Features

Digital Visual Effects, Spring 2007

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Outline

- Features
- Harris corner detector
- SIFT
- Applications

Features

Features

- Properties of features
- Detector: locates feature
- Descriptor and matching metrics: describes and matches features

Desired properties for features

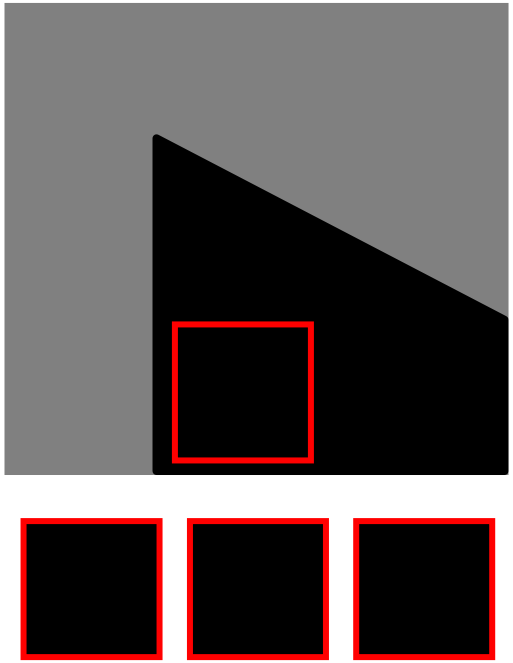
- Distinctive: a single feature can be correctly matched with high probability.
- Invariant: invariant to scale, rotation, affine, illumination and noise for robust matching across a substantial range of affine distortion, viewpoint change and so on. That is, it is repeatable.

Harris corner detector

Moravec corner detector (1980)

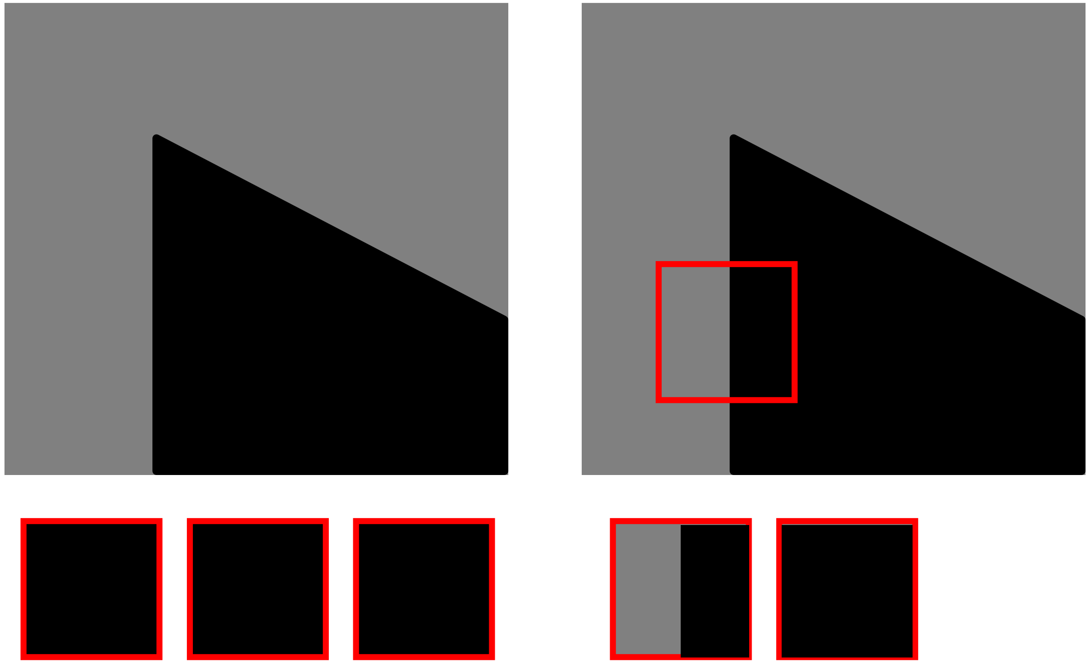
- We should easily recognize the point by looking through a small window
- Shifting a window in *any direction* should give *a large change* in intensity

Moravec corner detector



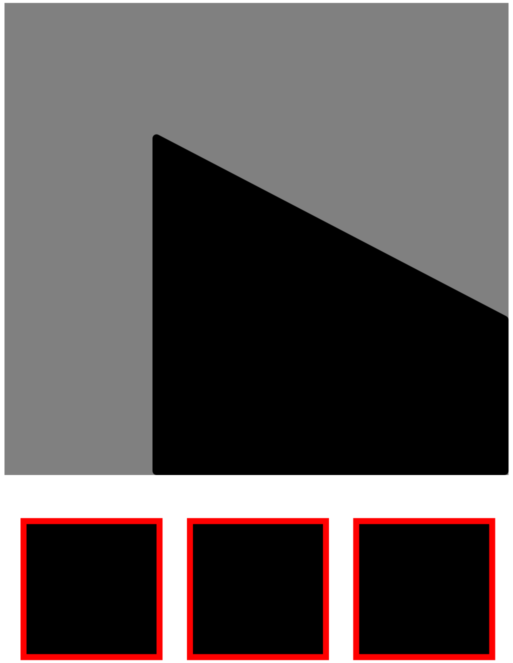
flat

Moravec corner detector

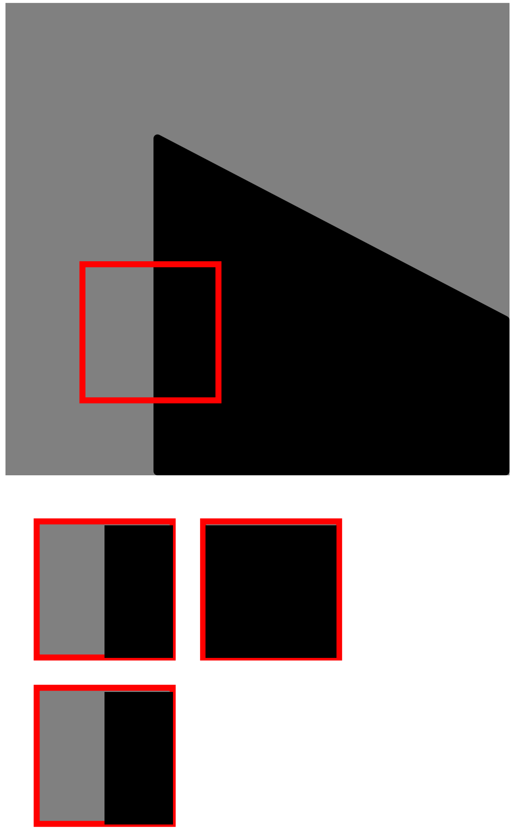


flat

Moravec corner detector

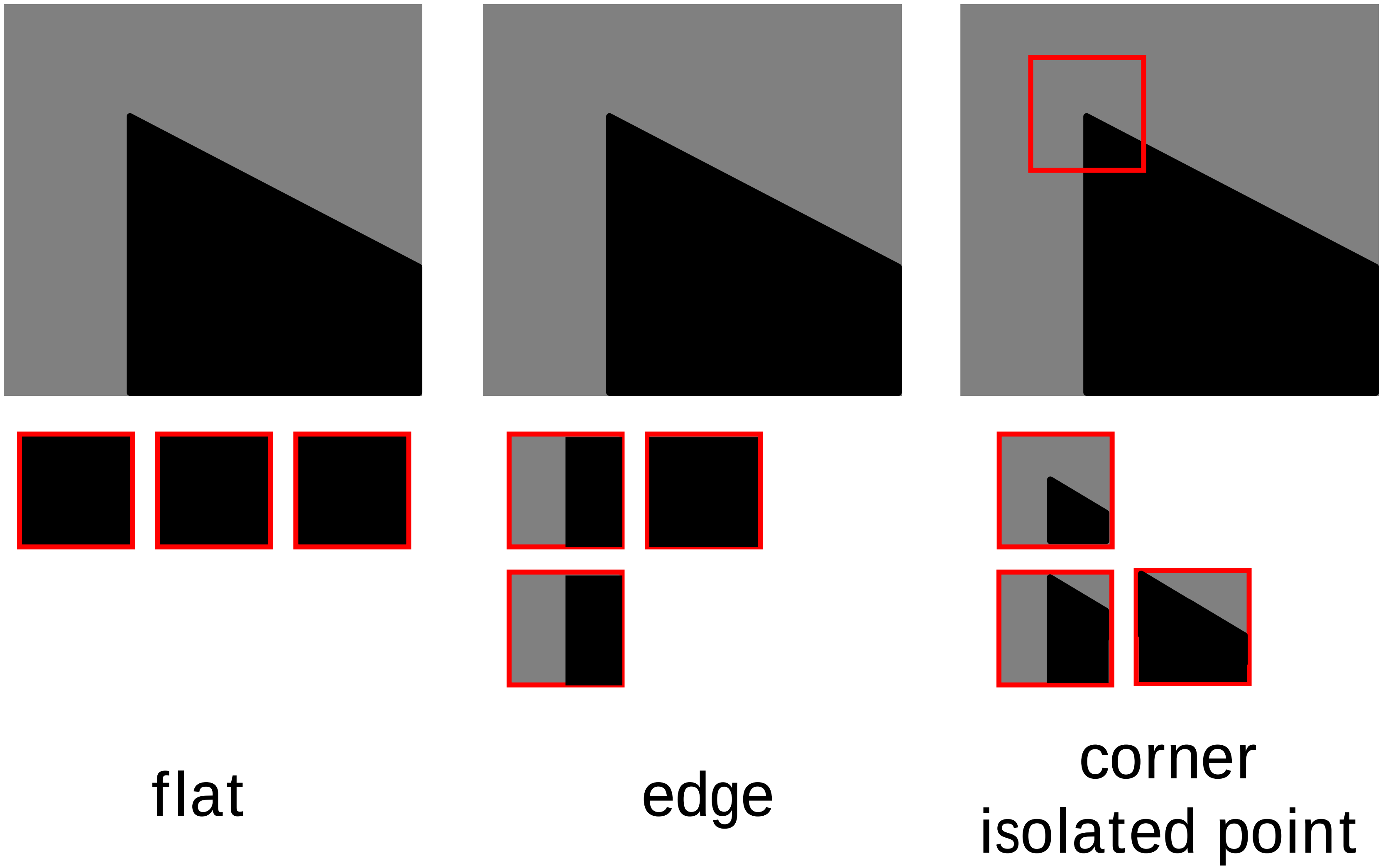


flat



edge

Moravec corner detector



Moravec corner detector

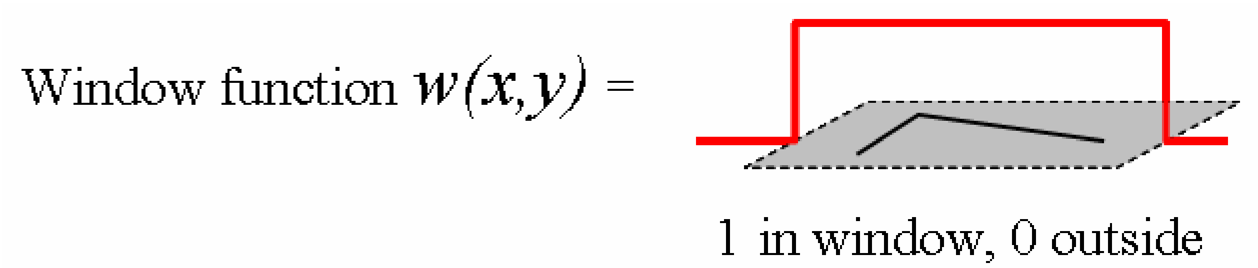
Change of intensity for the shift [*u,v*]:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

Window function

Shifted intensity

Intensity



Four shifts: (u,v) = (1,0), (1,1), (0,1), (-1, 1)

Look for local maxima in *min{E}*

Problems of Moravec detector

- Noisy response due to a binary window function
- Only a set of shifts at every 45 degree is considered
- Only minimum of E is taken into account

⇨ Harris corner detector (1988) solves these problems.

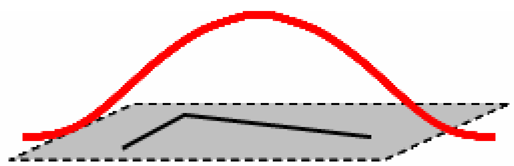
Harris corner detector

Noisy response due to a binary window function

- Use a Gaussian function

$$w(x,y) = \exp\left(-\frac{(x^2+y^2)}{2\sigma^2}\right)$$

Window function $w(x,y) =$



Gaussian

Harris corner detector

Only a set of shifts at every 45 degree is considered

- Consider all small shifts by Taylor's expansion

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

$$= \sum_{x,y} w(x,y)\left[I_x u + I_y v + O(u^2,v^2)\right]^2$$

$$E(u,v) = Au^2 + 2Cuv + Bv^2$$

$$A = \sum_{x,y} w(x,y) I_x^2(x,y)$$

$$B = \sum_{x,y} w(x,y) I_y^2(x,y)$$

$$C = \sum_{x,y} w(x,y) I_x(x,y) I_y(x,y)$$

Harris corner detector

Equivalently, for small shifts [u,v] we have a *bilinear* approximation:

$$E(u,v) \cong [u,v] \quad M \begin{bmatrix} u \\ v \end{bmatrix}$$

, where M is a 2×2 matrix computed from image derivatives:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Harris corner detector

Only minimum of E is taken into account

➢A new corner measurement

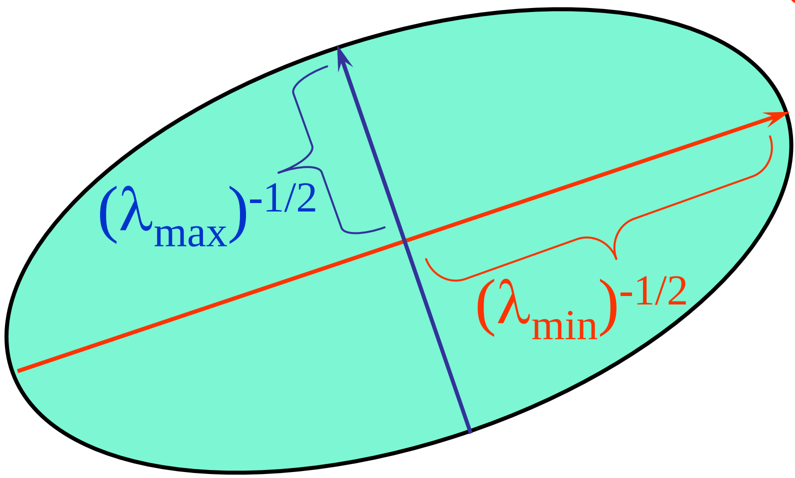
Harris corner detector

Intensity change in shifting window: eigenvalue analysis

$$E(u,v) \cong [u,v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

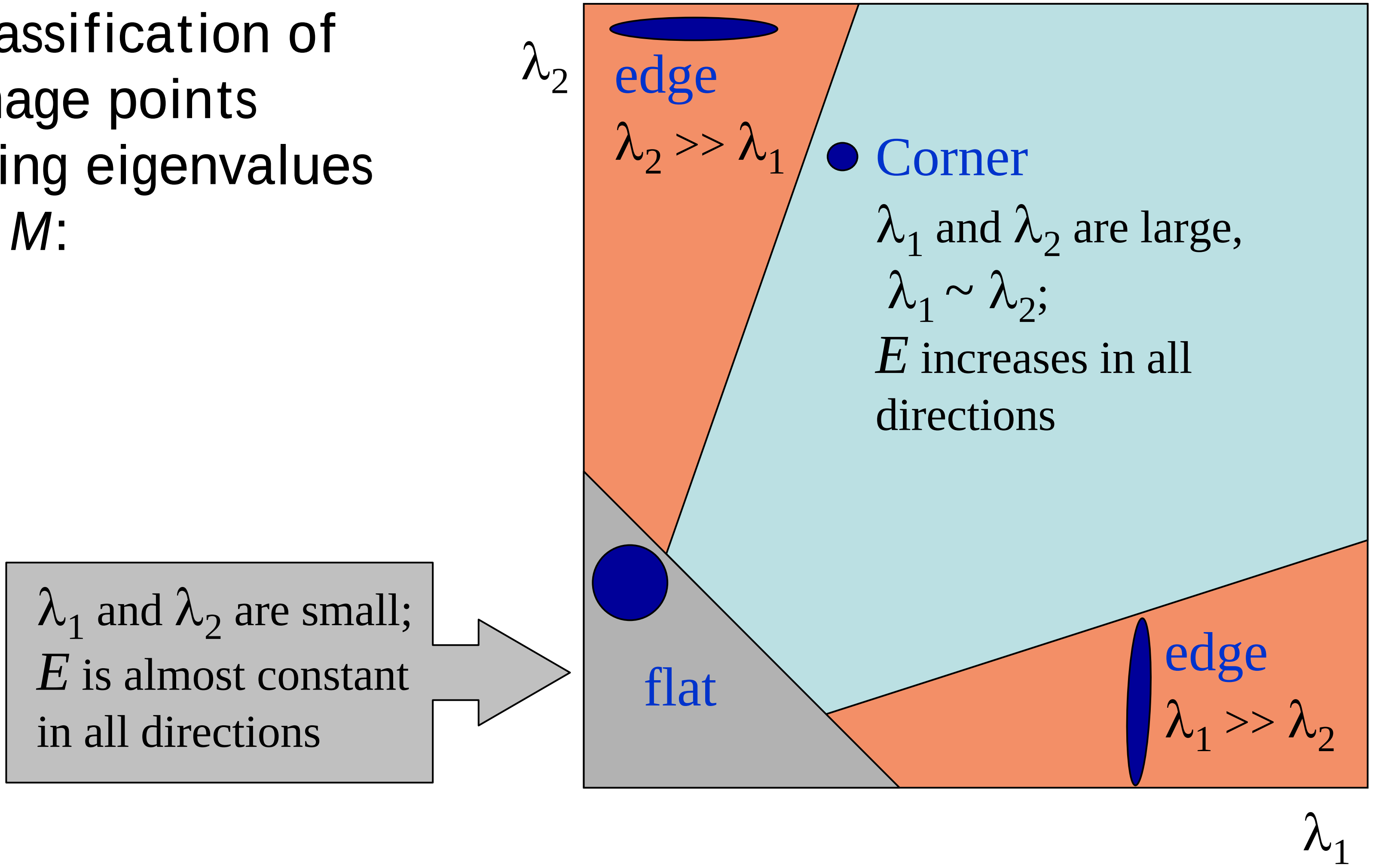
λ_1, λ_2 – eigenvalues of M

Ellipse $E(u,v)$ = const



Harris corner detector

Classification of image points using eigenvalues of *M*:



Harris corner detector

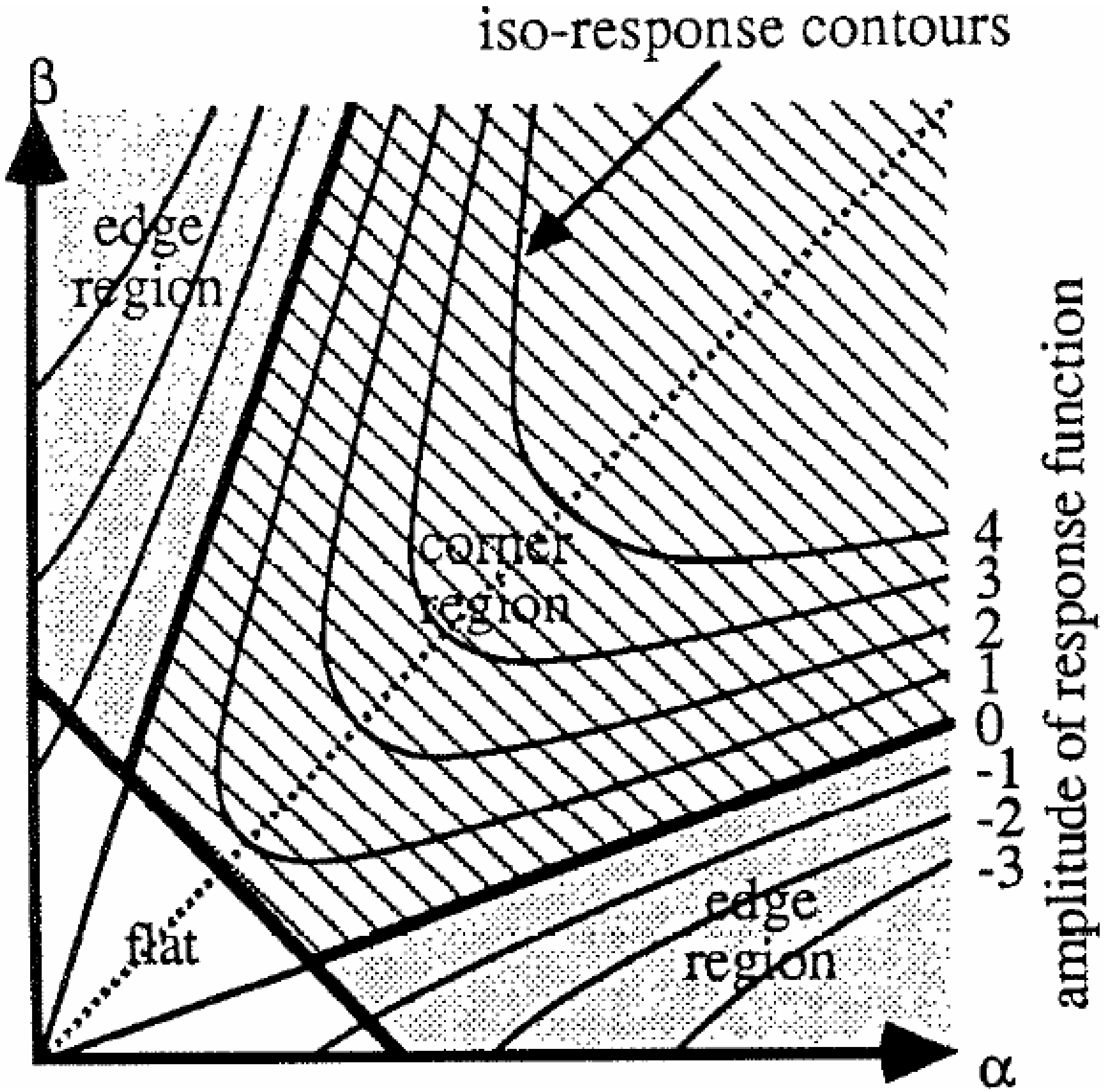
Measure of corner response:

$$R = \det M - k\left(\operatorname{trace} M\right)^2$$

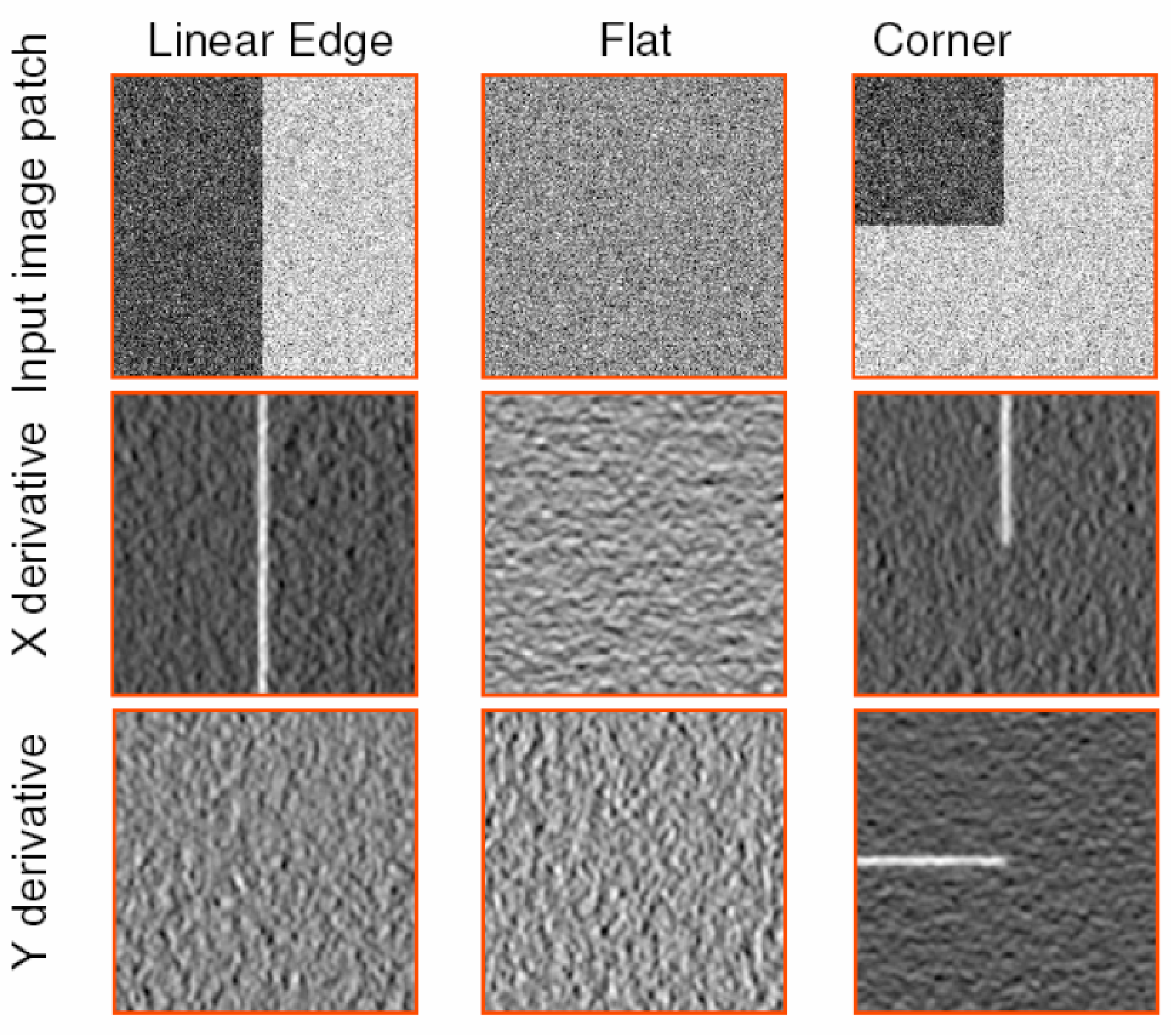
$$\det M = \lambda_1 \lambda_2$$
$$\operatorname{trace} M = \lambda_1 + \lambda_2$$

(k – empirical constant, k = 0.04-0.06)

Harris corner detector

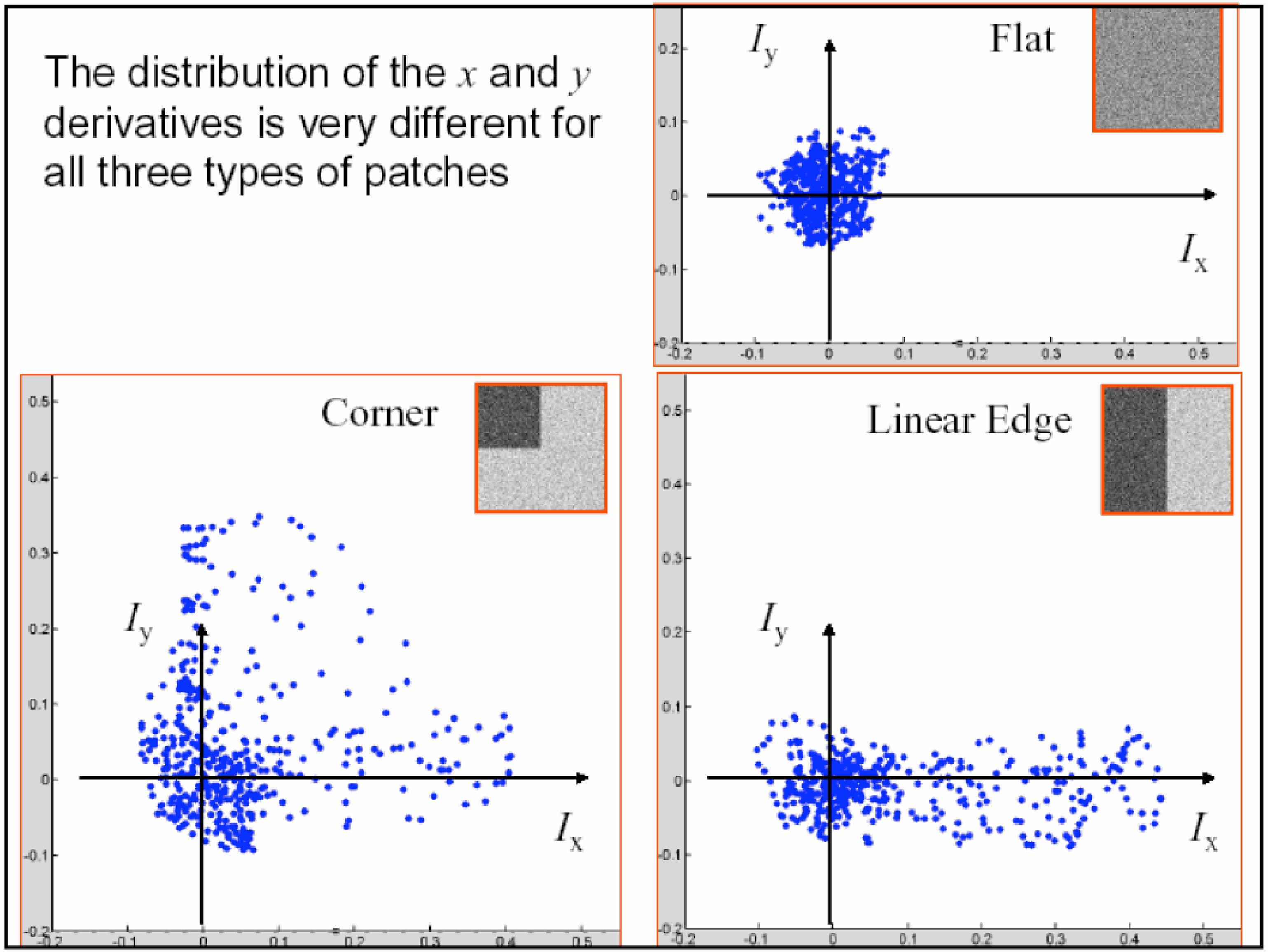


Another view



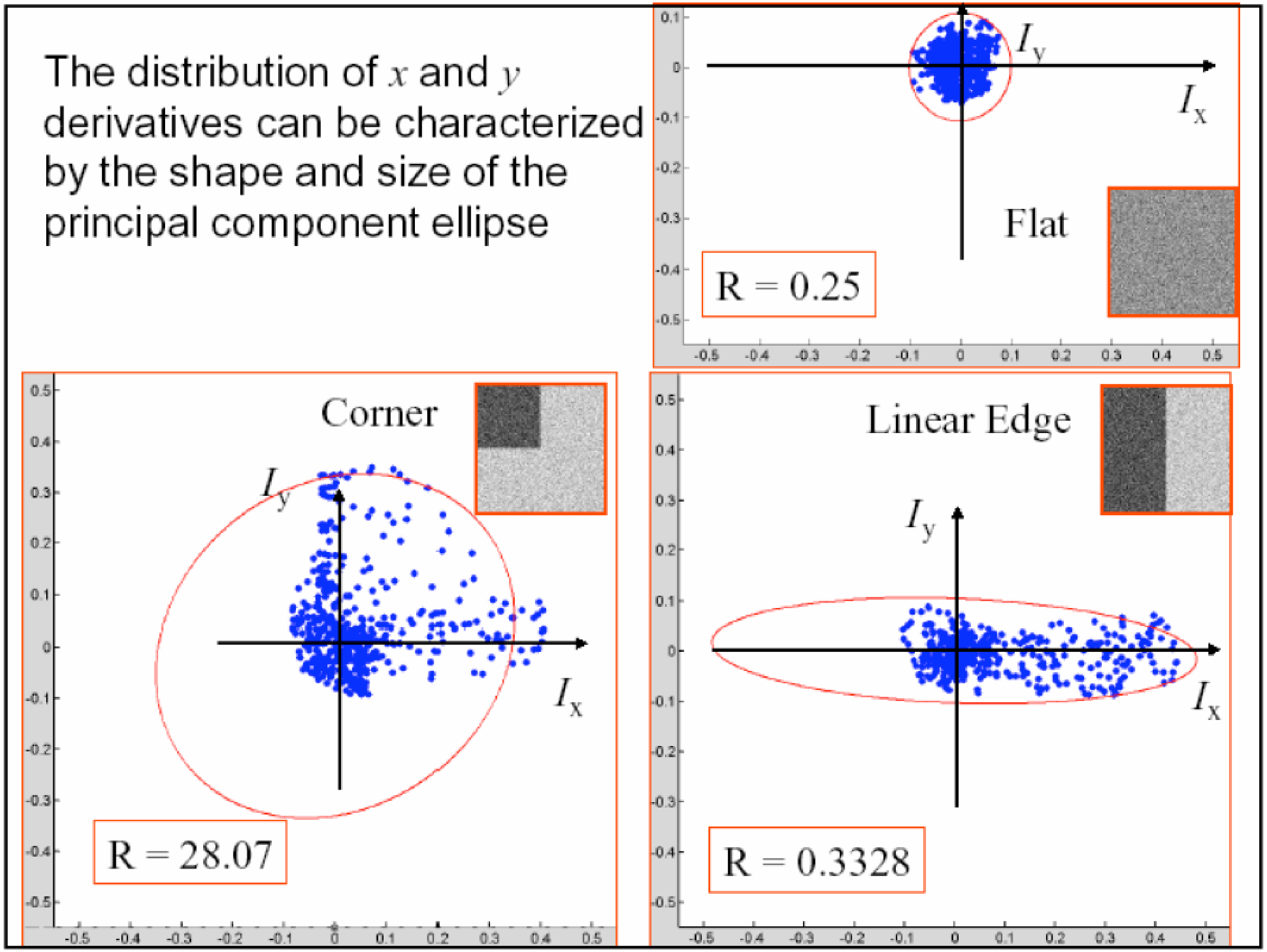
Another view

The distribution of the x and y derivatives is very different for all three types of patches



Another view

The distribution of x and y derivatives can be characterized by the shape and size of the principal component ellipse



Summary of Harris detector

1. Compute x and y derivatives of image

$$I_x = G_\sigma^x * I \quad I_y = G_\sigma^y * I$$

2. Compute products of derivatives at every pixel

$$I_{x2} = I_x.I_x \quad I_{y2} = I_y.I_y \quad I_{xy} = I_x.I_y$$

3. Compute the sums of the products of derivatives at each pixel

$$S_{x2} = G_{\sigma\prime} * I_{x2} \quad S_{y2} = G_{\sigma\prime} * I_{y2} \quad S_{xy} = G_{\sigma\prime} * I_{xy}$$

4. Define at each pixel (x, y) the matrix

$$H(x,y) = \begin{bmatrix} S_{x2}(x,y) & S_{xy}(x,y) \\ S_{xy}(x,y) & S_{y2}(x,y) \end{bmatrix}$$

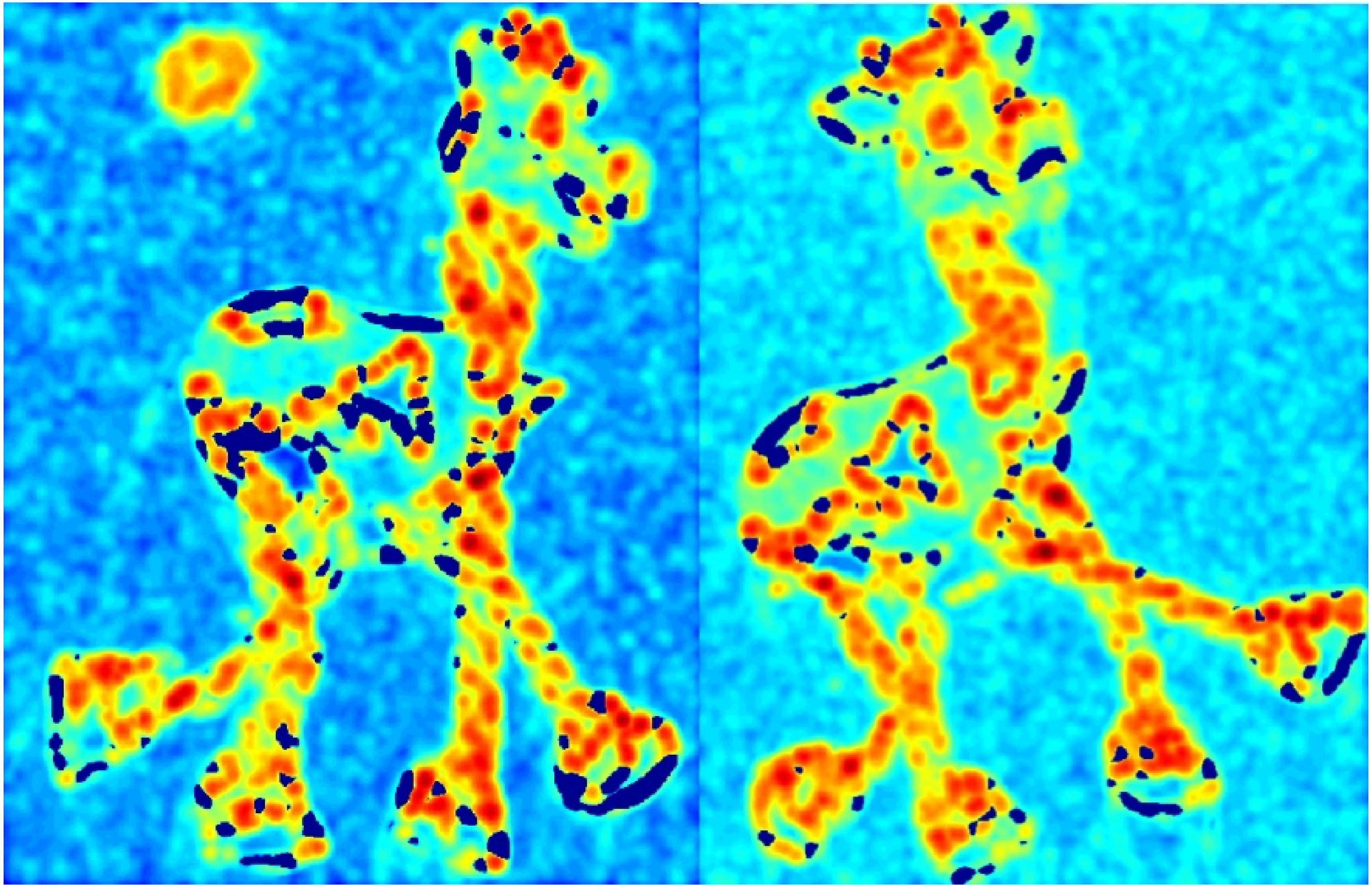
5. Compute the response of the detector at each pixel

$$R = Det(H) - k(Trace(H))^2$$

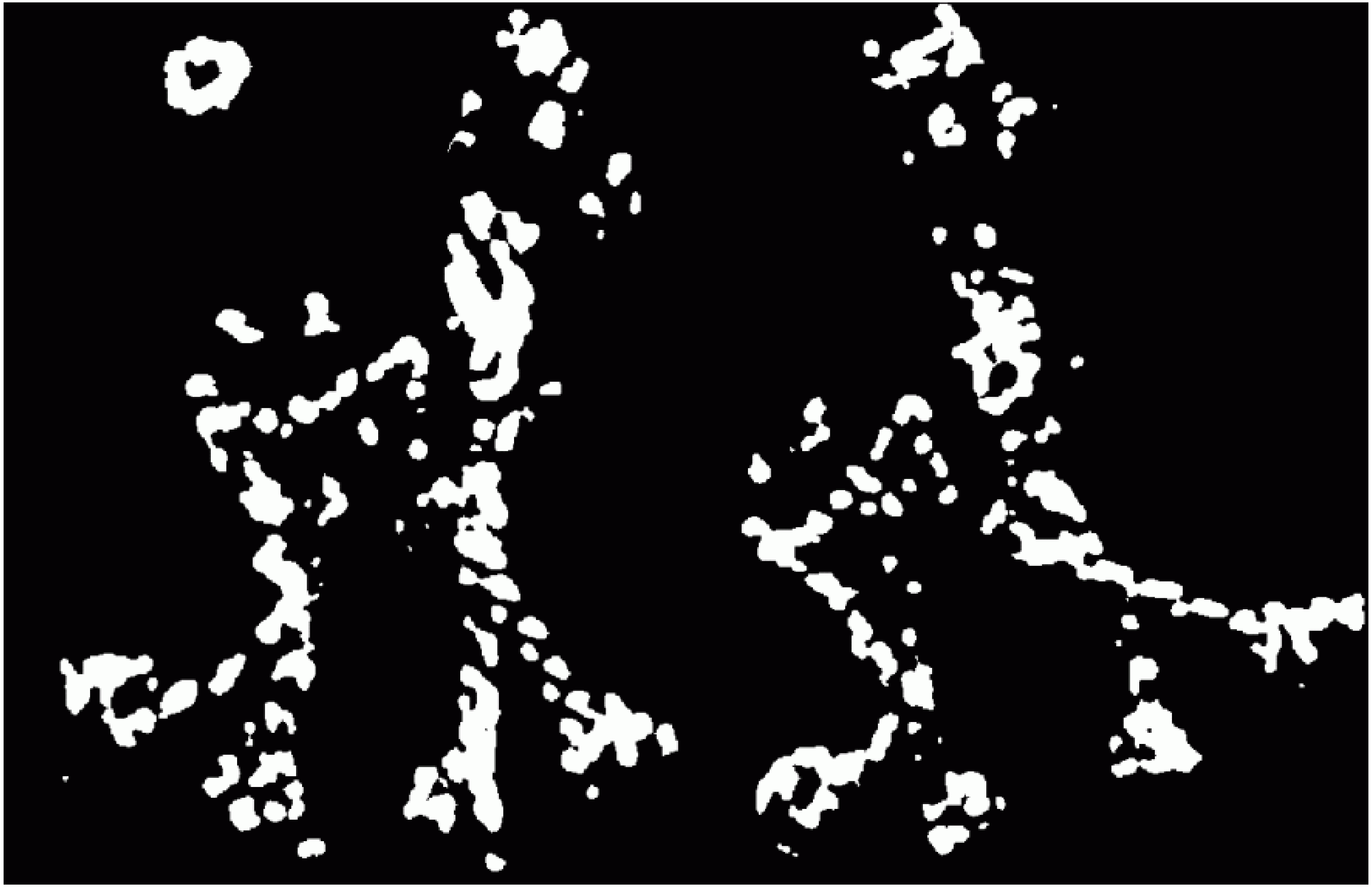
6. Threshold on value of R. Compute nonmax suppression.

Harris corner detector (input)

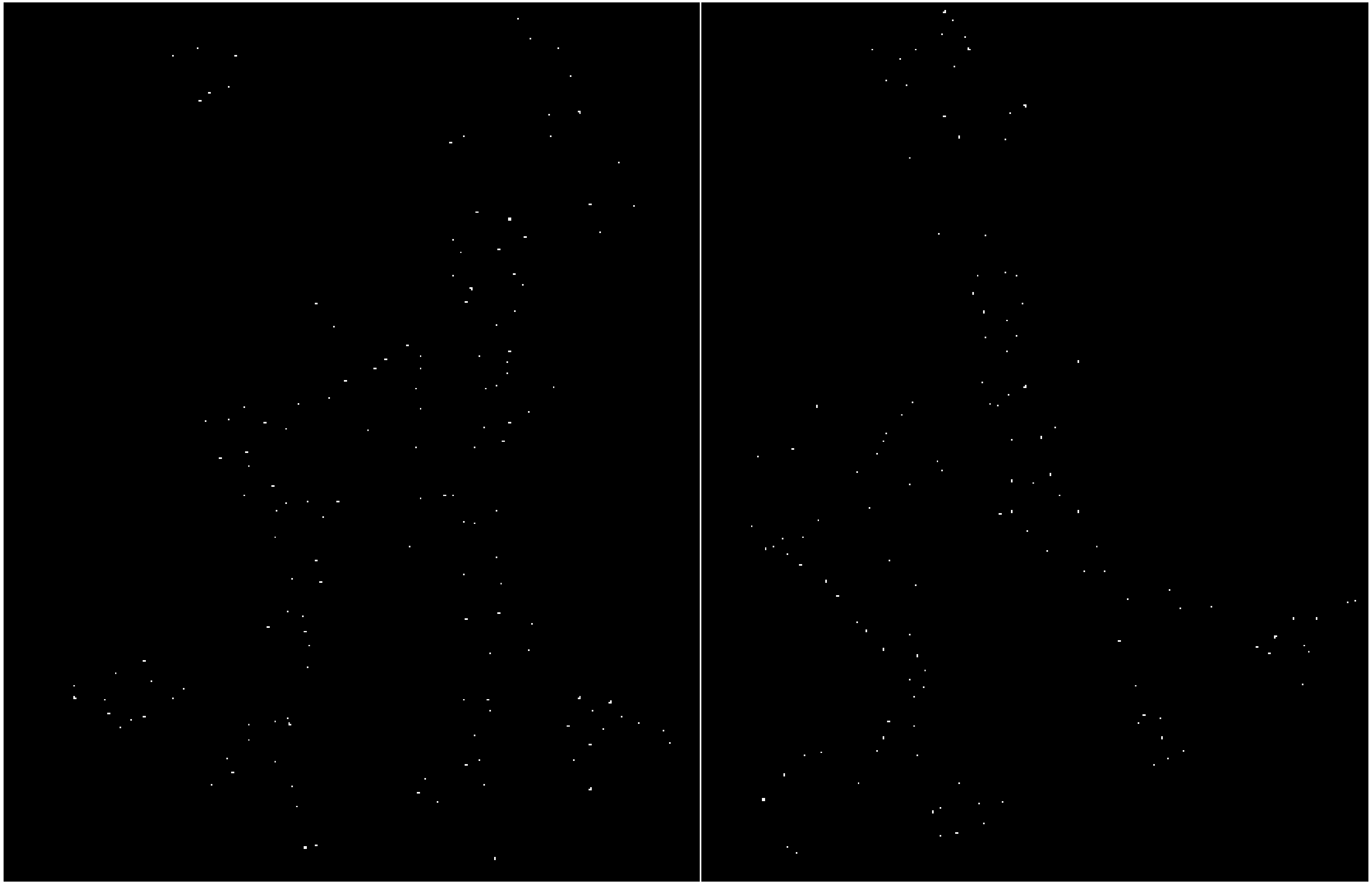
Corner response R



Threshold on R



Local maximum of R



Harris corner detector

Harris detector: summary

- Average intensity change in direction [u,v] can be expressed as a bilinear form:

$$E(u,v) \cong [u,v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

- Describe a point in terms of eigenvalues of M: *measure of corner response*

$$R = \lambda_1 \lambda_2 - k\left(\lambda_1 + \lambda_2\right)^2$$

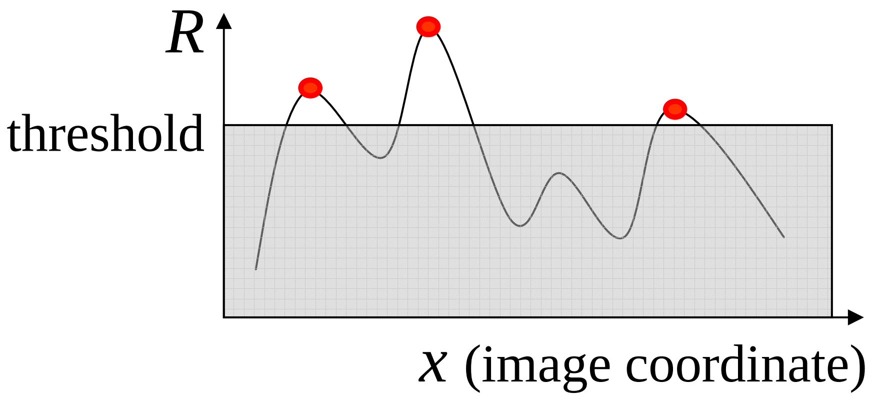
- A good (corner) point should have a *large intensity change* in *all directions*, i.e. R should be large positive

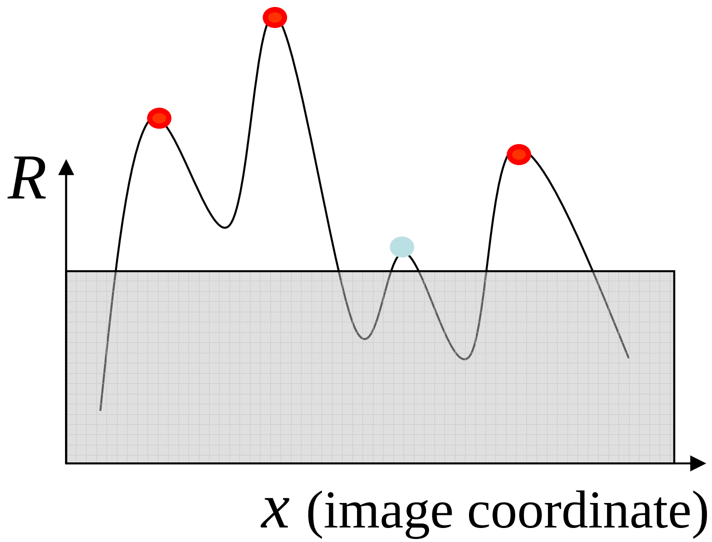
Harris detector: some properties

- Partial invariance to *affine intensity* change

✓ Only derivatives are used => invariance to intensity shift $I \to I + b$

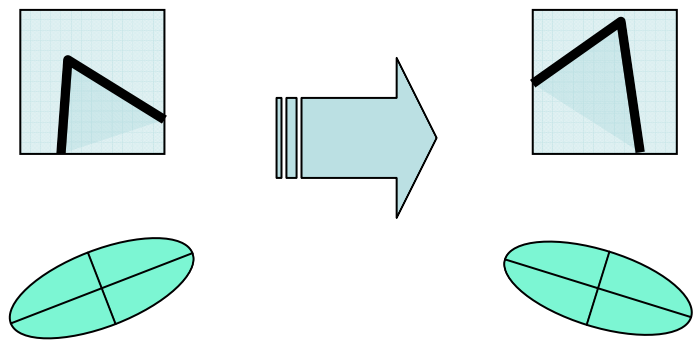
✓ Intensity scale: $I \to a\,I$





Harris Detector: Some Properties

- Rotation invariance



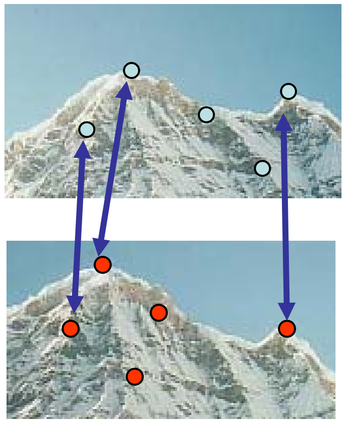
Ellipse rotates but its shape (i.e. eigenvalues) remains the same

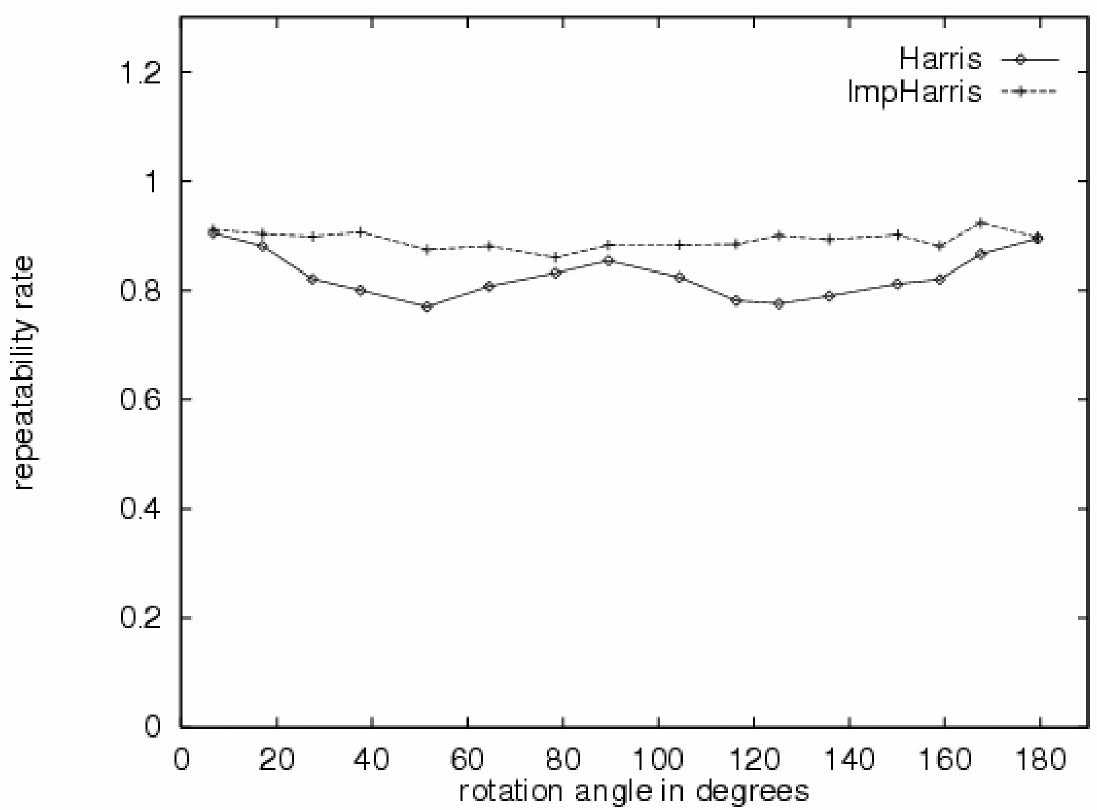
Corner response R is invariant to image rotation

Harris Detector is rotation invariant

Repeatability rate:

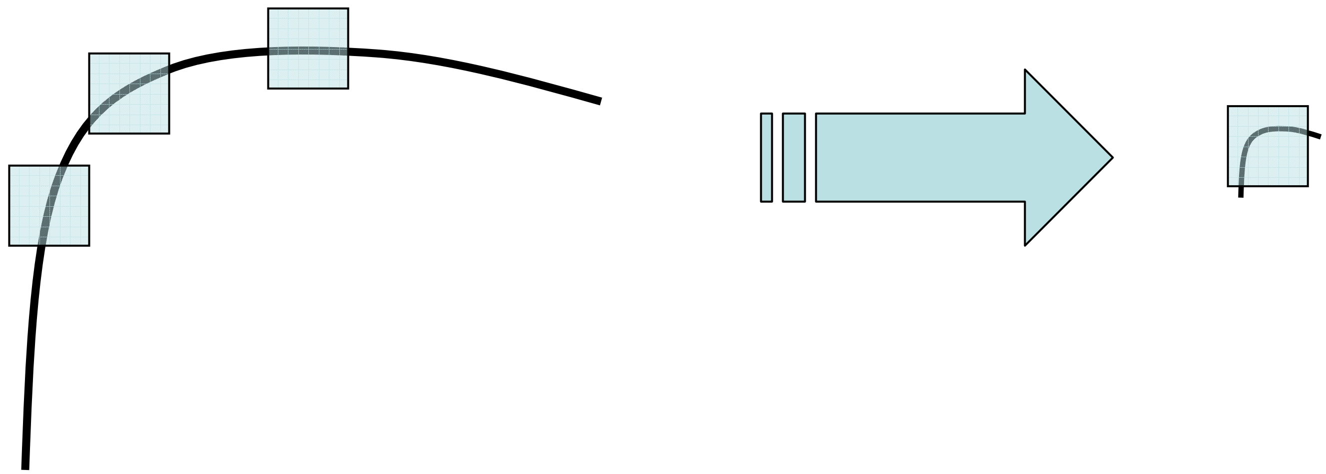
$$\frac{\text{\# correspondences}}{\text{\# possible correspondences}}$$





Harris Detector: Some Properties

- But: non-invariant to *image scale*!



All points will be classified as edges

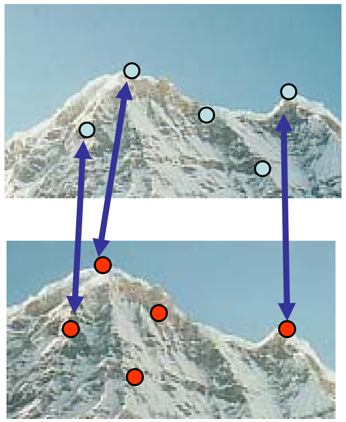
Corner !

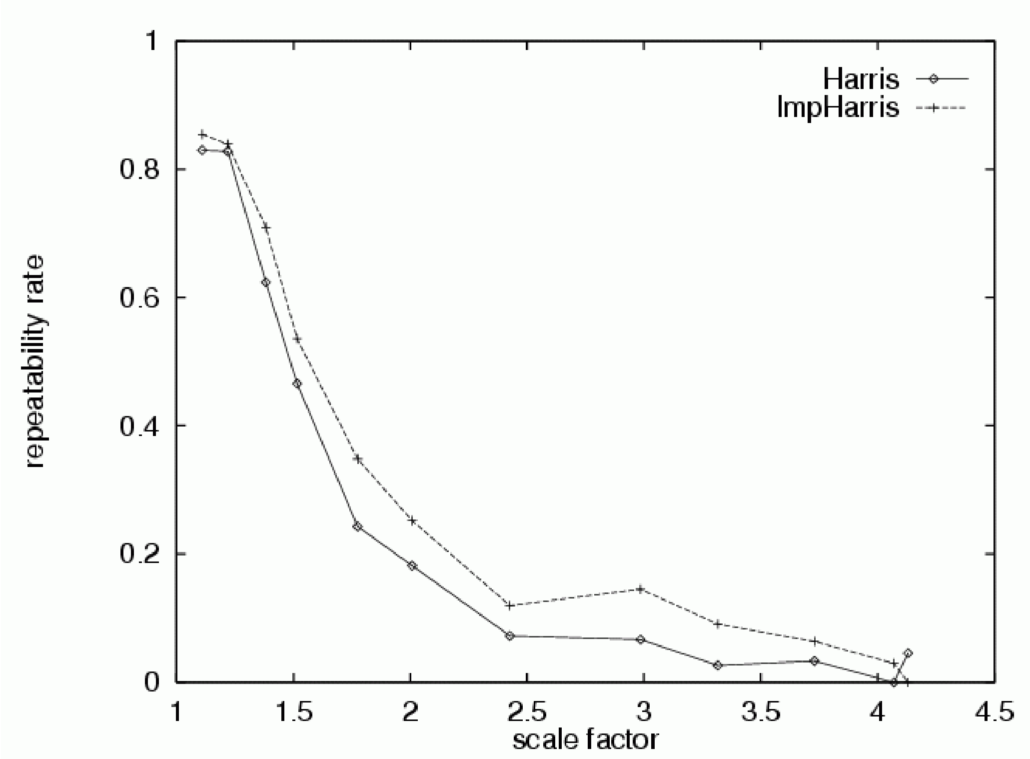
Harris detector: some properties

- Quality of Harris detector for different scale changes

Repeatability rate:

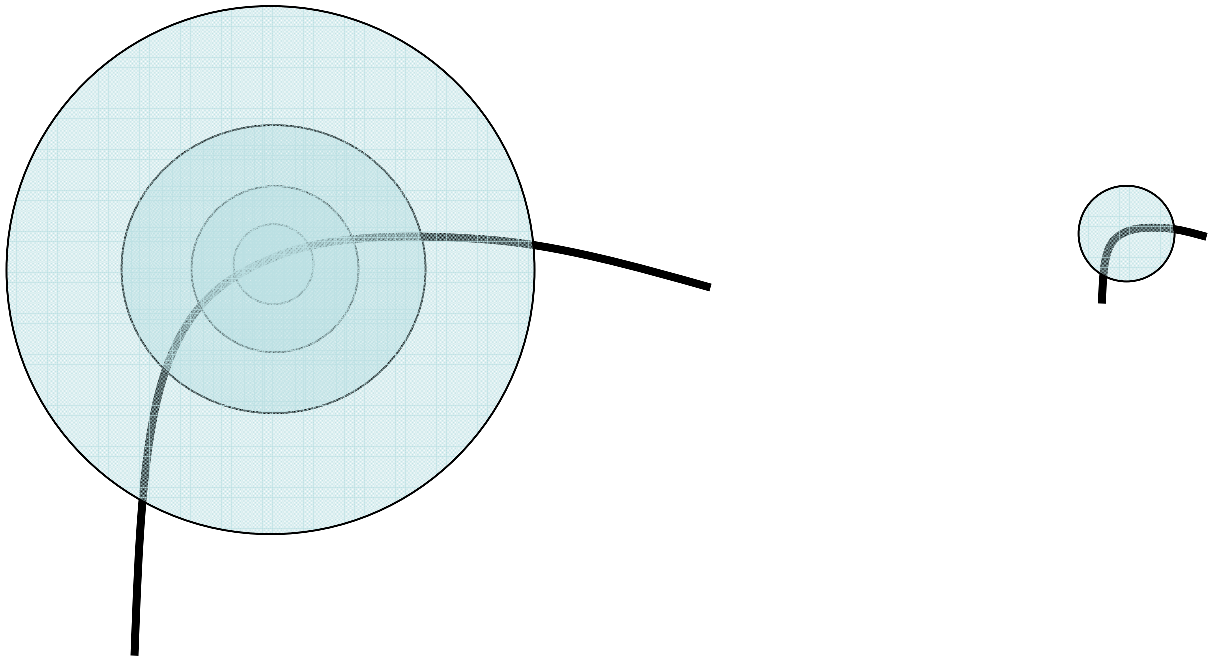
$$\frac{\text{\# correspondences}}{\text{\# possible correspondences}}$$





Scale invariant detection

- Consider regions (e.g. circles) of different sizes around a point
- Regions of corresponding sizes will look the same in both images



Scale invariant detection

- The problem: how do we choose corresponding circles *independently* in each image?
- Aperture problem