Name: Solutions

A vector $\mathbf{x}$ can be written as $\mathbf{x} = 2\mathbf{u}_1 + \mathbf{u}_2$ with respect to the basis $\{\mathbf{u}_1, \mathbf{u}_2\}$ having

$$\mathbf{u}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \text{and} \quad \mathbf{u}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

Determine c_1 and c_2 so that $\mathbf{x} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2$ with respect to the basis $\{\mathbf{v}_1, \mathbf{v}_2\}$ having

$$\mathbf{v}_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad \text{and} \quad \mathbf{v}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

In the standard basis, we have

$$\bar{x} = 2\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}.$$

So then we have

$$\begin{pmatrix} 3 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$\Rightarrow c_1 = 1, \; c_2 = 3.$$