

Remember to adequately label all plots and include any MATLAB scripts and functions with your solutions. A clear and complete presentation of your solutions is required for full credit.

1. The roots of the quadratic equation $ax^2 + bx + c = 0$ are

$$x_{\pm} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

We may also approximate these roots using fixed-point iteration by dividing through by x (assuming $x \neq 0$) to get the equivalent equation

$$ax + b + \frac{c}{x} = 0.$$

Now, we can rearrange this equation to get the following methods:

- (a) Method 1 (Forward Iteration): Consider the sequence

$$x_0 = \text{initial guess}, \quad x_{k+1} = -\frac{b}{a} - \frac{c}{ax_k}, k = 1, 2, 3, \dots$$

- (b) Method 2 (Backward Iteration): Consider the sequence

$$x_0 = \text{initial guess}, \quad x_{k+1} = -\frac{c}{b + ax_k}, k = 1, 2, 3, \dots$$

Write MATLAB functions **forward** and **backward** that implement the above iterations to return r which differs from either x_+ or x_- by no more than a tolerance **tol**. Your functions should have **(a,b,c,x0,tol)** as their argument lists and terminate when $|x_{k+1} - x_k| < \text{tol}$. Demonstrate the performance of your functions for the values

$$x_0 = 1, \quad a = 1, \quad b = c = -1, \quad \text{tol} = 10^{-10}.$$

Note that the two functions will return different approximate roots r_{forward} and r_{backward} . Verify directly that each is within the desired tolerance from x_{\pm} . Which root is found by each method? Does the answer change if you change x_0 ?

2. Use the bisection method and the MATLAB function **fzero** to compute a positive real number x satisfying $\sinh x = \cos x$. For each method, use a tolerance of 10^{-8} . List your initial approximation (or interval in the case of bisection) and the number of required iterations. Also print at least nine digits of the approximate roots.
3. Use the bisection method and the MATLAB function **fzero** to compute all three real numbers x satisfying $5x^2 - e^x = 0$. For each of the three roots and each method, use a tolerance of 10^{-8} . List your initial approximations (or intervals in the case of bisection) and the number of required iterations. Also print at least nine digits of each approximate roots.
4. Sauer, Section 1.2, Exercise 14.