

Computer problems from the textbook contain “CP” in the exercise number. For these problems, remember to adequately label all plots and include code that you have written along with your solutions. All code that you include should be properly explained. Do all other problems by hand and make sure to your work. A clear and complete presentation of your solutions is required for full credit.

1. (Sauer §5.1, #2) Use the three-point centered-difference formula to approximate $f'(0)$, where $f(x) = e^x$, for (a) $h = 0.1$ (b) $h = 0.01$ (c) $h = 0.001$.
2. (Sauer §5.1, #6) Use the three-point centered-difference formula for the second derivative to approximate $f''(0)$, where $f(x) = \cos x$, for (a) $h = 0.1$ (b) $h = 0.01$ (c) $h = 0.001$. Find the approximation error.
3. (Sauer §5.1, #8) Prove the second-order formula for the first derivative

$$f'(x) = \frac{-f(x+2h) + 4f(x+h) - 3f(x)}{2h} + O(h^2).$$

4. (Sauer §5.1, #10) Find the error term and order for the approximation formula

$$f'(x) \approx \frac{4f(x+h) - 3f(x) - f(x-2h)}{6h}.$$

5. (Sauer §5.1, #13) Develop a second-order method for approximating $f'(x)$ that uses the data $f(x-h)$, $f(x)$, and $f(x+3h)$ only.
6. (Sauer, §5.2, CP1) Use the composite Trapezoid Rule with $m = 16$ and 32 panels to approximate the definite integral. Compare with the correct integral and report the two errors.

(a) $\int_0^4 \frac{x \, dx}{\sqrt{x^2+9}}$

(b) $\int_0^1 \frac{x^3 \, dx}{x^2+1}$

(c) $\int_0^1 x e^x \, dx$

(d) $\int_1^3 x^2 \ln x \, dx$

(e) $\int_0^\pi x^2 \sin x \, dx$

(f) $\int_2^3 \frac{x^3 \, dx}{\sqrt{x^4-1}}$

(g) $\int_0^{2\sqrt{3}} \frac{dx}{\sqrt{x^2+4}}$

(h) $\int_0^1 \frac{x \, dx}{\sqrt{x^4+1}}$

7. (Sauer, §5.3, CP9) For the integrals in Problem 6, calculate the approximation error of the composite Trapezoid Rule for $h = b-a$, $(b-a)/2$, $(b-a)/4$, \dots , $(b-a)/2^8$, and plot. Make a log-log plot, using, for example, Matlab's `loglog` command. What is the slope of the plot, and does it agree with theory?