Programming Skills

Understand the basic control structures of Matlab, such as `if-else` blocks and `for` loops. Know the various ways in which Matlab indexes matrices and vectors. Be able to define a Matlab function and read an existing function to understand its behavior.

Bisection Method

Write down the bisection method. Use the intermediate value theorem to prove that $f(x) = x^3 + x - 1$ has a root on the interval $[0, 1]$. Starting with this interval, how many iterations must you perform in order to approximate the root with 10 correct decimal places?

Fixed Point Iteration

Find at least three functions that you can iterate on to find a root of $f(x) = x^3+x-1$. Define local convergence. Under what conditions does a fixed-point iteration locally converge? Evaluate the convergence properties of the iterations that you have found previously. What are three possible stopping criteria?

Newton's Method

Write down Newton's method. What does it mean for an iterative method to converge quadratically? Prove that Newton's method exhibits local quadratic convergence to a simple root. What happens in the case of a multiple root?

Secant Method

Write down the secant method. What are the convergence properties of the secant method? Give a case in which the secant method will fail to converge.