This homework is due on Thursday, February 18.

1. Consider the linear second-order PDE

$$u_{tt} + (5+2x^2)\,u_{xt} + (1+x^2)(4+x^2)\,u_{xx} = 0.$$

   (a) Show that this PDE is hyperbolic.

   (b) Show that the two families of characteristics are given by

$$\xi = \arctan x - t, \quad \eta = \frac{1}{2}\arctan\frac{x}{2} - t$$

   for arbitrary $\xi$ and $\eta$.

   (c) Plot[1] the domain of dependence of the point $(t,x) = (5,1)$.

   (d) Plot the domain of influence of the point $x = 1$.

   (e) Show that this PDE can be expressed in the canonical form

$$u_{\xi\eta} + \frac{2}{9}\,x(4+x^2)^2\,u_\xi + \frac{2}{9}\,x(1+x^2)^2\,u_\eta = 0$$

   for $x$ solving

$$\xi - \eta = \arctan x - \frac{1}{2}\arctan\frac{x}{2}.$$

2. Let $\alpha$ be a constant with $\alpha \neq -c$. Let $f$ and $g$ be functions of class $C^2$ for $x > 0$ and vanish near $x = 0$.

   (a) Find the solution[2] $u(x,t)$ of

$$u_{tt} - c^2 u_{xx} = 0$$

   in the quadrant $t > 0$, $x > 0$ for which

$$u(x,0) = f(x), \quad u_t(x,0) = g(x)$$

   if $x > 0$ and

$$u_t(0,t) = \alpha u_x(0,t)$$

   if $t > 0$.

   (b) Show that generally no solution exists when $\alpha = -c$.

[1] I recommend that you use a software package of some sort.

[2] *Hint*: You know that a solution has the form $u(x,t) = F(x-ct) + G(x+ct)$ for some $F$ and $G$.