1. Explain why subharmonic functions have the name that they do. That is, give a brief argument why the graphs of subharmonic functions must lie below (or at worst touching) the graphs of harmonic functions with the same boundary conditions.

2. Let U be a bounded, open subset of $\mathbb{R}^n$ and suppose that u is a smooth solution of

$$\begin{cases} -\Delta u = f & \text{in } U \\ u = g & \text{on } \partial U. \end{cases}$$

   (a) Show that $v(x) = u(x) + \frac{|x|^2}{2n}\lambda$ is subharmonic, where $\lambda = \max_{\bar{U}} |f|$.

   (b) Show that $w(x) = -u(x) + \frac{|x|^2}{2n}\lambda$ is subharmonic.

3. (Evans §2.5, #6) Let U be a bounded, open subset of $\mathbb{R}^n$. Prove that there exists a constant C, depending only on U, such that

$$\max_{\bar{U}} |u| \le C(\max_{\partial U} |g| + \max_{\bar{U}} |f|)$$

   whenever u is a smooth solution of

$$\begin{cases} -\Delta u = f & \text{in } U \\ u = g & \text{on } \partial U. \end{cases}$$

   Hint: Use your results from the last question, along with a result about subharmonic functions from last week's homework.

4. (Evans §2.5, #7) Use Poisson's formula for the ball to prove

$$r^{n-2}\,\frac{r-|x|}{(r+|x|)^{n-1}}\,u(0) \le u(x) \le r^{n-2}\,\frac{r+|x|}{(r-|x|)^{n-1}}\,u(0)$$

   whenever u is positive and harmonic in $B^0(0,r)$.

5. Use Poisson's formula for the half-plane to find the harmonic function $u(x)$ in $\mathbb{R}^2_+$ with

$$u(x_1, 0) = \begin{cases} -1 & \text{if } x_1 < -1 \\ x_1 & \text{if } -1 \le x_1 \le 1 \\ 1 & \text{if } x_1 > 1. \end{cases}$$