

Name: Solutions

Perform at least two iterations of the Jacobi method on the system $Ax = b$ with

$$A = \begin{pmatrix} 4 & 2 \\ 3 & 4 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

starting from the initial vector of your choice.

The solution will depend on the initial vector. I will use $\bar{x}^{(0)} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

$$\begin{aligned} \bar{x}^{(1)} &= D^{-1} \left(\bar{b} - (L+U) \bar{x}^{(0)} \right) \\ &= \begin{pmatrix} 1/4 & 0 \\ 0 & 1/4 \end{pmatrix} \left(\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 & 2 \\ 3 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right) \\ &= \begin{pmatrix} 1/4 \\ 1/4 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \bar{x}^{(2)} &= D^{-1} \left(\bar{b} - (L+U) \bar{x}^{(1)} \right) \\ &= \begin{pmatrix} 1/4 & 0 \\ 0 & 1/4 \end{pmatrix} \left(\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 & 2 \\ 3 & 0 \end{pmatrix} \begin{pmatrix} 1/4 \\ 1/4 \end{pmatrix} \right) \\ &= \begin{pmatrix} 1/8 \\ 1/16 \end{pmatrix}. \end{aligned}$$