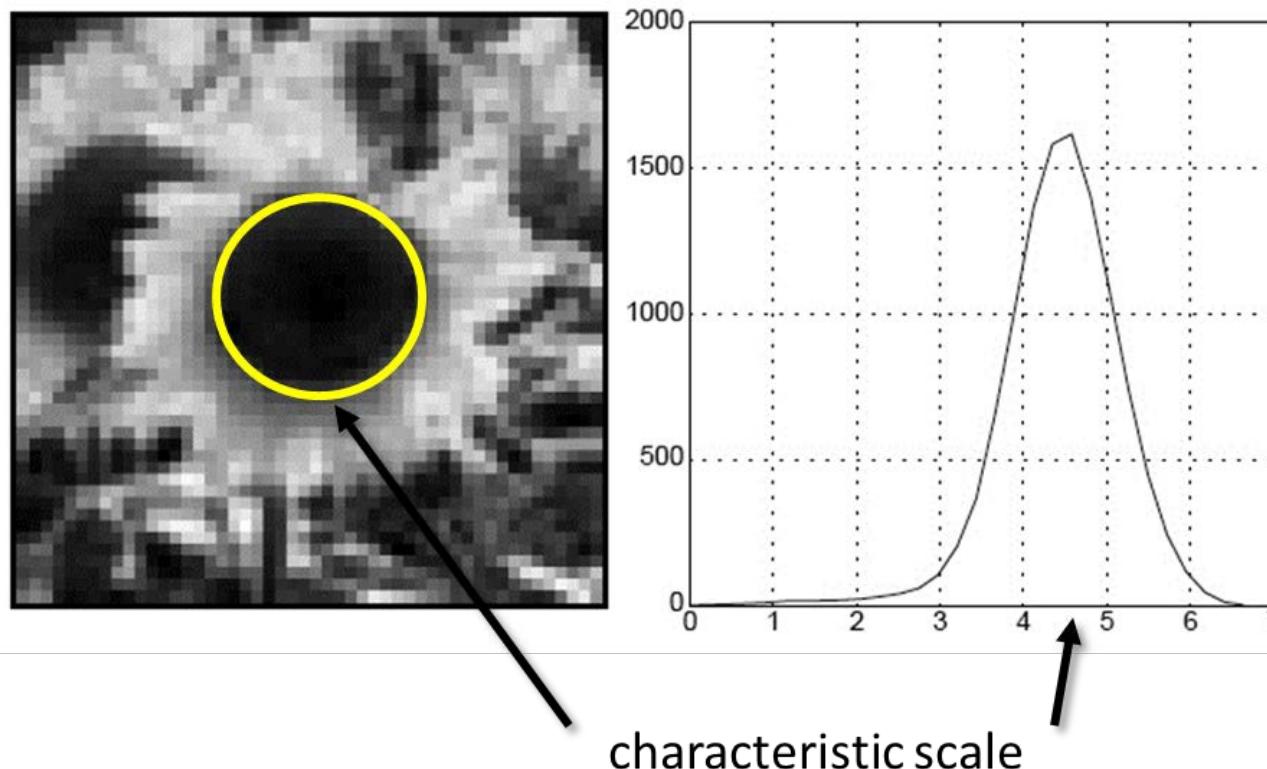
- Normalize to make the response independant of scale



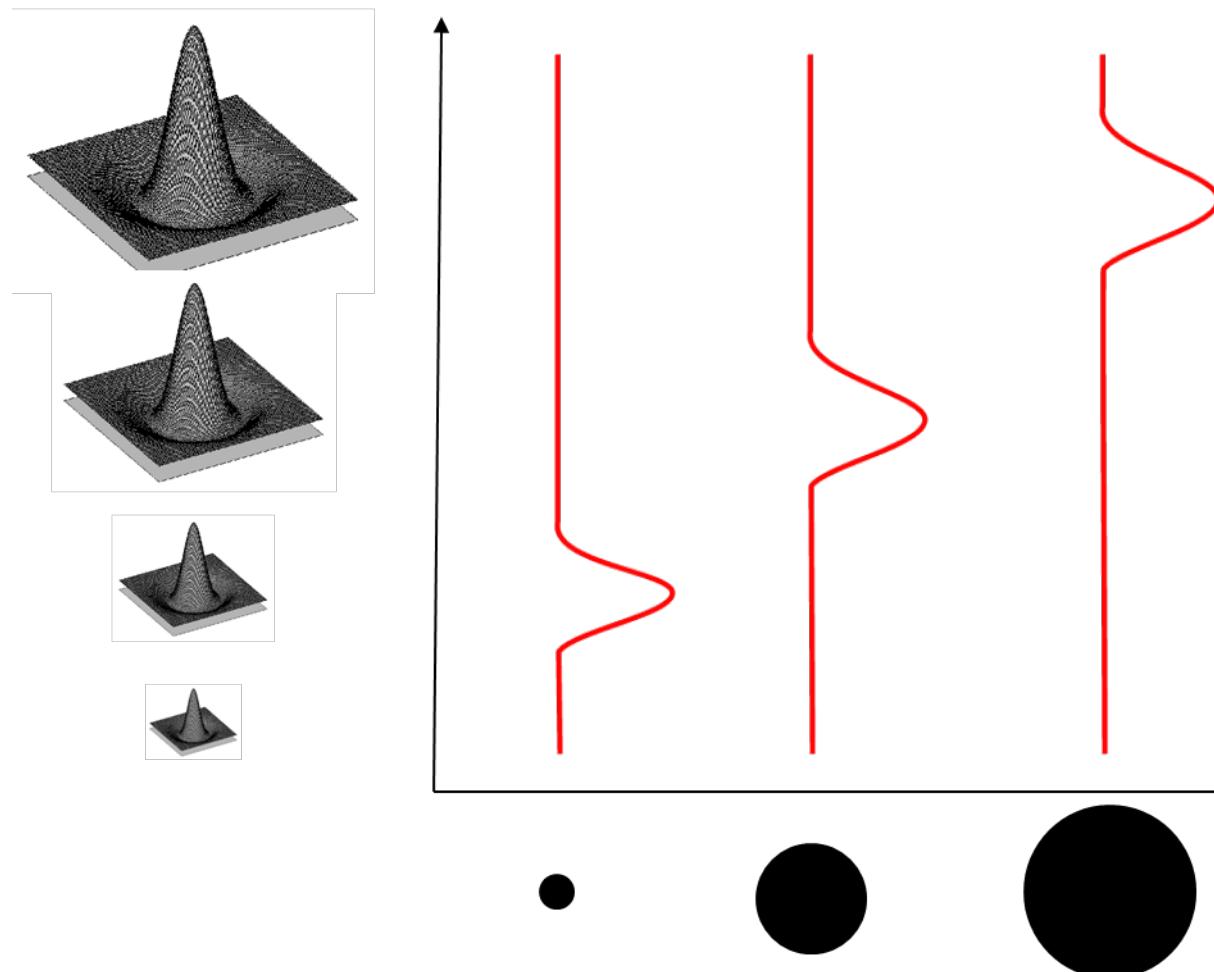
Scale-normalized:  $\nabla_{\text{norm}}^2 g = \sigma^2 \left( \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} \right)$

## Characteristic scale

- We define the characteristic scale as the scale that produces peak of Laplacian response



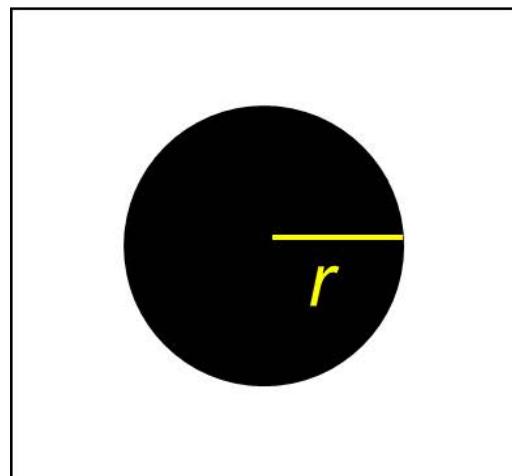
# Characteristic scale



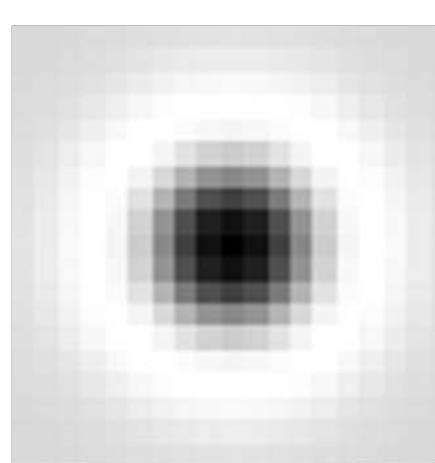
K. Grauman, B. Leibe

## Scale selection

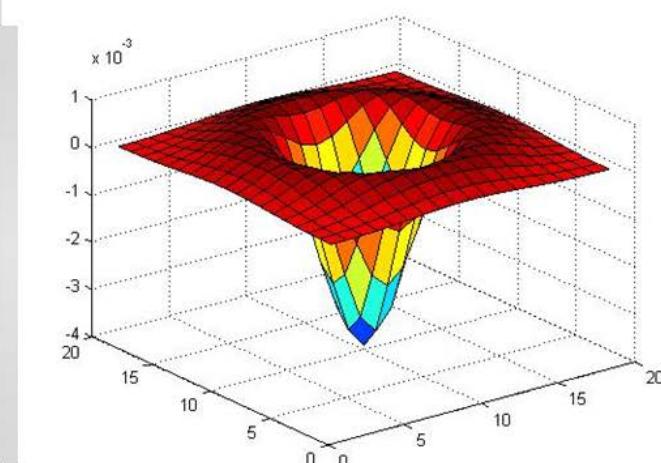
- At what scale does the Laplacian achieve a maximum response to a binary circle of radius  $r$ ?



image



Laplacian

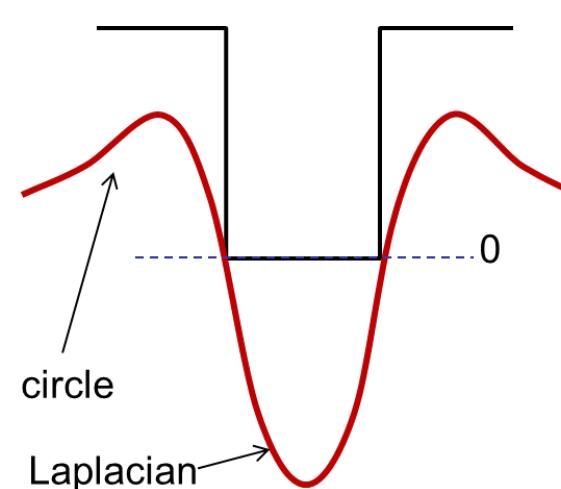
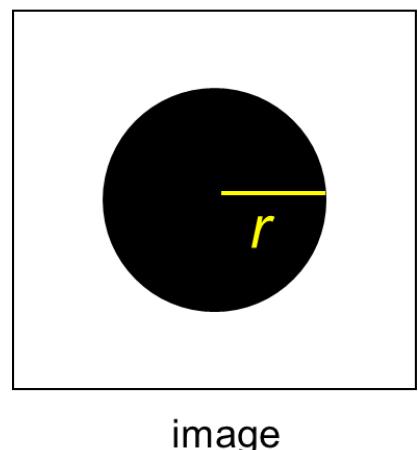


## Scale selection

- To get maximum response, the zeros of the Laplacian have to be aligned with the circle
- The Laplacian is given by (up to scale):

$$(x^2 + y^2 - 2\sigma^2) e^{-(x^2+y^2)/2\sigma^2}$$

- Therefore, the maximum response occurs at  $\sigma = r / \sqrt{2}$ .

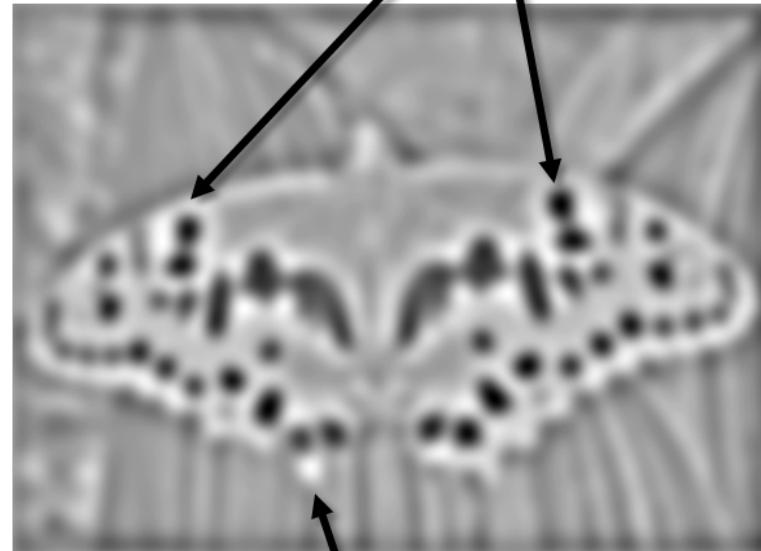


# Laplacian of Gaussian

- Circularly symmetric operator for blob detection in 2D



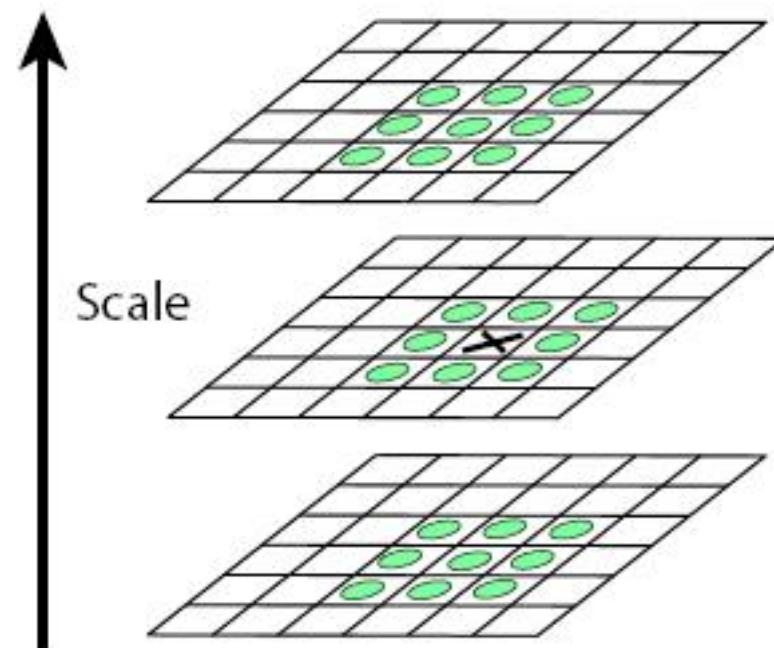
$$\ast \begin{array}{|c|}\hline \bullet \\ \hline \end{array} =$$



- Find maxima and minima of LoG operator in space and scale

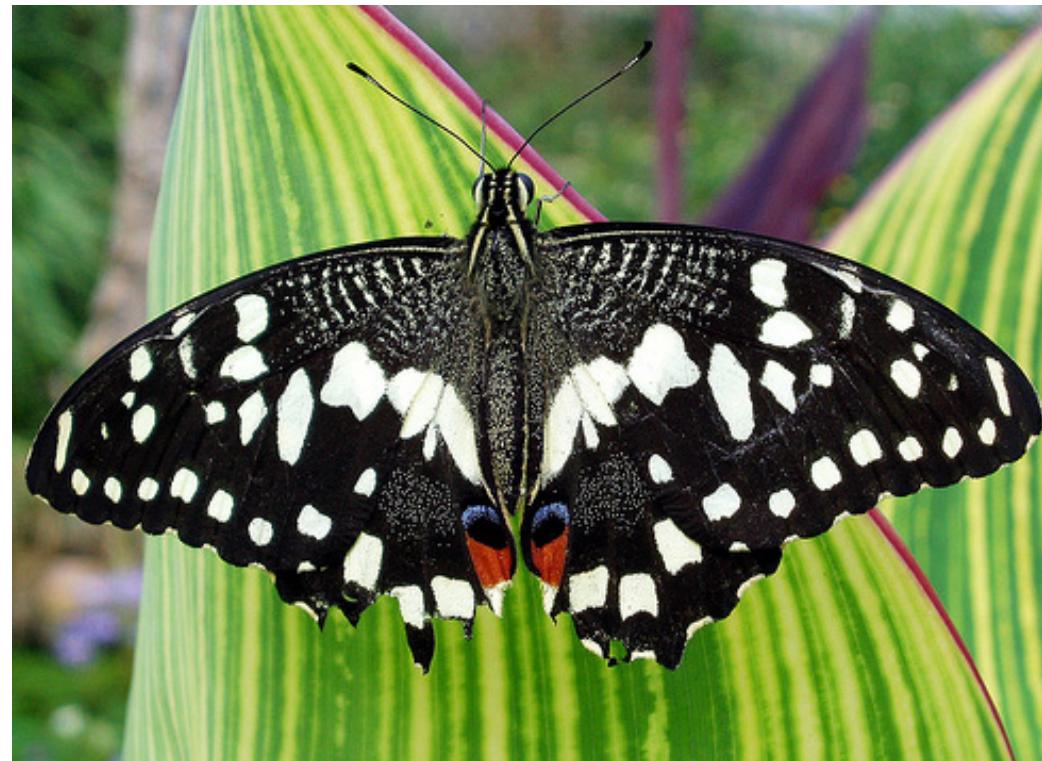
# LoG blob detector

- Convolve the image with scale-normalized LoG at several scales
- Find maxima of squared LoG response in scale-space



# LoG blob detector

- Example



# LoG blob detector

- Example: Squared Log response



$\sigma = 2$

# LoG blob detector

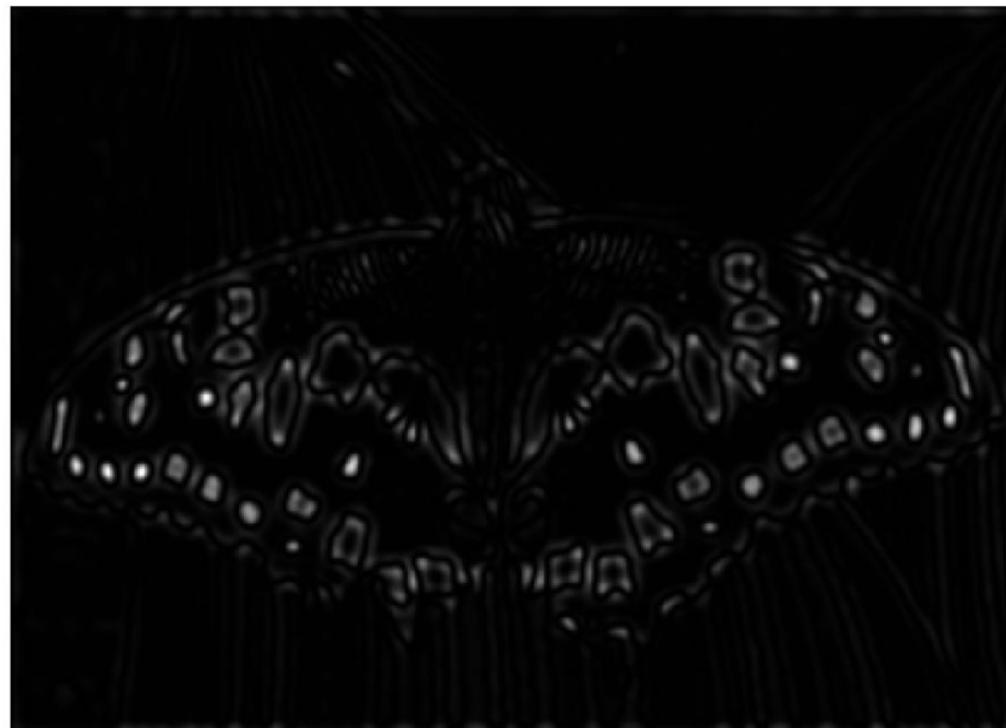
- Example: Squared Log response



sigma = 2.5018

# LoG blob detector

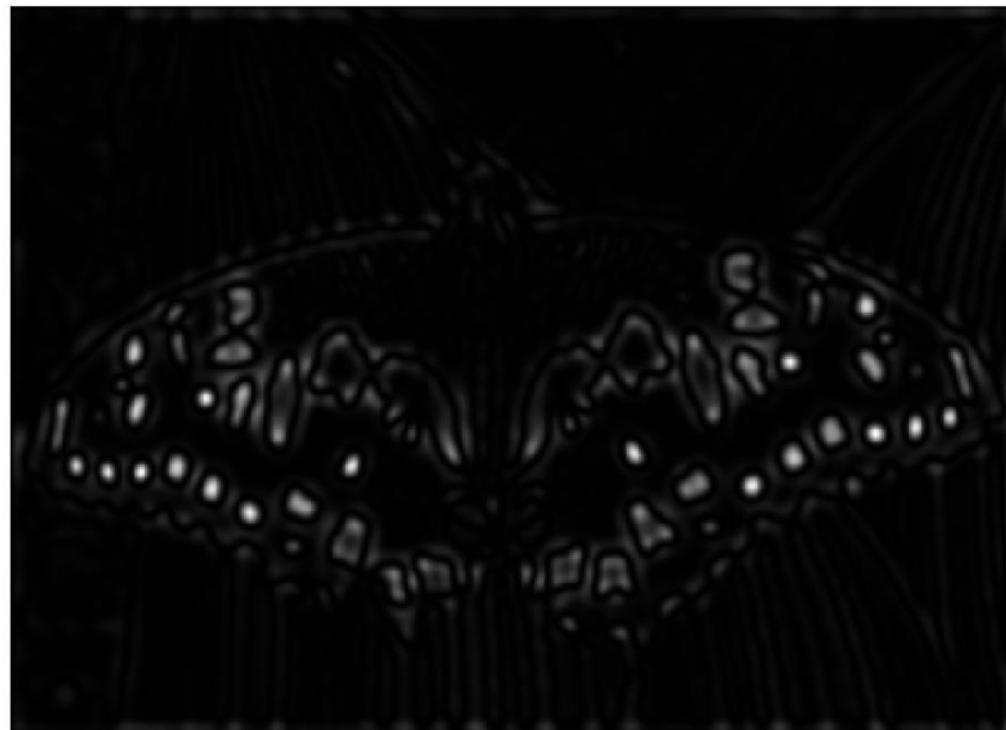
- Example: Squared Log response



$\sigma = 3.1296$

# LoG blob detector

- Example: Squared Log response



sigma = 3.9149

# LoG blob detector

- Example: Squared Log response



$\sigma = 4.8972$

# LoG blob detector

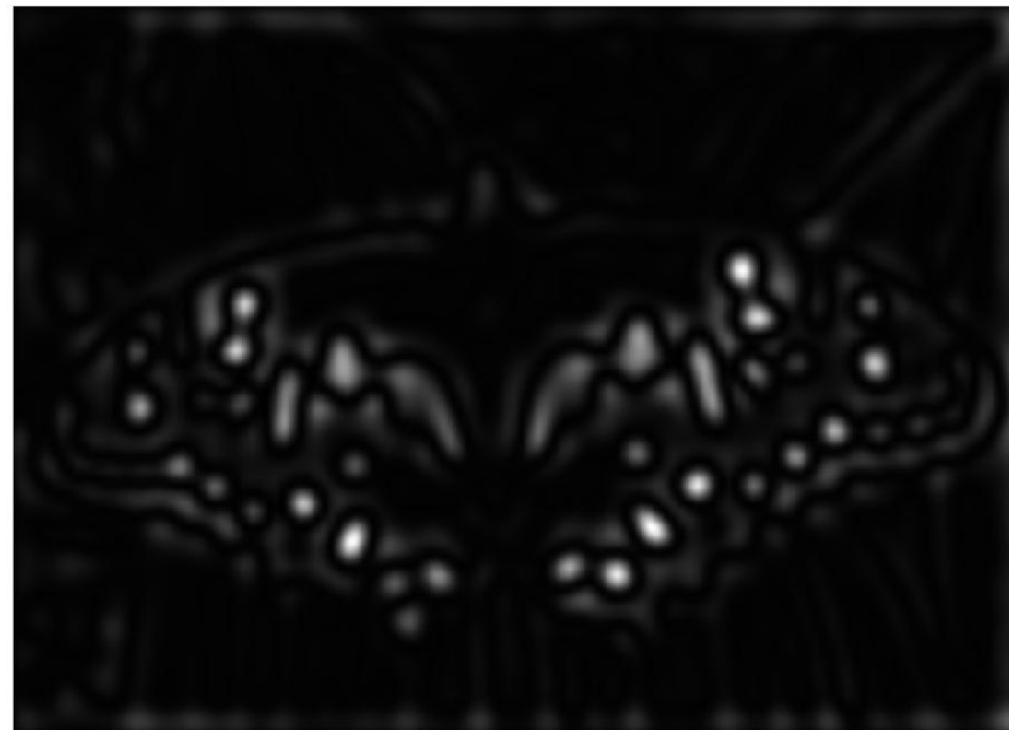
- Example: Squared Log response



$\sigma = 6.126$

# LoG blob detector

- Example: Squared Log response



sigma = 7.6631

# LoG blob detector

- Example: Squared Log response



sigma = 9.5859

# LoG blob detector

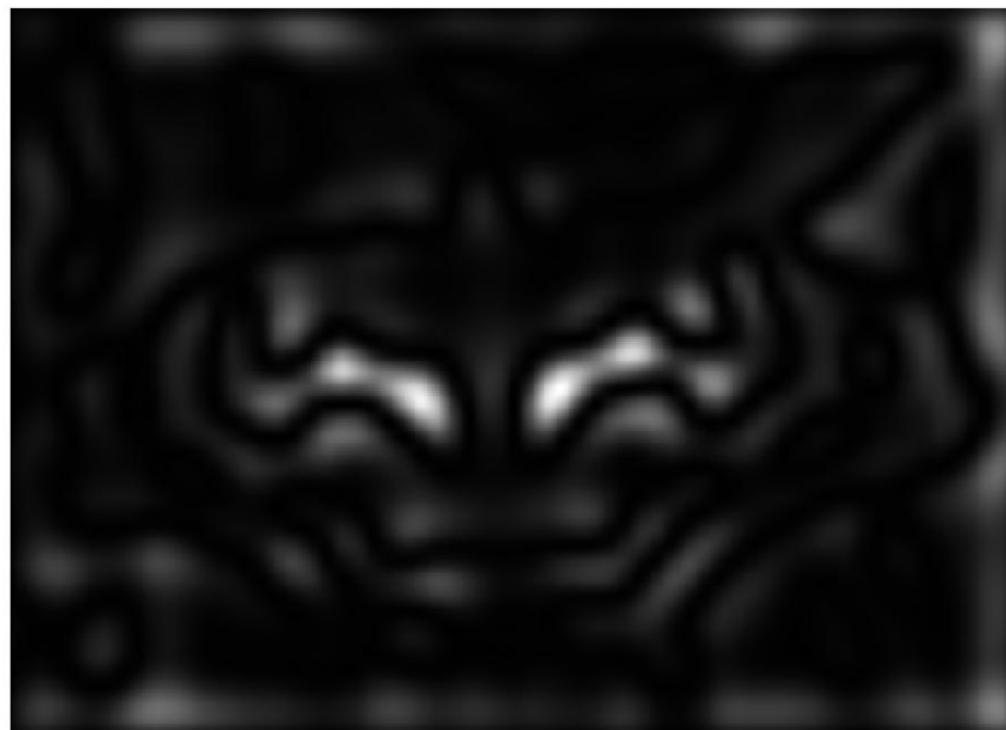
- Example: Squared Log response



$\sigma = 11.9912$

# LoG blob detector

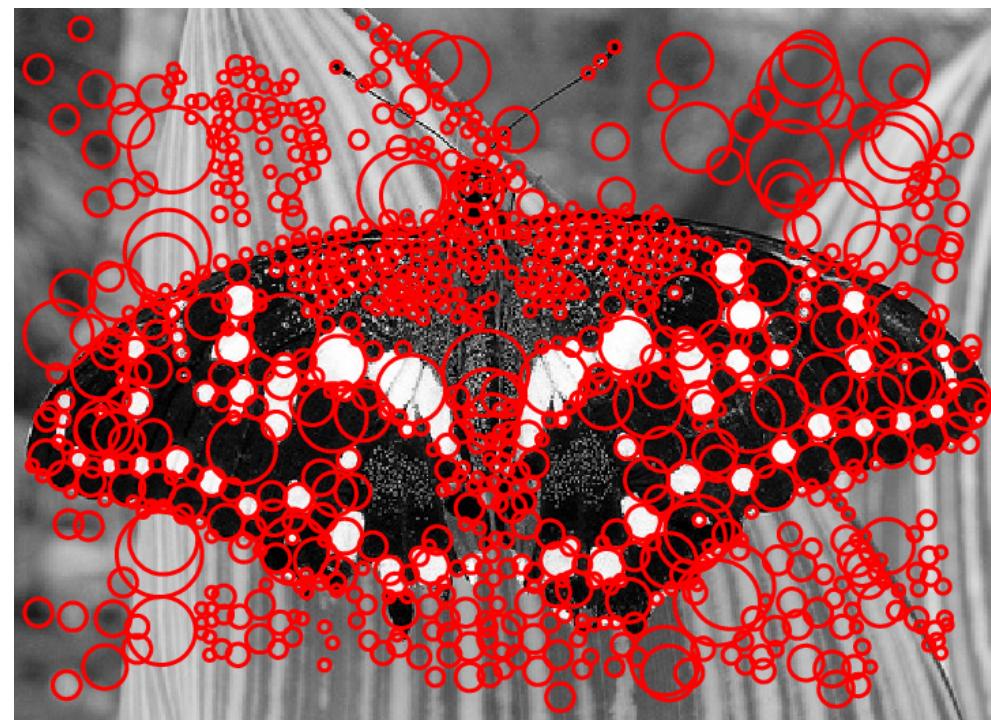
- Example: Squared Log response



$\sigma = 15$

# LoG blob detector

- Example: Detected blobs





# Efficient implementation

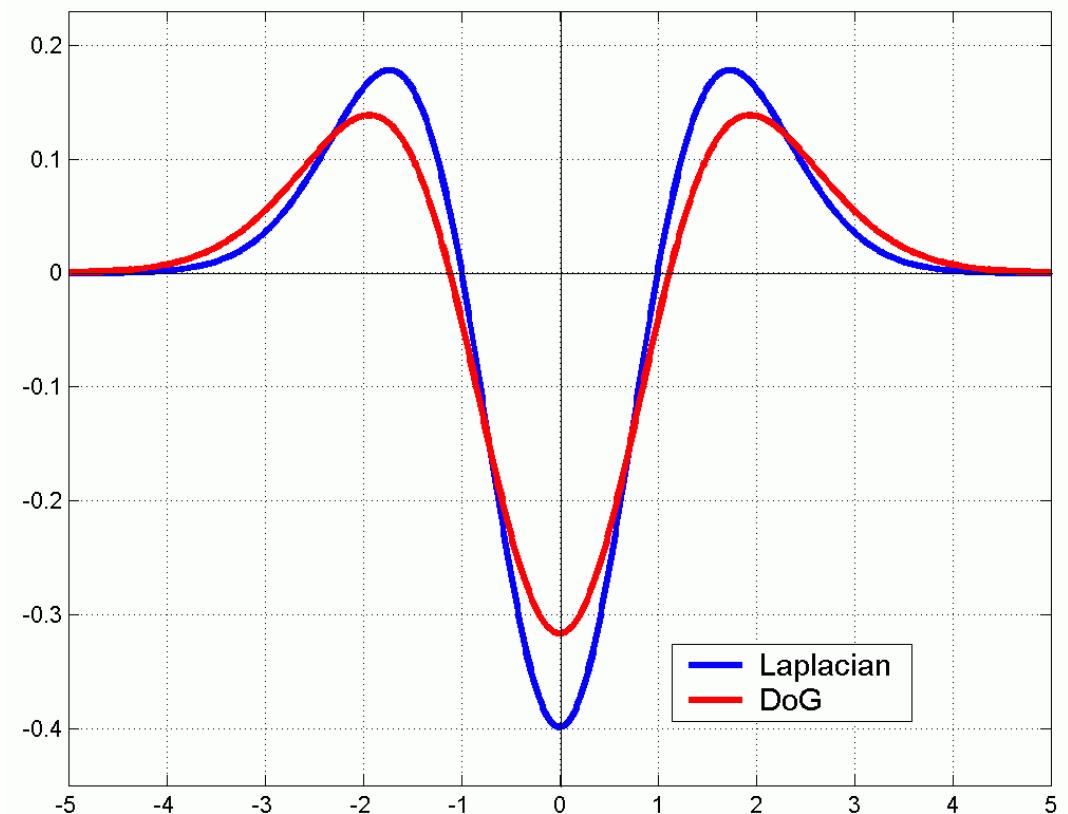
- Approximating the Laplacian with a difference of Gaussians (DoG)

$$L = \sigma^2 \left( G_{xx}(x, y, \sigma) + G_{yy}(x, y, \sigma) \right)$$

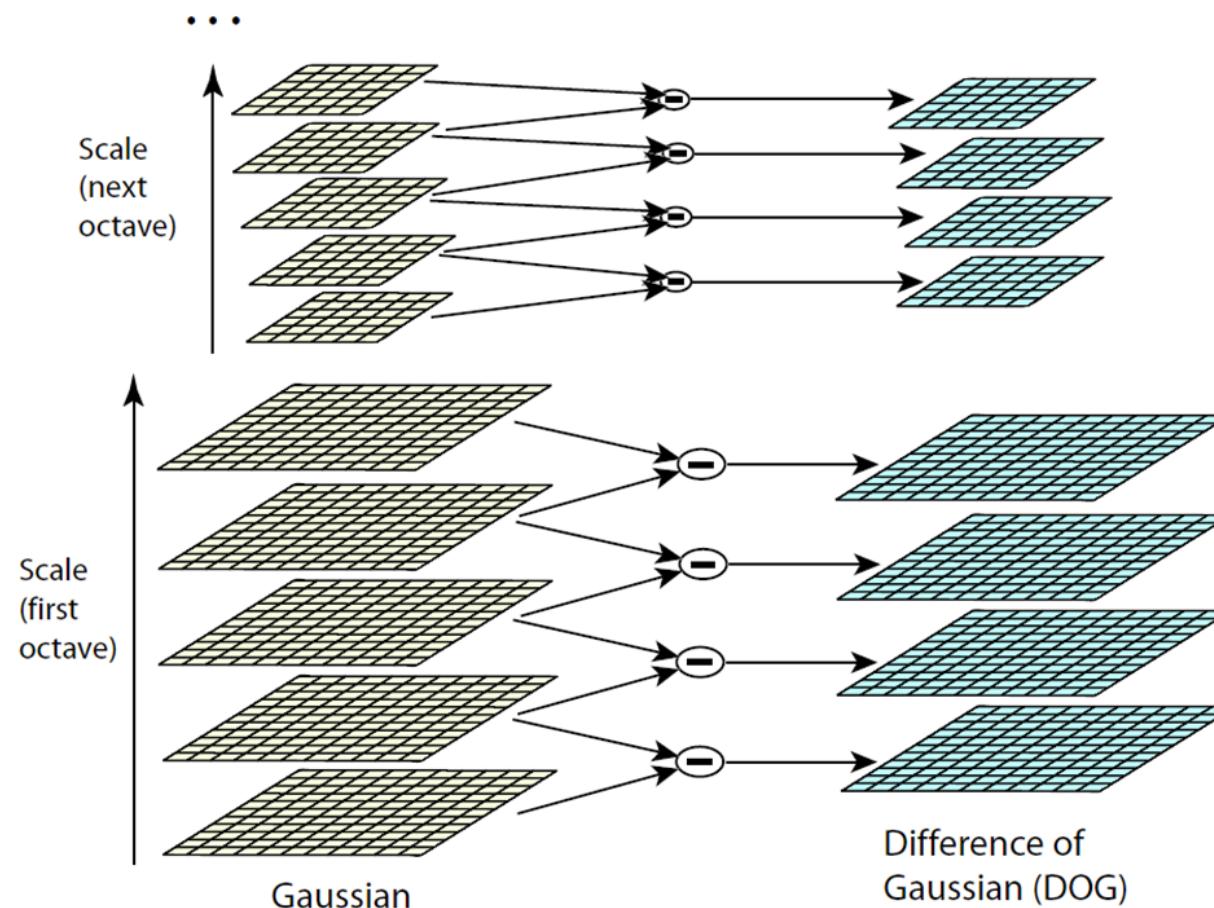
(Laplacian)

$$DoG = G(x, y, k\sigma) - G(x, y, \sigma)$$

(Difference of Gaussians)

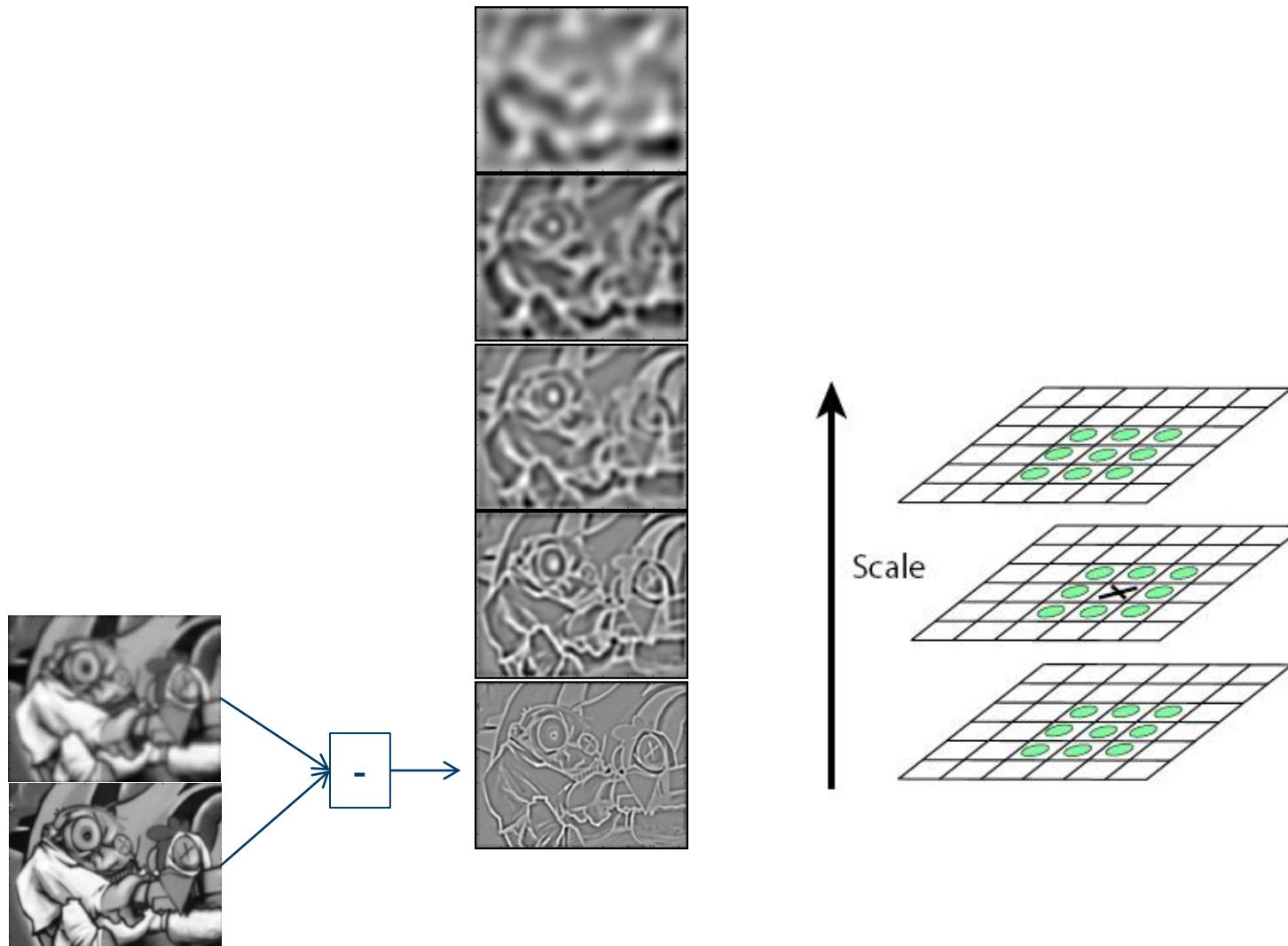


# Efficient implementation



David G. Lowe. "[Distinctive image features from scale-invariant keypoints.](#)"  
IJCV 60 (2), pp. 91-110, 2004.

# Difference of Gaussians (DoG)



# Summary

- Corner detectors
  - Stable in space
  - Min eigenvalue, Harris
- Blob detectors
  - Stable in scale and space
  - LoG, DoG
- Combine methods!
- Further reading
  - David G. Lowe, [“Distinctive image features from scale-invariant keypoints”](#)
  - T. Lindeberg, [“Feature detection with automatic scale selection”](#)
- Lab next week:
  - Implement and test feature detectors!