This homework is due on Tuesday, March 29.

1. Let us think a bit more about finding Green's functions for wedge-shaped domains.
   (a) Try to use the method from last week's homework to find the Green's function for the wedge $\Omega = \{\mathbf{x} \in \mathbb{R}^2 \mid x_1 > -x_2, x_2 > 0\}$. Can you find a satisfactory corrector function? If not, explain why.
   (b) Under what conditions on the angle at the vertex of the wedge can you use this method to find a Green's function?

2. Prove the following properties of the Fourier transform in $\mathbb{R}^n$.
   (a) $(u(\mathbf{x} - \mathbf{z}))\hat{} = e^{-i\,\mathbf{z}\cdot\mathbf{y}}\hat{u}(\mathbf{y})$.
   (b) $(u(a\mathbf{x}))\hat{} = \frac{1}{|a|}\hat{u}\left(\frac{\mathbf{y}}{a}\right)$.

3. Prove the following properties of the Fourier transform in $\mathbb{R}$.
   (a) If u is purely real, then $\hat{u}(-y) = \overline{\hat{u}(y)}$.
   (b) If u is purely real and even, then $\hat{u}$ is purely real and even.

4. Find the Fourier transform of the function

$$u(x) = \begin{cases} 1 - \frac{|x|}{a} & \text{if } |x| \leq a \\ 0 & \text{if } |x| > a \end{cases}$$

5. Use the Fourier transform to find a solution of

$$u'' + xu' + u = 0, \quad u(0) = 1, \quad u'(0) = 0.$$

   Hint: First show that $(x\,u'(x))\hat{} = -(\hat{u}(y) + y\,\hat{u}'(y))$.

6. Use the Fourier transform to find an explicit formula for the solution of

$$\begin{cases} u_t - u_{xx} + \alpha u = 0 & \text{in } \mathbb{R} \times (0, \infty) \\ u = g & \text{on } \mathbb{R} \times \{t = 0\}, \end{cases}$$

   where $g \in L^2(\mathbb{R})$ and $\alpha \in \mathbb{R}$ is a constant.