

Linear Least Squares

Suppose that you are given an overdetermined system $A\mathbf{x} = \mathbf{b}$. If \mathbf{x} is a least squares solution to this system, what conditions must it satisfy? Write the overdetermined system for fitting the data given in the previous two questions with a polynomial of degree 1. What are the normal equations associated with this overdetermined system? What is a QR factorization? How can you use a QR factorization to solve this least squares problem? Find the QR factorization of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}.$$

Numerical Differentiation

Write down Taylor's theorem. What are the standard forward, backward, and centered difference methods for approximating the first derivative of a function? What are their order of accuracy? Why might you use a method based on the forward or backward difference when the order of the centered difference is higher? What is a disadvantage of using a higher order method?

Newton-Cotes Quadrature

What is the main idea behind Newton-Cotes quadrature? Show how the trapezoid rule, Simpson's rule, and the midpoint rule are constructed. What are their orders of accuracy? How can you estimate the error? Estimate the integral

$$\int_0^1 \sin(x) dx$$

with each method using 2 subintervals.

Ordinary Differential Equations

Write down Euler's method. What is its order of accuracy? How is an explicit method different from an implicit method? How can you determine the region of stability of a method? Draw the region of stability for Euler's method. What is the maximum step size h required for stability if $\lambda = -5 + 5i$? How can you use a method designed for a first-order equation to integrate a higher-order equation?