1. (Evans §1.5, #1) Classify each of the partial differential equations in §1.2 as follows:

   (a) Is the PDE linear, semilinear, quasilinear or fully nonlinear?

   (b) What is the order of the PDE?

2. (Evans §1.5, #2) Prove the *multinomial theorem*

$$(x_1 + x_2 + \cdots + x_n)^k = \sum_{|\alpha|=k} \binom{|\alpha|}{\alpha} x^\alpha$$

   where $\binom{|\alpha|}{\alpha} = \frac{|\alpha|!}{\alpha!}$, $\alpha! = \alpha_1!\alpha_2!\ldots\alpha_n!$, and $x^\alpha = x_1^{\alpha_1} x_2^{\alpha_2} \ldots x_n^{\alpha_n}$. The sum is taken over all multiindices α with order $|\alpha| = k$.

   Hint: Use the binomial theorem along with induction on n.

3. (Evans §2.5, #1) Write down an explicit formula for a function u solving the initial-value problem

$$\begin{cases} u_t + b \cdot Du + cu = 0 & \text{in } \mathbb{R}^n \times (0, \infty) \\ u = g & \text{on } \mathbb{R}^n \times \{t = 0\}. \end{cases}$$

   Here $c \in \mathbb{R}$ and $b \in \mathbb{R}^n$ are constants. Describe how your solution differs from the solution of the initial value problem for the transport equation.

   Hint: Calculate $\dot{z}$ with z as defined in §2.1. What ODE does z solve?

4. (Evans §2.5, #2) Prove that Laplace's equation $\Delta u = 0$ is rotation invariant; that is, if O is an orthogonal $n \times n$ matrix and we define

$$v(x) = u(Ox) \quad \text{for } x \in \mathbb{R},$$

   then $\Delta v = 0$.

   Hint: Compute Δv using the multivariable chain rule, taking advantage of the orthogonality of O.