This homework is due on Thursday, February 25.

1. Find all harmonic functions u which have the form $u(x, y) = F(x/y)$.

2. Let Ω be the open unit square $(0, 1) \times (0, 1)$. Consider the problem of trying to find u with

$$\Delta u = 0 \text{ in } \Omega, \quad \frac{du}{dn} = h \text{ on } \partial\Omega$$

for $h = 0$ on all sides but the bottom, where $h = h(x)$ with $\int_0^1 h(x)\, dx = 0$. Find a family of solutions to this problem using separation of variables.

3. Let Ω be the annulus with inner radius 1 and outer radius 2. Consider the problem of trying to find u with

$$\Delta u = 0 \text{ in } \Omega, \quad u = h \text{ on } \partial\Omega$$

for $h = 0$ on the inner radius and $h = h(\theta)$ on the outer radius. Find a solution to this problem using separation of variables.

4. A function $u \in C^2(\Omega)$ is called *subharmonic* if $\Delta u \geq 0$ in Ω and *superharmonic* if $\Delta u \leq 0$ in Ω. Following the proof for the mean-value property of harmonic functions, show that

$$u(x) \leq \fint_{\partial B(x,r)} u(y)\, dS_y$$

for subharmonic functions u and

$$u(x) \geq \fint_{\partial B(x,r)} u(y)\, dS_y$$

for superharmonic functions u if $B(x, r) \subset \Omega$.