Remember to adequately label all plots and include any MATLAB scripts and functions with your solutions. A clear and complete presentation of your solutions is required for full credit.

1. *Jacobi Method.*
   (a) Write a function with the declaration `function x = jacobi(A,x0,b,tol)` which implements the Jacobi method for solving a linear system of equations. Its inputs are the matrix `A`, an initial guess `x0`, the right-hand-side `b`, and an absolute error tolerance `tol`. Its output `x` is an approximate solution of the linear system.
   (b) Consider the matrices stored in `jacobi1.mat`, `jacobi2.mat`, and `jacobi3.mat`. Are any of these matrices strictly diagonally dominant? With this information, what can you conclude about the convergence of the Jacobi method?
   (c) Use the `eig` command to determine the spectral radius of the iteration matrix in each case. With this information, what can you conclude about the convergence of the Jacobi method?
   (d) Use the Jacobi method to solve the linear system of equations in each case, with a right-hand-side so that the solution is $(1, 1, \ldots, 1)$. Use the initial guess $(0, 0, \ldots, 0)$ and the tolerance 10^{-6}. For each of the three matrices, record the true error at each iteration (using the known solution). Plot the error as a function of iteration number on a single set of axes using the `semilogy` command. You should find that the matrices each require a different number of iterations. Explain your findings. What does this plot tell you about the convergence rate of the Jacobi Method?

2. *Gauss-Seidel Method.* This is another stationary iterative method with (perhaps) enhanced convergence properties. The iteration is given by

$$\boldsymbol{x}_{k+1} = D^{-1}\big(\boldsymbol{b} - U\boldsymbol{x}_k - L\boldsymbol{x}_{k+1}\big)$$

   which means that you will need to solve a lower triangular system of equations for $\boldsymbol{x}_{k+1}$ at each step.
   (a) Write a function with the declaration `function x = gauss_seidel(A,x0,b,tol)` which implements the Gauss-Seidel method for solving a linear system of equations. Its inputs and outputs are the same as in the previous problem. *Important*: Do not find the inverse of the matrix $D + L$. Instead, use the \ operator to solve using forward substitution.
   (b) Repeat part (d) of the previous problem using the Gauss-Seidel method. For each of the three matrices, record the true error at each iteration (using the known solution). Plot the error as a function of iteration number on a single set of axes using the `semilogy` command. You should find that the matrices each require a different number of iterations. Explain your findings. What does this plot tell you about the convergence rate of the Gauss-Seidel method?
   (c) Compare the convergence properties of the Gauss-Seidel method with the Jacobi method. What are some of the pros and cons of using one method versus the other?