

Name: Solutions

Consider the  $3 \times 3$  matrix  $A = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ -1 & -2 & -3 \end{pmatrix}$ .

1. Find  $A^{-1}$  (the inverse of  $A$ ).

$$\begin{aligned}
 & \left( \begin{array}{ccc|ccc} \boxed{1} & 0 & 1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 & 1 & 0 \\ -1 & -2 & -3 & 0 & 0 & 1 \end{array} \right) \Rightarrow \left( \begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & \boxed{1} & 2 & 1 & 1 & 0 \\ 0 & -2 & -2 & 1 & 0 & 1 \end{array} \right) \\
 & \Rightarrow \left( \begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 2 & 3 & 2 & 1 \end{array} \right) \Rightarrow \left( \begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & \boxed{1} & 3/2 & 1 & 1/2 \end{array} \right) \\
 & \Rightarrow \left( \begin{array}{ccc|ccc} 1 & 0 & 0 & -1/2 & -1 & -1/2 \\ 0 & 1 & 0 & -2 & -1 & -1 \\ 0 & 0 & 1 & 3/2 & 1 & 1/2 \end{array} \right) \Rightarrow A^{-1} = \begin{pmatrix} -1/2 & -1 & -1/2 \\ -2 & -1 & -1 \\ 3/2 & 1 & 1/2 \end{pmatrix}
 \end{aligned}$$

2. What does the fact that  $A$  is invertible tell you about the solution set of  $A\mathbf{x} = \mathbf{b}$ ?

The system  $A\bar{\mathbf{x}} = \bar{\mathbf{b}}$  has exactly one solution, which is  $\bar{\mathbf{x}} = A^{-1}\bar{\mathbf{b}}$ .