

Name: Solutions

Determine whether the following linear system is consistent or inconsistent. If the system is consistent, find all of its solutions.

$$\begin{cases} x_1 + 2x_2 + x_3 = 4 \\ x_1 + 4x_2 + 3x_3 = 6 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 1 & 4 & 3 & 6 \end{array} \right)$$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 2 & 2 & 2 \end{array} \right)$$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 1 & 1 & 1 \end{array} \right) \quad \begin{array}{l} \text{row} \\ \text{echelon} \\ \text{form} \end{array}$$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & 1 & 1 \end{array} \right) \quad \begin{array}{l} \text{reduced} \\ \text{row echelon} \\ \text{form} \end{array}$$

↑
 x_3 is free

$$\Rightarrow \text{If } x_3 = \alpha, \text{ then } x_1 = 2 + \alpha \text{ and} \\ x_2 = 1 - \alpha.$$