Name: Solutions

Another approximation to the first derivative of a twice continuously differentiable function $f(x)$ at $x = x_0$ is the backward difference

$$f'(x_0) \approx \frac{f(x_0) - f(x_0 - h)}{h}.$$

Assuming $|f''(x)| < M$ for x near x_0, show that this method is accurate first order in h by making a substitution using Taylor's theorem of the term $f(x_0 - h)$.

By Taylor's theorem:

$$f(x_0-h) = f(x_0) - hf'(x_0) + \frac{h^2}{2} f''(c)$$

where c is between x_0 and x_0-h.

$$\Rightarrow \frac{f(x_0) - f(x_0-h)}{h} = \frac{\cancel{f(x_0)} - \left(\cancel{f(x_0)} - hf'(x_0) + \frac{h^2}{2} f''(c)\right)}{h}$$

$$= f'(x_0) + \frac{h}{2} f''(c).$$

$$\Rightarrow \left| f'(x_0) - \frac{f(x_0) - f(x_0-h)}{h} \right| = \frac{h}{2} |f''(c)|$$

$$\leq \frac{Mh}{2} \Rightarrow O(h).$$