1. Solve each of the following Cauchy problems using the method of characteristics by completing the Cauchy data[1] and solving the characteristic equations.
   (a) $u_x^2 + u_y = y, \quad u(x,0) = 0$
   (b) $u_x^3 - u_y = 0, \quad u(x,0) = 2x^{3/2}$
   (c) $xu_x + yu_y + \frac{1}{2}(u_x^2 + u_y^2) = u, \quad u(x,0) = \frac{1}{2}(1 - x^2)$

2. Consider the Cauchy problem for $u_x^2 + u_y^2 = 1$ subject to the boundary condition $u = 0$ on the circle of radius 1 in the xy-plane.
   (a) You may parameterize the Cauchy data $\Gamma : (f, g, h, \phi, \psi)$ with
   $$f(s) = \cos(s), \quad g(s) = \sin(s), \quad h(s) = 0.$$
   Determine compatible functions $\phi(s)$, $\psi(s)$. You should get two sets of solutions.
   (b) Find the solution $u(x, y)$ for each set of Cauchy data using the characteristic equations. Be careful to make sure your solution satisfies the boundary condition.

3. Recall that
   $$F(x, y, z, p, q) = a(x, y, z)\, p + b(x, y, z)\, q - c(x, y, z)$$
   for a quasilinear first-order equation. Determine admissible functions ϕ, ψ in terms of f, g, h for the Cauchy data $\Gamma : (f, g, h, \phi, \psi)$. You should be able to express each function as a quotient. What condition guarantees that the denominator remains nonzero?

[1] Recall that the Cauchy data are compatible if $F(f, g, h, \phi, \psi) = 0$ and $h' = \phi f' + \psi g'$.