Assignment 2: Specification and Prediction of Lunar Probe Satellite

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Launch Vehicle: Atlas IIIB (DEC) Rocket

Primary Launch System Specifications

The Atlas IIIB consists of two rocket stages, an interstage adapter and a large payload fairing (LPF). The first stage consists of a single stage Atlas rocket whilst the second stage is the Dual Engine Centaur (DEC) rocket. Combined they are capable of putting a maximum payload of $10.7 \times 10^3 kg$ into a Low Earth Orbit (LEO) (Space Launch Report, 2005).

Table 1: Stage Specific Data

	Stage 1 (Atlas)	Stage 2 (DEC)	Interstage Adapter	Standard Package[1]	Large Payload Fairing (LPF) adapter: Type E
Gross Mass (kg)	195628	22940	470	4	98
Empty Mass (kg)	13725	2130	-	-	-
Thrust (vac) (kN)	4148.722	198.319	-	-	-
ISP (vac sec)	337.8	450.5	-	-	-
Burn time (seconds)	184	464	-	-	-
No. Engines	1	2	-	-	-

Inclination of the Lunar Plane

Given that the inclination of the lunar plane is 5.15 degrees from the ecliptic (http://nssdc.gsfc.nasa.gov/planetary/factsheet/moonfact.html), the equatorial plane is at an angle of -23.5 degrees from the ecliptic (Auld, 2013) and the launch of a satellite from Cape Canaveral will go into orbit at an inclination of 28.5 degrees to the equatorial plane (ILS,2001), a satellite launched from Cape Canaveral will have an inclination 0.45 degrees below the lunar plane but this is only the case when the right ascension of the ascending node (RAAN) of the moon is on the vernal equinox vector.

[1] The Standard Package consists of a flights termination system and airborne harness

However, this is not always the case as the RAAN of the moon perturbates around the **ecliptic** plane in a cycle which takes 18.6 years, and as the ecliptic and celestial planes are at a fixed angle it will also take 18.6 years to rotate around the celestial plane (Vincent, F, 2013).

There is also a minor oscillating cycle which occurs on top of this caused by the gravitational effects of the sun, the effects of which are ignored in this model.

The RAAN of the moon in the celestial plane in January 2000 was $125.08\ degrees$ (NASA, 2013) and as it completes a full rotation every 18.6 years = 223.2 months, then it will perturb by -1.61 degrees per month (perturbation is in the westerly direction). As of October 2013, 165 months have passed so the RAAN of the moon is approximately $-140.6 = 219.4$ degrees.

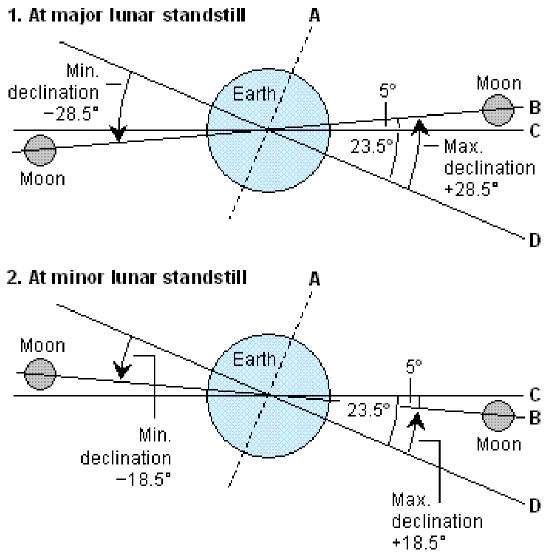


Figure 1: The migration of the right ascension of the moon. Note how the moon's inclination to the celestial plane changes over time.

Because of the angle between the celestial and ecliptic planes the inclination of the moon's orbit with respect to the celestial plane will also change. Thus, as of October 2013, its inclination will be 19.45 degrees[2].

Thus the satellite launched will need to have a RAAN of 219.4 degrees and an inclination of 19.45 degrees.

[2] It has completed $\frac{88}{223} = 0.390$ of the cycle, therefore $5.14cos(0.39 \times 2\pi) = -4.05$ less that mean.

Launch Requirements

From the Atlas Launch System Mission Planner's Guide (ILS, 2001) the Atlas IIIB (DEC) can carry payloads according to the table below.

Thus it is possible to put a 10.3 tonne payload into circular 310 km LEO at 28.5 degrees.

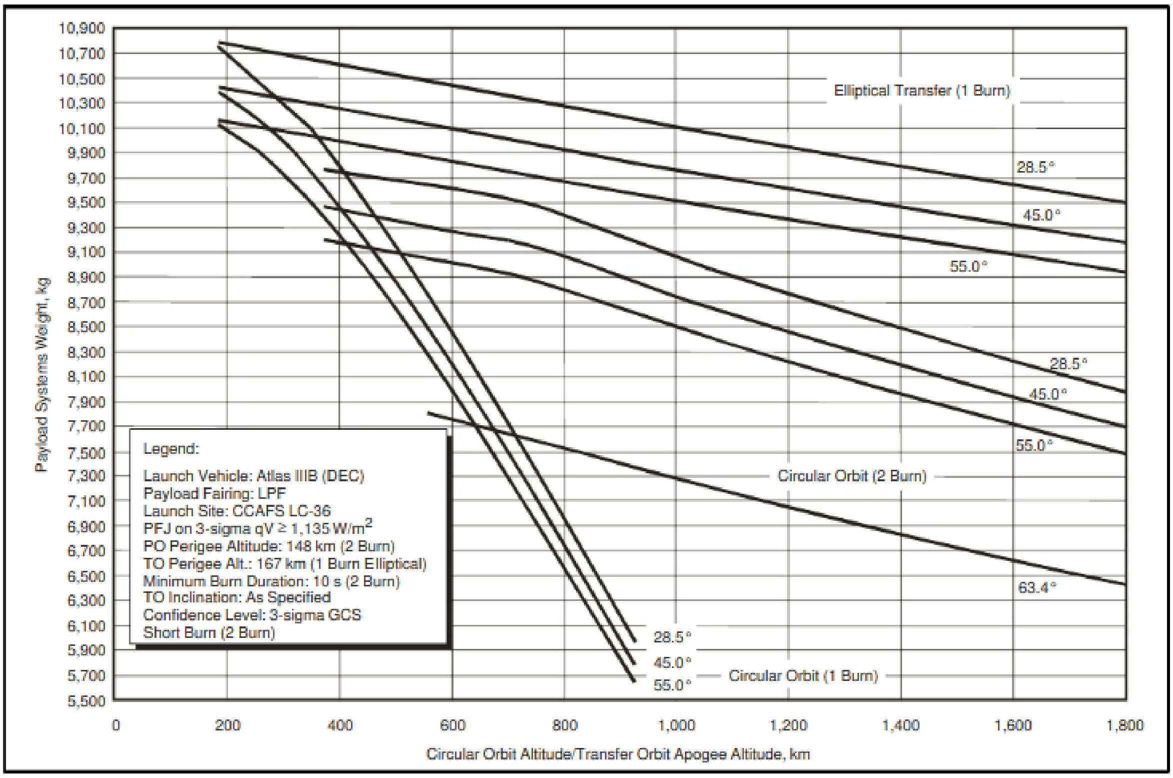


Figure 2: Payload vs circular orbit altitude page 2-95 (ILS,2001)

The launch necessitates a direct ascent mission (DAM) where the Centaur main engines are ignited just after separation with minimal coasting, and puts the payload into LEO. Atlas has flown 15 missions using this design.

RAAN Control

The Atlas rockets are highly capable of controlling the RAAN of the spacecraft. The table below shows injection accuracies for LEOs for Atlas IIAS and Atlas IIIA, but unfortunately not the Atlas IIIB as it is less commonly used for LEO. However, the very similar Atlas IIIA rocket has an RAAN error of 0.11 degrees so it is safe to say that the Atlas IIIB will perform to similar specifications.

Table 2.3.3-1 Typical Injection Accuracies at Spacecraft Separation

Atlas IIAS/IIIA										
Orbit at Centaur Spacecraft Separation				**± 3-sigma Errors**						
Mission	**Apogee, km (nmi**	**Perigee, km (nmi)**	**Inclin-ation, °**	**Semi-major Axis, km (nmi)**	**Apogee, km (nmi)**	**Perigee, km (nmi)**	**Inclin-ation, °**	**Eccentricity**	**Argument of Perigee, °**	**RAAN, °**
GTO	35,941 (19,407)	167 (90)	27.0	N/A	117 (63.2)	2.4 (1.3)	0.02	N/A	0.23	0.26
GTO	35,949 (19,411)	167 (90)	22.1	N/A	109 (58.9)	2.2 (1.2)	0.02	N/A	0.19	0.21
Super-synchron-ous	123,500 (66,685)	167 (90)	27.5	625 (337)	1,250 (675)	2.8 (1.5)	0.01	N/A	0.17	0.26
Inter-mediate Circular Orbit	10,350 (5,589)	10,350 (5,589)	45.0	42.7 (23.0)	N/A	N/A	0.07	0.002	N/A	0.08
Elliptical Transfer	10,350 (5,589)	167 (90)	45.0	N/A	40.0 (21.6)	2.40 (1.3)	0.07	N/A	0.07	0.08
LEO (Circular)	1,111 (600)	1,111 (600)	63.4	19.4 (10.5)	N/A	N/A	0.15	0.004	N/A	0.11
Legend: N/A = Not Applicable or Available										

Figure 3: Typical Injection Accuracies at Spacecraft Separation (ILS, 2001)

Inclination Accuracies

The inclination derived from the table above has impressive accuracies and there is a 95% chance that it will be within 0.15 degrees of the specified inclination.

Eccentricity Accuracies

The eccentricty has a 3 sigma error of 0.004 for a 1111 km LEO.

Altitude Accuracies

The altitude has a 3 sigma error of 19.4 for a 1111 km orbit (Figure 3) which is an error of $\pm 1.7\%$ therefore for an orbit of 310 km the expected error is ± 5.41km.

Payload Volume

The LPF has a diameter of 4.2 meters, therefore an approximate volume of $4.2^3 = 74$ cubic meters. As our payload is up to $10300kg$ it will have to have a density of $135kg/m^3$ for it to fit, which is easily enough room given the density of water is $1000kg/m^3$.

Mission Requirements and Expectations

From the estimated errors detailed in the Atlas Launch System Mission Planner's Guide (ILS, 2001) the mission parameters the satellite requires are:

Apogee Altitude: 310 ± 5.41 km
Perigee Altitude: 310 ± 5.41 km
Inclination: 28.5 ± 0.15 degrees
Argument of Perigee: N/A
RAAN: 219.4 ± 0.11 degrees
Eccentricity: 0 ± 0.004
Payload Mass: 10.3×10^3 kg

Satellite Mass: 500 kg
Percent Structure: 15%
Fuel Mass: 8330 kg
Structural Mass: 1470 kg

Table 2: Movement of RAAN and inclination of the lunar plane over time

Date	**RAAN**	**Inclination of lunar plane to ecliptic**	**Inclination change needed**
June 2005	0	28.64	0.14
February 2006	346.62	28.5	0
January 2010	270.00	23.5	-5
October 2013	220	19.45	-9.05[3]
November 2013	199.28	18.64	-9.86
May 2019	90.00	23.5	-5
December 2022	13.38	28.5	0
August 2023	0	28.64	0.14

Movement to Lunar Plane

As of September 2013, the launch trajectory of the Atlas IIIB rocket puts the satellite in a plane which is 9.05 ± 0.15 degrees above the lunar plane.

Assuming the rocket has launched us into a circular orbit, the velocity needed to move to this new inclination is given by:

$$\Delta v = 2v\sin(\frac{\Delta i}{2})$$

[3] Inclination of launch is 28.5 degrees and inclination of the moon is 19.45 degrees as of September 2013

Where $v = \sqrt{\frac{GM}{Re+(310\pm5.41)\times10^3}} = 7700 \pm 130m/s$

So $\Delta v = -1215 \pm 40m/s$

When it is applied perpendicular to the plane (south) at the ascending node between the two planes.

Note: if there is an eccentricity it will be given by;

$$\Delta v = \frac{2v\sin\left(\frac{\Delta i}{2}\right)\sqrt{1-e^2}\cos(\omega+f)\,na}{1+e\cos(f)}$$

Where e=eccentricity, ω = argument of periapsis, f= true anomaly, n=mean motion and a=semimajor axis.

Fuel Consumed

By using the rocket equation, the amount of fuel consumed in a change of velocity of $1215 \pm 40m/s$ is;

$$F_{consumed} = F_{init} - ((M_{struct} + M_{payload} + M_{fuel})/e^{\frac{\Delta v}{ISP*g0}} - M_{struct} - M_{payload})$$

So $F_{consumed} = 3480 \pm 110$ kg, given $ISP = 300$ and $g0 = 9.81$

And the new mass, $M_{new} = 6820 \pm 110$kg

Further velocity change

Note that the RAAN that the satellite will be launched into is 219.4 ± 0.11 degrees. If a small error occurs it will have to be corrected before the LTO begins, resulting in increased fuel consumption which will have to be accounted for.

The maximum possible error is 0.11 degrees which will require a change in velocity given by.

$$\Delta v = 2v\sin(\frac{\Delta i}{2})$$

Where $v = \sqrt{\frac{GM}{Re+(310\pm5.41)\times10^3}} = 7700 \pm 130m/s$

so the velocity will change by $14.8 \pm 0.2\ m/s$ which will require $12.4 \pm 0.2kg$ of fuel.

Minimum fuel left: 6810 ± 110 kg

Modeling the Lunar Transfer Orbit (LTO)

Once the satellite is in the lunar plane at 310 $\pm$5.41 km above the earth the LTO must be achieved.

The model used a moon located at a right ascension of 65.15 degrees to the solar vectors projection onto the lunar plane and moving at a speed of $1023m/s$ in the anti-clockwise direction as seen from above. The model uses a system where the position of the moon is defined at $\theta = 0$ degrees when it is located purely on the x axis.

The sun located at $(0,149600000000,2213000)$ which moves at a speed of $(30000t, 0,2213000\sin(\theta + 40))$, where θ is the right ascension of the moon from the x axis.

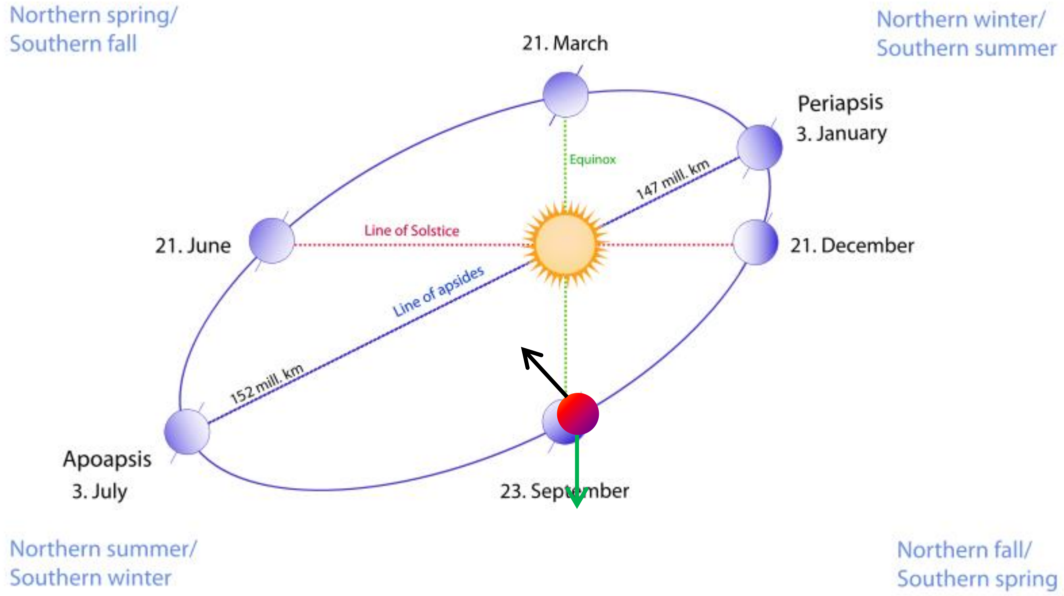


Figure 4: The counter-clockwise orbit of the earth around the sun showing the vernal equinox vector (green line) and the RAAN of the moon in the celestial plane (black). The October 2013 position of the earth is shown in red. Note that the moon rotates around the earth in an anti-clockwise direction as seen from above.

The coordinate system is defined with the x and y plane corresponding to the lunar plane, and the earth located at the origin, with the 2D layout shown in figure 5 below.

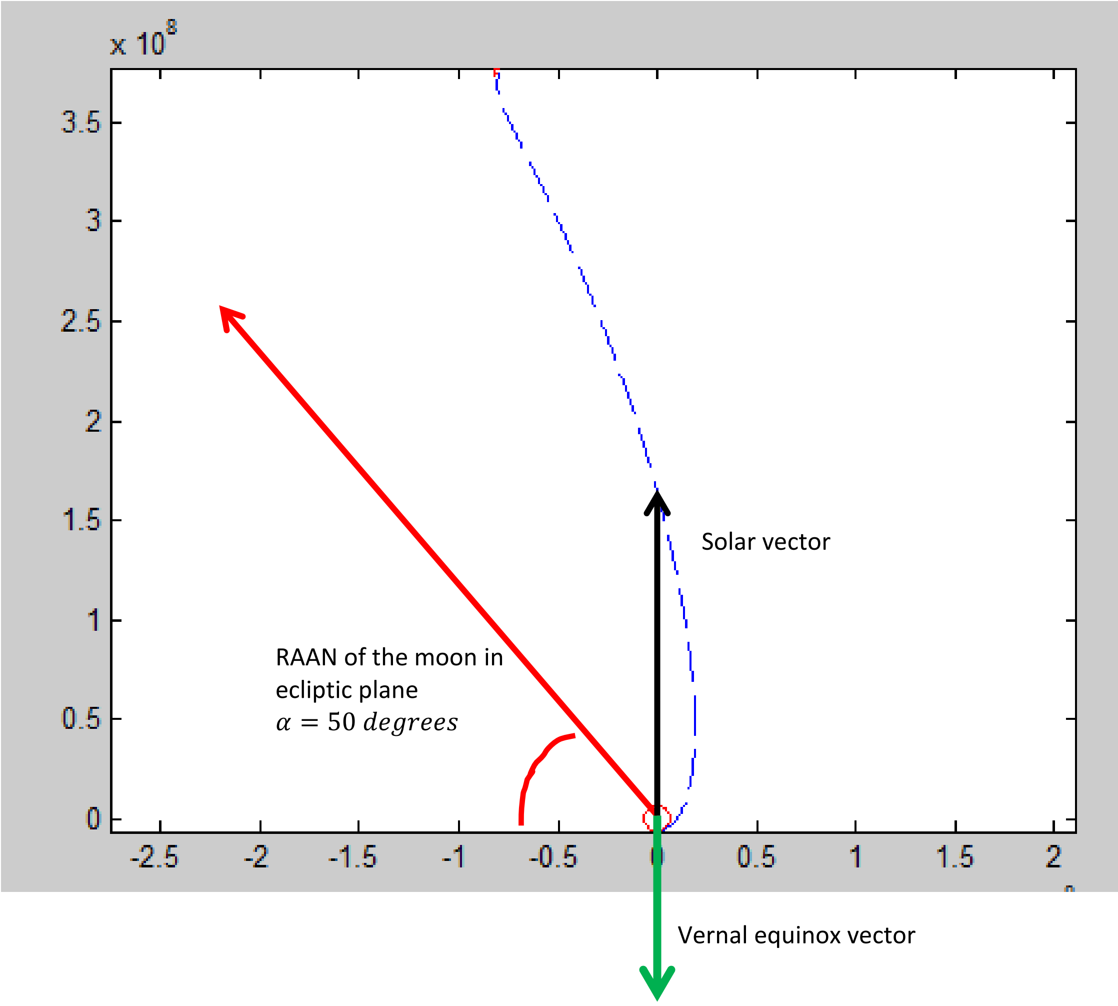


Figure 5: The coordinate system referred to and used to make the models

Movement of the Sun

Z-coordinate of the Sun

The moon has an orbit 5.14 degrees away from the ecliptic plane and as of September 2013 the RAAN on the **ecliptic** plane is 140 degrees (Vincent, F, 2013) and as we are at the autumnal equinox, the sun is located at 180 degrees to the vernal equinox vector, thus the angle between the RAAN vector and the solar vector is 40 degrees (as seen above).

The maximum declination of the lunar orbit occurs at 90 degrees to the ascending node, at which point it has dropped $24700\sin(5.14)$ km below the plane of the sun.

Therefore the z position of the sun is at a maximum $z = 2213$ km when angle between the moon and the sun is 50 degrees and a minimum $z = 2213$ km when the angle between the moon and the sun is 130 degrees (and the moon is in the third quadrant).

Thus the movement of the z position of the sun relative to the moon can be given by the equation; $z = 2213000\sin(\theta + 40)$.

Y-coordinate of the Sun

As the satellite takes approximately 2 days to get to the moon the y coordinate of the sun does not change significantly during this time.

X-coordinate of the Sun

As the earth is moving around the sun it is moving with an orbital velocity of $30000m/s$, thus for small time scales when the earth is moving purely parallel to the sun (near the ”bottom” of its orbit, the sun is moving relative to the earth at a rate of $-30000m/s$ in the x direction.

Mapping the gravitational field

For a given position (x,y,z) of the spacecraft, a gravitational field was obtained by considering the gravity of the Earth, Moon and Sun in the x, y and z directions.

This made use of the following formulas:

$$g_x = \frac{GM}{r^2}\frac{x}{r}$$

$$g_y = \frac{GM}{r^2}\frac{y}{r}$$

$$g_z = \frac{GM}{r^2}\frac{z}{r}$$

Mapping the spacecraft thrust

The thrust of the spacecraft was obtained by the formulas

$$a = \frac{F_{thrust}}{m}$$

where

$$m = m_{initial} - \frac{t\,F_{thrust}}{g0\,ISP}$$

and was assumed to be pointing in the same direction as the movement of the spacecraft.

Mapping the spacecraft path

The spacecraft’s position, velocity and acceleration were calculated every second using the formulas

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at$$

for velocity and position. Acceleration was determined from the gravitational field at any given position plus the thrust given by the engine at the start of the simulation. Air resistance was ignored as it is negligible at >200 km above the Earth (National Weather Service, 2013).

Model of the LTO

The lunar transfer orbit starts when the satellite is at a right ascension of 0 degrees to the vernal equinox vector and a declination of +19.45 degrees (as it is located on the lunar plane) and the moon is located at $\theta = 1.1399 \pm 0.0001$ radians so $\theta = 65.310 \pm 0.005$ degrees (see Table 3: Model Results).

A new moon occurs on October 5, 2013, 0034:00 GMT (Calendar 365, 2013) and the moon travels at 1023 m/s at a radius of 384400 km, so it will take from 161922 ± 14 seconds for the moon to reach the solar vector which is equal to 1 day 20 hours, 58 mins, 42 ± 14 seconds.

Thus, to make use of the optimal position of the sun the satellite will have to fire its engines on October 3, 0335:18 GMT ± 14 seconds (and it must be located on the vernal equinox vector).

The rocket turns on at t=0, and provide a thrust of 400kN at an ISP of 300 for a total burn time of 320 seconds.

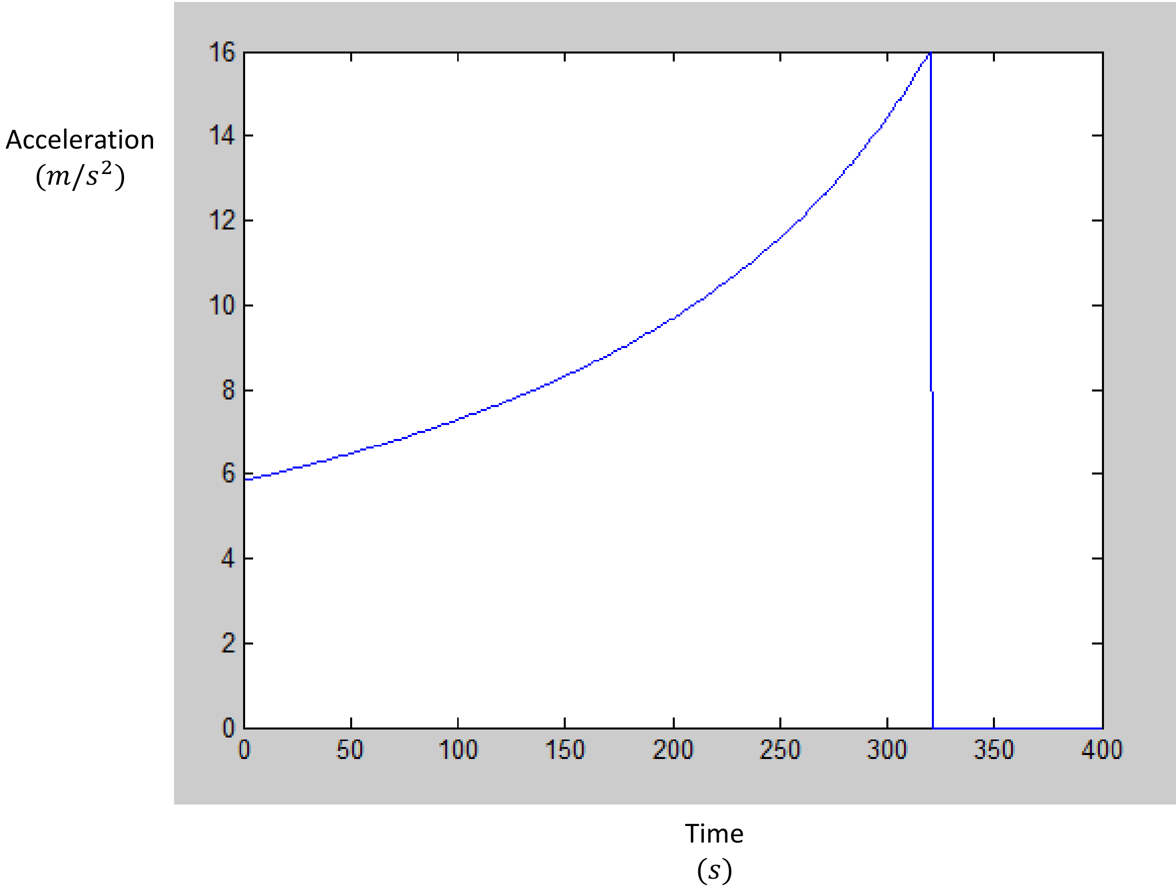
At t=233771 seconds or October 5, 2031:29 ± 14 sec the satellite is closest to the moon at a distance of ± 51 km above the surface (see Table 3: Model Results).

Table 3: Model Results

Start point of moon	Altitude (km)	Angle from any orbital plane (degrees)	Velocity (m/s)	Travel time (s)
1.1401	-4	-	-	-
1.14005	9	14.4	2069	233741
1.1400	23	11.5	2063	233751
1.1399	51	14.7	2052	233771
1.1398	79	14.8	2041	233791
1.1397	107	15.0	2030	233811
1.1395	164	15.3	2009	233850
1.390	306	16.15	1962	233949

Here the green numbers are the optimal start points and velocities whilst the yellow ones are within error ranges.

Figure 6: Acceleration due to thrust profile



Total Change in velocity

as $\Delta v = \int a dt = 2964 m/s$

Fuel Consumed

By the spacecraft thrust equation the final mass of the spacecraft was;

$$m = m_{initial} - \frac{t\, F_{thrust}}{g0\ ISP}$$

So

$$m = 2500kg$$

Of which 1970kg is payload and structure therefore;

Fuel consumed = 4350 kg

Fuel remaining = 530 kg

Plot of LTO

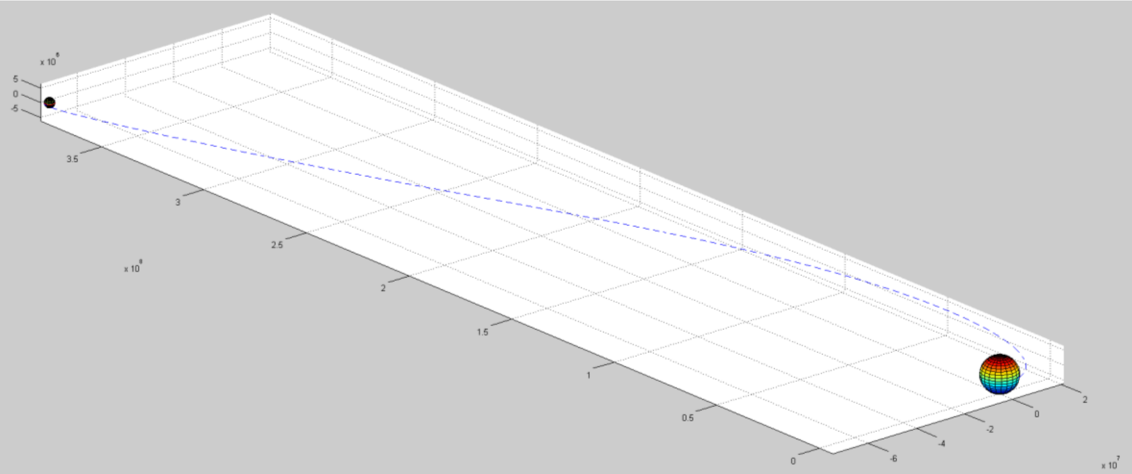


Figure 7: LTO with the moon in the top left corner and the earth into the bottom left corner

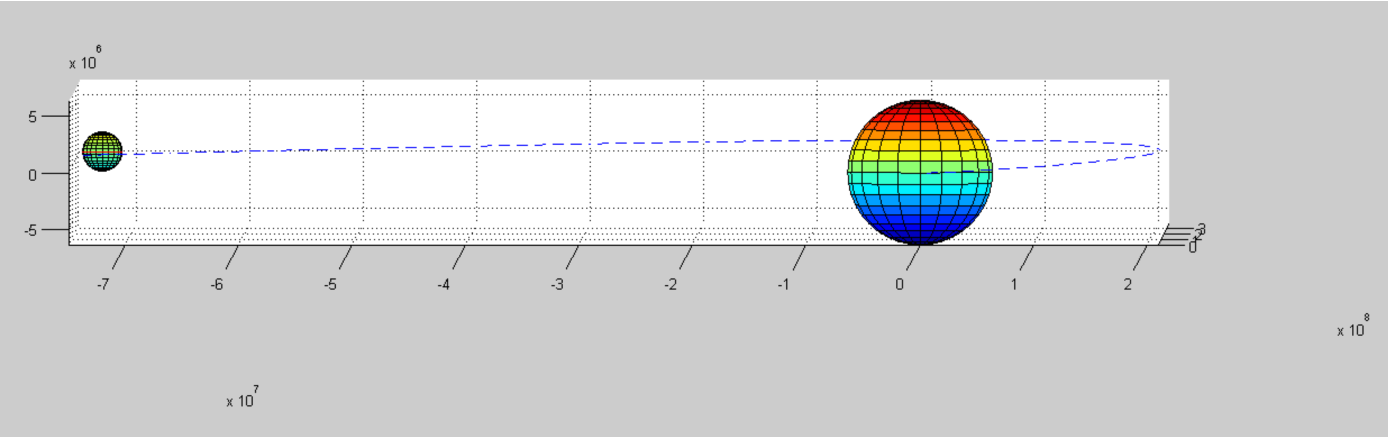


Figure 8: LTO as seen from the lunar plane.

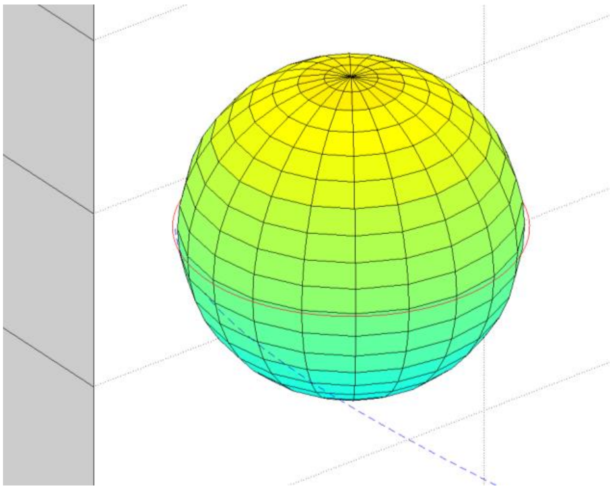


Figure 9: Satellite 51km above the moon at an inclination of 14.7 degrees.

Lunar Orbit (LO)

On October 5, 2031:29$\pm 20 sec$ GMT the satellite will reach the atmosphere of the moon at which time it will commence the lunar orbit burn.

From the LTO the spacecraft will enter the LO at anywhere from 100 km above the moon's surface to 50 km above the moon surface at angles to an orbital plane varying from 11.5 degrees to 14.8 degrees, and at velocities from 2063 m/s to 2041 m/s.

As there is 530 kg of remaining fuel the satellite has enough to undergo a Δv of 701m/s by the rocket equation.

$$F_{consumed} = F_{init} - (M_{struct} + M_{payload} + M_{fuel})/e^{\frac{\Delta v}{ISP*g0}} - M_{struct} - M_{payload}$$

The speed required to stay in orbit above the moon is $v = \sqrt{\frac{GM}{Rm+(50\pm 20)\times 10^3}} = 1645 \pm 20\ m/s.$

Thus from a lunar starting point of 1.1399, the satellite must gain a velocity radially towards the moon of $2052 \sin(14.7) = 520.7 m/s$ and slow down by $2050 \cos(14.7) - 1645 = 330 m/s$ so total $\Delta v = \sqrt{520^2 + 330^2} = 615 m/s$, which is within fuel limitations.

Table 4: LO Results

Start point of moon	Altitude (km)	Angle from any orbital plane (degrees)	Velocity (m/s)	Change in velocity needed (m/s)	Velocity to Orbit	Travel time (s)	Fuel Burnt (kg)
1.1401	-4	-	-	-	-	-	
1.14005	9	14.4	2069	617	1664	233741	473
1.1400	23	11.5	2063	554	1658	233751	429
1.1399	51	14.7	2052	615	1644	233771	471
1.1398	79	14.8	2041	644	1632	233791	491
1.1397	107	15.0	2030	656	1619	233811	500
1.1395	164	15.3	2009	677	1595	233850	514
1.390	306	16.2	1962	730	1539	233949	549

Here the green numbers are the optimal start points and velocities whilst the yellow ones are within quoted error ranges.

Specifications of motor on satellite

Table 3: Data for satellite (all for vacuum)

Engine	Merlin 1D rocket engine (Space X)
Engine Mass	220 kg
Thrust	40000 N (capable of 690000N)
ISP	300 sec (capable of 310 seconds)
Thrust to weight ratio	159.9
Payload Mass	500 kg
Burn Time LTO	320 sec
Propellant Mass	8330 kg
Structural Mass	1470 kg
Gross Mass	10300 kg

Data from Space Launch Report (2013) and SpaceX (2013)

Summary

Specifications of LEO

Apogee Altitude: 310 ± 5.41 km
Perigee Altitude: 310 ± 5.41 km
Inclination: 28.5 ± 0.15 degrees
Argument of Perigee: N/A
RAAN: 220.71 ± 0.11 degrees
Eccentricity: 0 ± 0.004

Lunar Plane transfer

Inclination of lunar plane to celestial plane: 9.05 degrees (as of October 2013)
Fuel used: 3480 ± 110 kg
Change in velocity: $\Delta v = -1215 \pm 40 m/s$

RAAN corrections

Change in velocity: $14.8 \pm 0.2\ m/s$
Fuel used: $12.4 \pm 0.2 kg$

Payload Mass Breakdown

Total Payload Mass: 10.3×10^3 kg
Fuel Mass: 8330 kg
Structural Mass: 1470 kg
Satellite Mass: 500 kg
Percent Structure: 15%
Engine Mass: 220 kg
Satellite Engine: Merlin 1D rocket (SpaceX)

LTO Specifications

Burn Date: October 3, 0335:18 GMT ± 14 seconds
Position: On the vernal equinox vector
Initial Altitude: 310 km
Burn time: 320 seconds
Fuel Burnt: 4350 kg
Fuel remaining: 530 kg
Thrust: 400 kN
Change in velocity: 2964 m/s
Peak acceleration: $16m/s^2$
Time to moon: 233771 ± 20 seconds
Final velocity at moon: $2052 \pm 11m/s$
Altitude: $51 \pm 28km$
Inclination to LO: $14.7 \pm 0.3\ degrees$

LO Specifications

Burn Date: October 5, 2031:29$\pm 20sec$ GMT
Fuel Burnt: 471 ± 42 kg
Fuel Remaining: 58 ± 42 kg
Initial Velocity: $2052 \pm 11m/s$
Change in Velocity: $615 \pm 2m/s$
Final Velocity: $1645 \pm 20\ m/s$
Final Altitude: 51 ± 20km

Errors

All errors are calculated using the formula: $\frac{\Delta u}{u} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$, where $u = ab$.

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