

1. *Jacobi's Method and Stationary Iterative Methods.* Write down the step equation for Jacobi's method. What are the conditions for convergence? What are the conditions of convergence for an arbitrary stationary iterative method? What is the rate of convergence? What is the computational cost of Jacobi's method? Why might Jacobi's method be a good choice for a sparse matrix?
2. *Polynomial Interpolation.* Suppose that you are given the data

$$(x, y) = (0, 1), (1, 2), (2, 0), (3, 1).$$

Write down a system of equations using the Vandermonde matrix that you could solve for the coefficients of the degree at most 3 polynomial $P(x)$ interpolating these data points. Evaluate $P(0.5)$ using the Lagrange interpolating polynomials. Evaluate $P(0.5)$ using Newton's divided differences. What are the advantages and disadvantages of each approach? Suppose furthermore that the data points are sampled from a function $f(x)$. Find an upper bound the error $\varepsilon(x) = |f(x) - P(x)|$ at $x = -\pi/4$ when interpolating $f(x) = \cos(x)$ at 5 equally spaced points between $-\pi$ and π . Discuss the advantages and disadvantages of using the chebyshev points for interpolation.

3. *Spline Interpolation* Evaluate $L(0.5)$ where L is the linear spline interpolating the data above. What is the form of a cubic spline? What are the regularity properties that a cubic spline must satisfy? What types of boundary conditions are possible for cubic splines? List some of the trade-offs when using spline interpolation instead of global polynomial interpolation.