General PDE Topics

- Know how to determine the order of a PDE.
- Be able to determine if a PDE is linear, semilinear, pseudolinear, or fully nonlinear.
- Know how to perform an arbitrary change of variables on a PDE.
- Be able to apply integration by parts and the Green's formulas.
- Understand how to formulate basic proofs (*e.g.* direct proof, proof by contradiction, proof by induction).

Laplace and Poisson's Equations

- Know the basic properties of the fundamental solution for Laplace's equation in $\mathbb{R}^n$, and how to solve Poisson's equation in $\mathbb{R}^n$ using the fundamental solution, but not necessarily the precise expression for the solution itself.
- Understand the physical interpretation of Laplace's equation.
- Know both mean-value properties for harmonic functions, and understand the basic ideas of the proof.
- Know the maximum principle for harmonic functions, and understand how the mean-value property is used to prove it.
- Understand how to establish uniqueness of solutions to Poisson's equation using the maximum principle.
- Be able to explain how the Green's function for the Poisson equation is obtained from the fundamental solution using a corrector, and know how to express the solution of Poisson's equation on a bounded domain in terms of the Green's function.
- Understand how to construct the Green's function for the half-space using reflection, and how to construct the Green's function for the unit ball using inversion.

Heat Equations

- Understand how to construct the parabolic cylinder and parabolic boundary from a bounded domain.
- Know the basic properties of the fundamental solution for the heat equation in $\mathbb{R}^n$, and how to solve the initial value problem for the heat equation in $\mathbb{R}^n$ using the fundamental solution, but not necessarily the precise expression for the solution itself.
- Understand the physical interpretation of the heat equation.
- Know how to obtain the solution for the inhomogeneous heat equation from the initial value problem using Duhamel's principle.

- Know the mean-value property for solutions to the heat equations, and understand the basic ideas of the proof, but not necessarily the precise definition of the heat ball.
- Know the maximum principle for solutions to the heat equation, and understand how the mean-value property is used to prove it.
- Understand how to establish uniqueness of solutions to the heat equation using the maximum principle.