

Computer problems from the textbook contain “CP” in the exercise number. For these problems, remember to adequately label all plots and include code that you have written along with your solutions. All code that you include should be properly explained. Do all other problems by hand and make sure to your work. A clear and complete presentation of your solutions is required for full credit.

1. (Sauer §1.3, #2) Find the forward and backward error for the following functions, where the root is $1/3$ and the approximate root is $x_a = 0.3333$:
 - (a) $f(x) = 3x - 1$
 - (b) $f(x) = (3x - 1)^2$
 - (c) $f(x) = (3x - 1)^3$
 - (d) $f(x) = (3x - 1)^{1/3}$
2. (Sauer §1.3, #4)
 - (a) Find the multiplicity of the root $r = 0$ of $f(x) = x^2 \sin x^2$.
 - (b) Find the forward and backward errors of the approximate root $x_a = 0.01$.
3. (Sauer §1.3, CP1) Let $f(x) = \sin x - x$.
 - (a) Find the multiplicity of the root $r = 0$.
 - (b) Use Matlab’s `fzero` command with initial guess $x = 0.1$ to locate a root. What are the forward and backward errors of `fzero`’s response?
4. (Sauer §1.3, CP3)
 - (a) Use `fzero` to find the root of $f(x) = 2x \cos x - 2x + \sin x^3$ on $[-0.1, 0.2]$. Report the forward and backward errors.
 - (b) Run the Bisection Method with initial interval $[-0.1, 0.2]$ to find as many correct digits as possible, and report your conclusion.
5. (Sauer §1.4, #2) Apply two steps of Newton’s Method with initial guess $x_0 = 1$.
 - (a) $x^3 + x^2 - 1 = 0$
 - (b) $x^2 + 1/(x + 1) - 3x = 0$
 - (c) $5x - 10 = 0$
6. (Sauer §1.4, #6) Sketch a function f and initial guess for which Newton’s Method diverges.
7. Write a function with the declaration `function xa = newton(f, fp, x0, k, tol)` which implements Newton’s Method. The inputs `f` and `fp` are function handles to implementations of $f(x)$ and $f'(x)$ respectively. The input `x0` is the initial guess x_0 . The iteration should terminate if the estimated absolute error is less than `tol`, and an error message should be displayed if the iteration has not converged after `k` steps. The output `xa` is an approximation to the root x_a of $f(x)$. Include a listing of your function.

8. (Sauer §1.4, CP2) Each equation has one real root. Use Newton's Method to approximate the root to eight correct decimal places.

(a) $x^5 + x = 1$

(b) $\sin x = 6x + 5$

(c) $\ln x + x^2 = 3$