1. *Linear Least Squares.* Suppose that you are given an overdetermined system $A\mathbf{x} = \mathbf{b}$. If $\mathbf{x}$ is a least squares solution to this system, what conditions must it satisfy? Write the overdetermined system for fitting the data given in the previous two questions with a polynomial of degree 1. What are the normal equations associated with this overdetermined system? What is a QR factorization? How can you use a QR factorization to solve this least squares problem? Find the QR factorization of the matrix
$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}.$$

2. *Numerical Differentiation.* Write down Taylor's remainder theorem. What are the standard forward, backward, and centered difference methods for approximating the first derivative of a function? What are their order of accuracy? Why might you use a method based on the forward or backward difference when the order of the centered difference is higher? What is a disadvantage of using a higher order method? How can you obtain approximations of the derivative using interpolating polynomials?

3. *Newton-Cotes Quadrature.* What is the main idea behind Newton-Cotes quadrature? Show how the trapezoid rule, Simpson's rule, and the midpoint rule are constructed. What are their orders of accuracy? How can you estimate the error? Construct a quadrature method using polynomials of degree three, where the interval $[a, b]$ is split into m subintervals with m divisible by three. This should be analogous to Simpson's rule. What is its order of accuracy?

4. *Ordinary Differential Equations.* Write down Euler's method. What is its order of accuracy? How is an explicit method different from an implicit method? How can you determine the region of stability of a method? Draw the region of stability for Euler's method. What is the maximum step size h required for stability if $\lambda = -5 + 5i$? How can you use a method designed for a first-order equation to integrate a higher-order equation?