

## Floating Point Numbers

Explain how floating point numbers are represented in a computer in terms of the sign, mantissa and exponent. Find the binary representation of the number 123. Find the exact binary representation of the number  $5/9$ . Use the round-to-nearest rule to determine the double precision floating point representation of the number  $5/9$ .

## Programming Skills

Understand the basic control structures of Matlab, such as `if-else` blocks and `for` loops. Know the various ways in which Matlab indexes matrices and vectors. Be able to define a Matlab function and read an existing function to understand its behavior.

## Bisection Method

Write down the bisection method. Use the intermediate value theorem to prove that  $f(x) = x^3 + x - 1$  has a root on the interval  $[0, 1]$ . Starting with this interval, how many iterations must you perform in order to approximate the root with 10 correct decimal places?

## Fixed Point Iteration

Find at least three functions that you can iterate on to find a root of  $f(x) = x^3 + x - 1$ . Define local convergence. Under what conditions does a fixed-point iteration locally converge? Evaluate the convergence properties of the iterations that you have found previously. What are three possible stopping criteria?

## Newton's Method

Write down Newton's method. What does it mean for an iterative method to converge quadratically? Prove that Newton's method exhibits local quadratic convergence to a simple root. What happens in the case of a multiple root?