Name: Solutions

Using the method of your choice, find the coefficients a_0, a_1, and a_2 of the polynomial $P(x) = a_0 + a_1x + a_2x^2$ of degree at most 2 interpolating the points (1,2), (2,4), and (4,2).

Using the Vandermonde matrix:

$$\left(\begin{array}{ccc|c} 1 & 1 & 1^2 & 2 \\ 1 & 2 & 2^2 & 4 \\ 1 & 4 & 4^2 & 2 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} \boxed{1} & 1 & 1 & 2 \\ 1 & 2 & 4 & 4 \\ 1 & 4 & 16 & 2 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & \boxed{1} & 3 & 2 \\ 0 & 3 & 15 & 0 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 6 & -6 \end{array}\right)$$

$$\Rightarrow a_2 = -1, \; a_1 = 5, \; a_0 = -2$$

$$\Rightarrow P(x) = -2 + 5x - x^2$$