1. (Evans §2.5, #12) Suppose $u$ is smooth and solves $u_t - \Delta u = 0$ in $\mathbb{R}^n \times (0, \infty)$.
   (a) Show that $u_\lambda(x,t) = u(\lambda x, \lambda^2 t)$ also solves the heat equation for each $\lambda \in \mathbb{R}$.
   (b) Use (a) to show $v(x,t) = x \cdot Du(x,t) + 2tu_t(x,t)$ solves the heat equation as well.
   *Hint*: Differentiate $u_\lambda$ with respect to $\lambda$.

2. (Evans §2.5, #13) Assume $n = 1$ and $u(x,t) = v(\frac{x}{\sqrt{t}})$.
   (a) Show
   $$u_t = u_{xx}$$
   if and only if
   $$(*) \qquad v'' + \frac{z}{2}v' = 0.$$
   Show that the general solution of $(*)$ is
   $$v(z) = c\int_0^z e^{-s^2/4}\, ds + d.$$
   (b) Differentiate $u(x,t) = v(\frac{x}{\sqrt{t}})$ with respect to x and select the constant $c$ properly, to obtain the fundamental solution $\Phi$ for $n = 1$. Explain why this procedure produces the fundamental solution.
   *Hint*: What is the initial condition for $u$?

3. (Evans §2.5, #14) Write down an explicit formula for a solution of
   $$\begin{cases} u_t - \Delta u + cu = f & \text{in } \mathbb{R}^n \times (0,\infty) \\ u = g & \text{on } \mathbb{R}^n \times \{t = 0\}, \end{cases}$$
   where $c \in \mathbb{R}$.

   *Hint*: Try a solution of the form $u(x,t) = v(x,t)e^{-ct}$.