1. The partial differential equation $u_{tt} = c^2\Delta t - q(x)u$ arises in the study of wave propagation in a nonhomogeneous elastic medium, where $q(x)$ is non-negative and represents the coefficient of elasticity at x.
   (a) Define an appropriate notion of energy for solutions, and justify your choice by showing that the energy is constant under reasonable assumptions about boundary conditions.
   (b) Use the energy result to show that solutions are uniquely determined by their initial condition.

2. Consider a flexible beam with clamped ends at $x = 0$ and $x = 1$. Small amplitude waves in this beam satisfy the problem

$$\begin{cases} u_{tt} + \gamma^2 u_{xxxx} = 0 & \text{in } (0,1) \times (0,\infty) \\ u(x,t) = u_x(x,t) = 0 & \text{on } \{x = 0,1\} \times (0,\infty), \end{cases}$$

   where γ^2 is a constant depending on the shape of the beam and the elastic material in the beam. Show that the following energy function is conserved:

$$e(t) = \frac{1}{2}\int_0^1 (u_t^2 + \gamma^2 u_{xx}^2)\, dx.$$

3. (Evans §2.5, #17) Let $u \in C^2(\mathbb{R} \times [0,\infty))$ solve the initial-value problem for the wave equation in one dimension:

$$\begin{cases} u_{tt} - u_{xx} = 0 & \text{in } \mathbb{R} \times (0,\infty) \\ u = g, u_t = h & \text{on } \mathbb{R} \times \{t = 0\}. \end{cases}$$

   Suppose g, h have compact support. The *kinetic energy* is $k(t) = \frac{1}{2}\int_{-\infty}^{\infty} u_t^2(x,t)\, dx$ and the *potential energy* is $p(t) = \frac{1}{2}\int_{-\infty}^{\infty} u_x^2(x,t)\, dx$. Prove
   (i) $k(t) + p(t)$ is constant in t,
   (ii) $k(t) = p(t)$ for all large enough times t

4. (Evans §2.5, #18) Let u solve

$$\begin{cases} u_{tt} - \Delta u = 0 & \text{in } \mathbb{R}^3 \times (0,\infty) \\ u = g, u_t = h & \text{on } \mathbb{R}^3 \times \{t = 0\}, \end{cases}$$

   where g, h are smooth functions and have compact support. Show there exists a constant C such that

$$|u(x,t)| \le C/t \quad (x \in \mathbb{R}^3,\ t > 0).$$