

Name: Solutions

Find the vector projection \mathbf{p} of $\mathbf{x} = (3, 5)^T$ onto $\mathbf{y} = (1, 1)^T$ and verify that \mathbf{p} and $\mathbf{x} - \mathbf{p}$ are orthogonal.

$$\begin{aligned}\bar{\mathbf{p}} &= \frac{\bar{\mathbf{x}}^T \bar{\mathbf{y}}}{\bar{\mathbf{y}}^T \bar{\mathbf{y}}} \bar{\mathbf{y}} = \frac{3+5}{1+1} \bar{\mathbf{y}} = 4\bar{\mathbf{y}} \\ &= \begin{pmatrix} 4 \\ 4 \end{pmatrix}.\end{aligned}$$

$$\Rightarrow \bar{\mathbf{x}} - \bar{\mathbf{p}} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\Rightarrow \bar{\mathbf{p}}^T (\bar{\mathbf{x}} - \bar{\mathbf{p}}) = (-4) + 4 = 0.$$