Recurrent Neural Networks

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Motivation

MLTS Exercise 10

- How can we utilize neural networks to process sequential data?
- Passing complete sequences to a conventional network is computationally very expensive
  - long training
  - gradients converge very slowly
  - Temporal context gets lost
- **Solution**: Adapt the time related characteristic into the architecture of the net

Name Entity Recognition

MLTS Exercise 10

Input: Harry Potter invented a new spell.

Dictionary:

$$\begin{pmatrix} \text{a} \\ \text{aaron} \\ \dots \\ \text{harry} \\ \dots \\ \text{potter} \\ \dots \\ \text{spell} \\ \dots \\ \text{zoo} \end{pmatrix} \begin{matrix} 1 \\ 2 \\ \dots \\ 2{,}039 \\ \dots \\ 6{,}453 \\ \dots \\ 8{,}940 \\ \dots \\ 10{,}000 \end{matrix}$$

Name Entity Recognition

MLTS Exercise 10

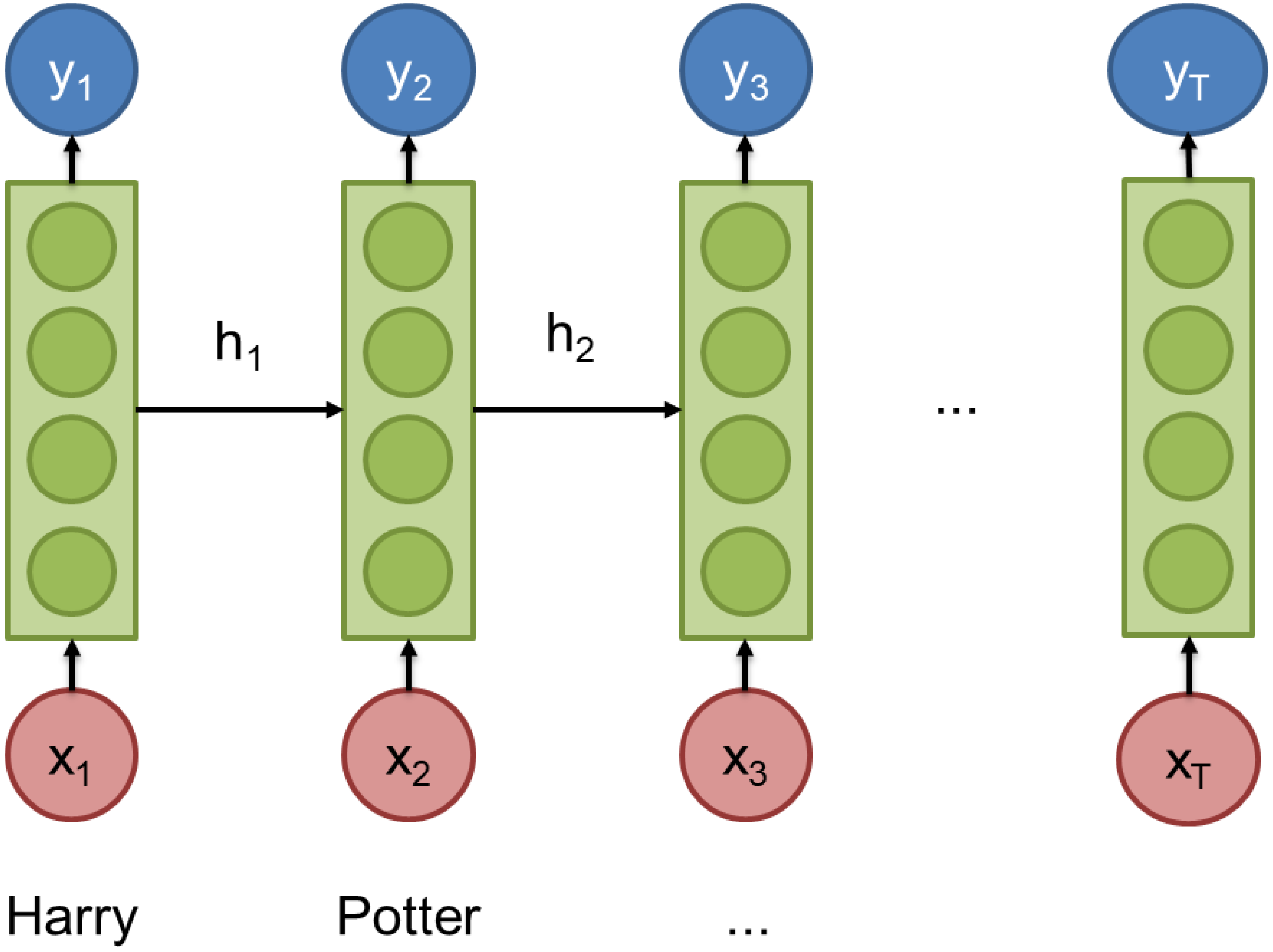
Input: <u>Harry</u> Potter invented a new <u>spell</u>.

Dictionary:

$$
\begin{pmatrix} \text{a} \\ \text{aaron} \\ \dots \\ \text{harry} \\ \dots \\ \text{potter} \\ \dots \\ \text{spell} \\ \dots \\ \text{zoo} \end{pmatrix}
\begin{matrix} 1 \\ 2 \\ \dots \\ 2{,}039 \\ \dots \\ 6{,}453 \\ \dots \\ 8{,}940 \\ \dots \\ 10{,}000 \end{matrix}
\qquad
x^1: \begin{pmatrix} 0 \\ \dots \\ 0 \\ 1 \\ 0 \\ \dots \\ \dots \\ \dots \\ \dots \\ 0 \end{pmatrix}
\qquad
x^6: \begin{pmatrix} 0 \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}
$$

Recurrent Neural Networks (RNN)

MLTS Exercise 10



RNN Forward Propagation

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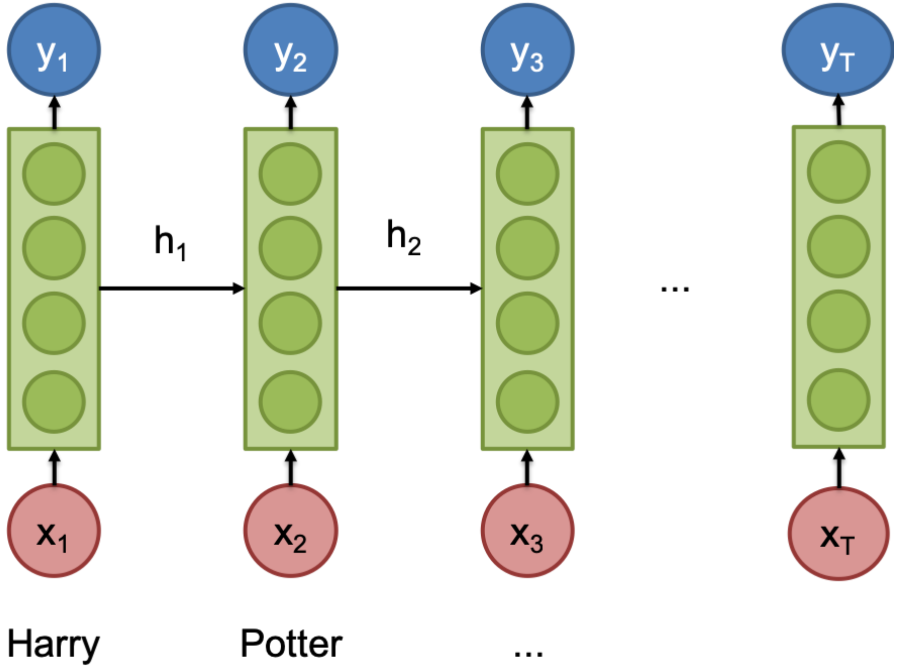
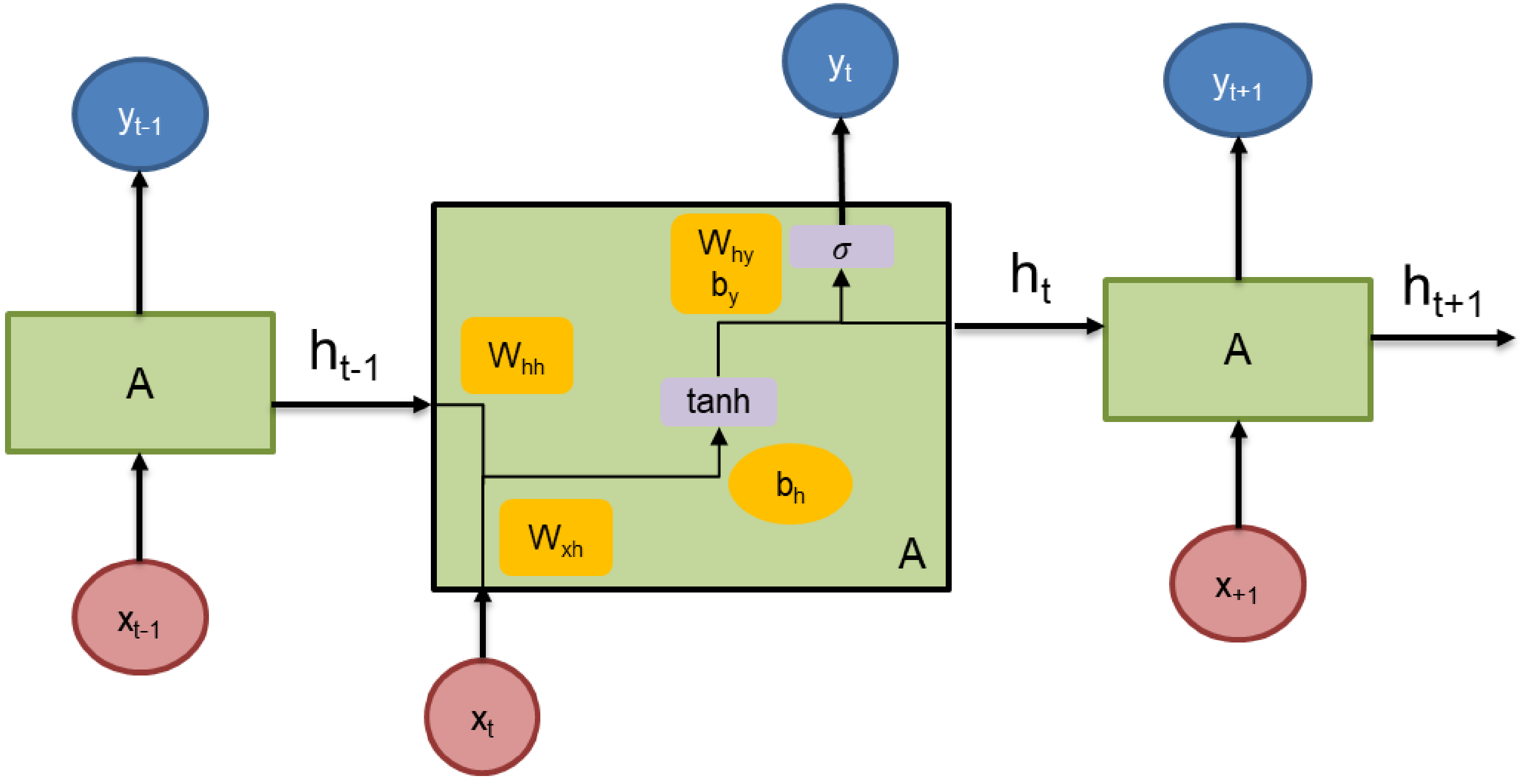
$$h_t = tanh(W_{hh}h_{t-1} + W_{xh}x_t + b_h)$$
$$y_t = \sigma(W_{hy}h_t + b_y)$$

σ: sigmoid / softmax



The RNN Cell Up Close

MLTS Exercise 10

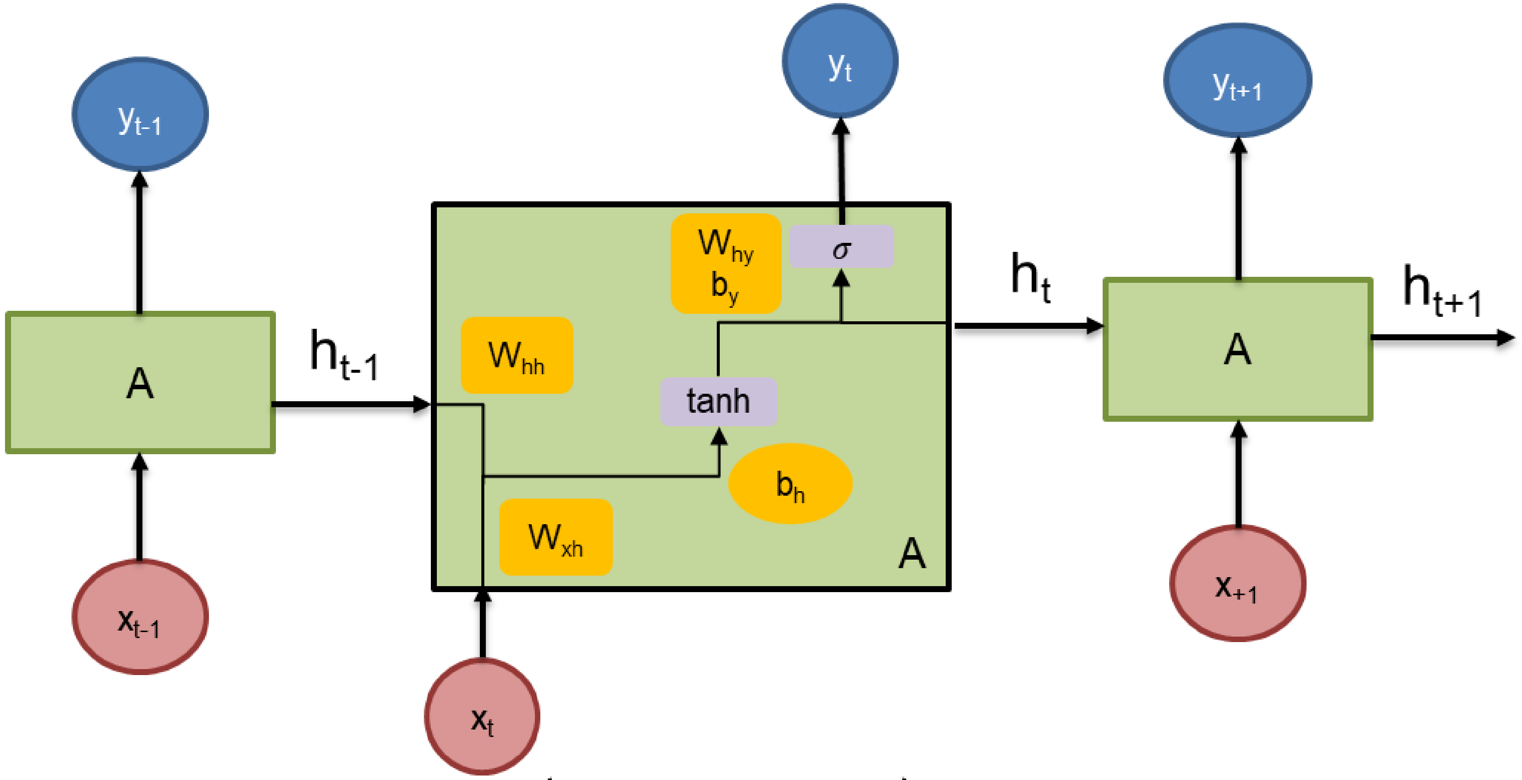


Hidden state: $h_t = tanh(W_{hh}h_{t-1} + W_{xh}x_t + b_h)$

W_{hh}: Weights of previous hidden state
W_{xh}: Weights of the input
b_h: Bias for state update

The RNN Cell Up Close

MLTS Exercise 10



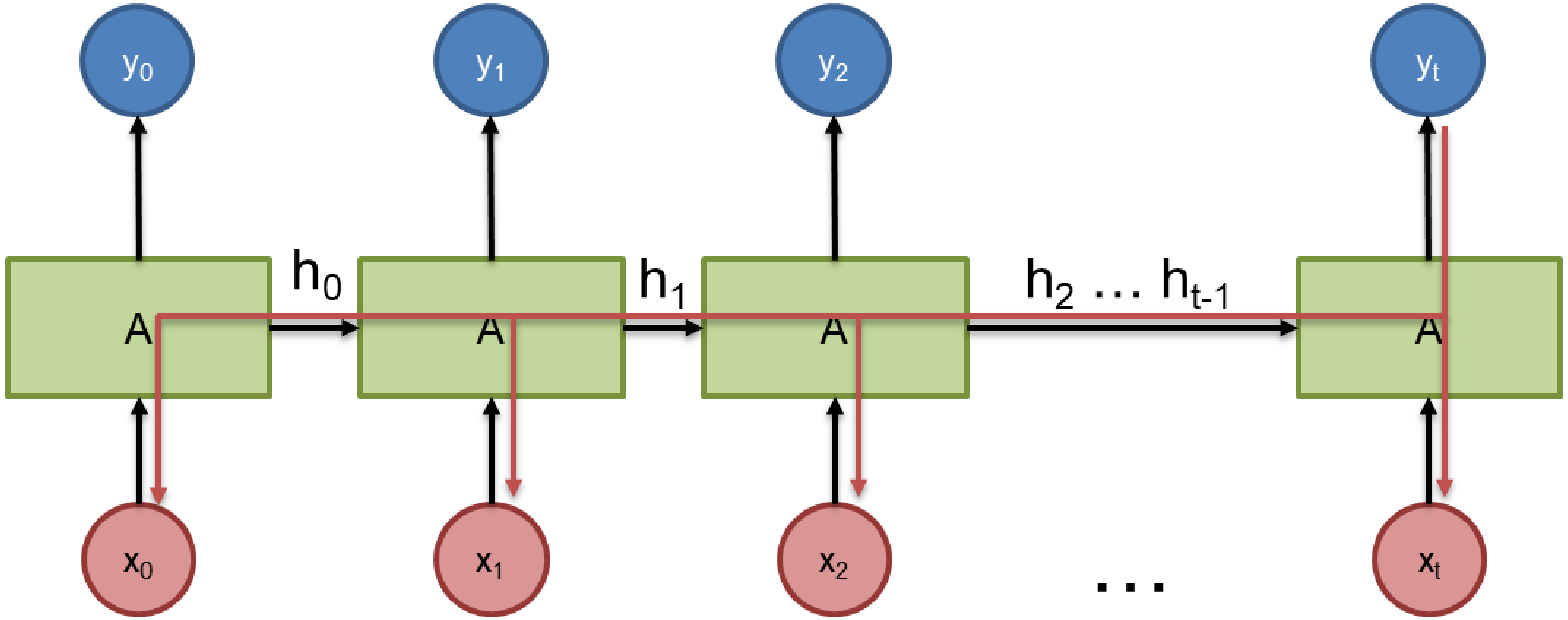
Output: $$y_t = \sigma(W_{hy} h_t + b_y)$$

W_{hy}: Weights of current hidden state

b_y: Bias for output update

Backpropagation Through Time (BPTT)

MLTS Exercise 10



- The backward pass takes activations off the stack to compute the error derivatives at each time step
- Backpropagate through time all the way to the initial states to get the gradient of the cost function with respect to each initial state

(BPTT) Calculate The Cost

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Total cost is the sum of losses at each time step

$$C(y, \hat{y}) = \sum_t C_t$$

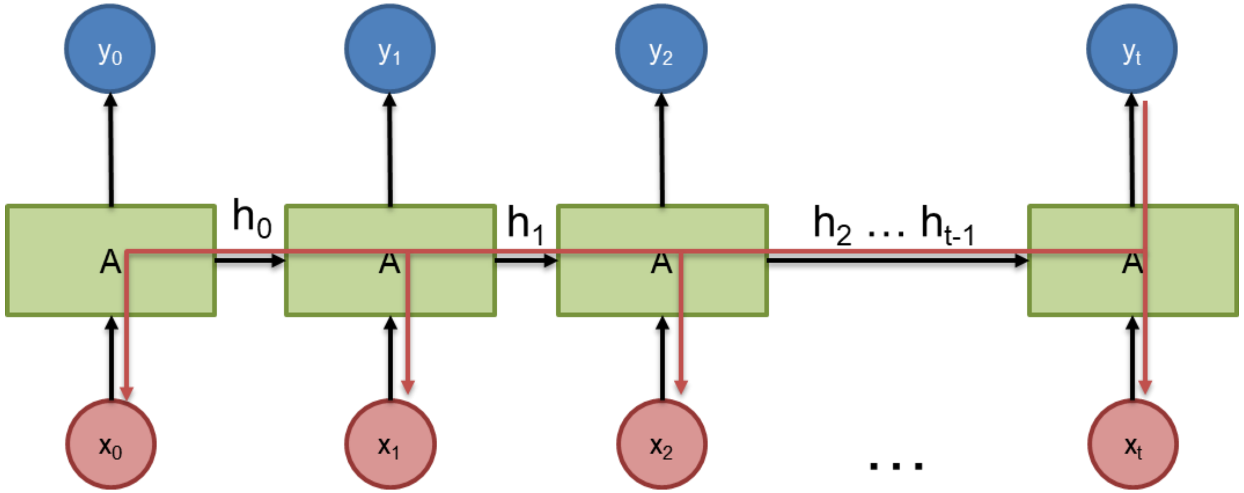
$$C_t = \frac{1}{2}||y_t - \hat{y_t}||^2$$

Compute the gradient of the loss

$$\nabla C = [\nabla W_{xh}, \nabla W_{hh}, \nabla W_{hy}, \nabla b_h, \nabla b_y, \nabla h]$$

Backpropagation Through Time (BPTT)

MLTS Exercise 10



$$h_t = tanh(W_{xh}x_t + W_{hh}h_{t-1} + b_h)$$
$$tanh'(x) = 1 - tanh(x)^2$$
$$\nabla W_{xh} = tanh'(W_{hx}x_t + W_{hh}h_{t-1} + b_h) \cdot x_t^T$$
$$\nabla W_{hh} = tanh'(W_{hx}x_t + W_{hh}h_{t-1} + b_h) \cdot h_{t-1}^T$$
$$\nabla b_h = \sum_{batch} tanh'(W_{hx}x_t + W_{hh}h_{t-1} + b_h)$$
$$\nabla x_t = W_{hx}^T \cdot tanh'(W_{hx}x_t + W_{hh}h_{t-1} + b_h)$$
$$\nabla h_{t-1} = W_{hh}^T \cdot tanh'(W_{hx}x_{t-1} + W_{hh}h_{t-1} + b_h)$$

(BPTT) Update Weights and Biases

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- Given the learning rate η update the weights and biases in the matrix W in the negative gradient direction

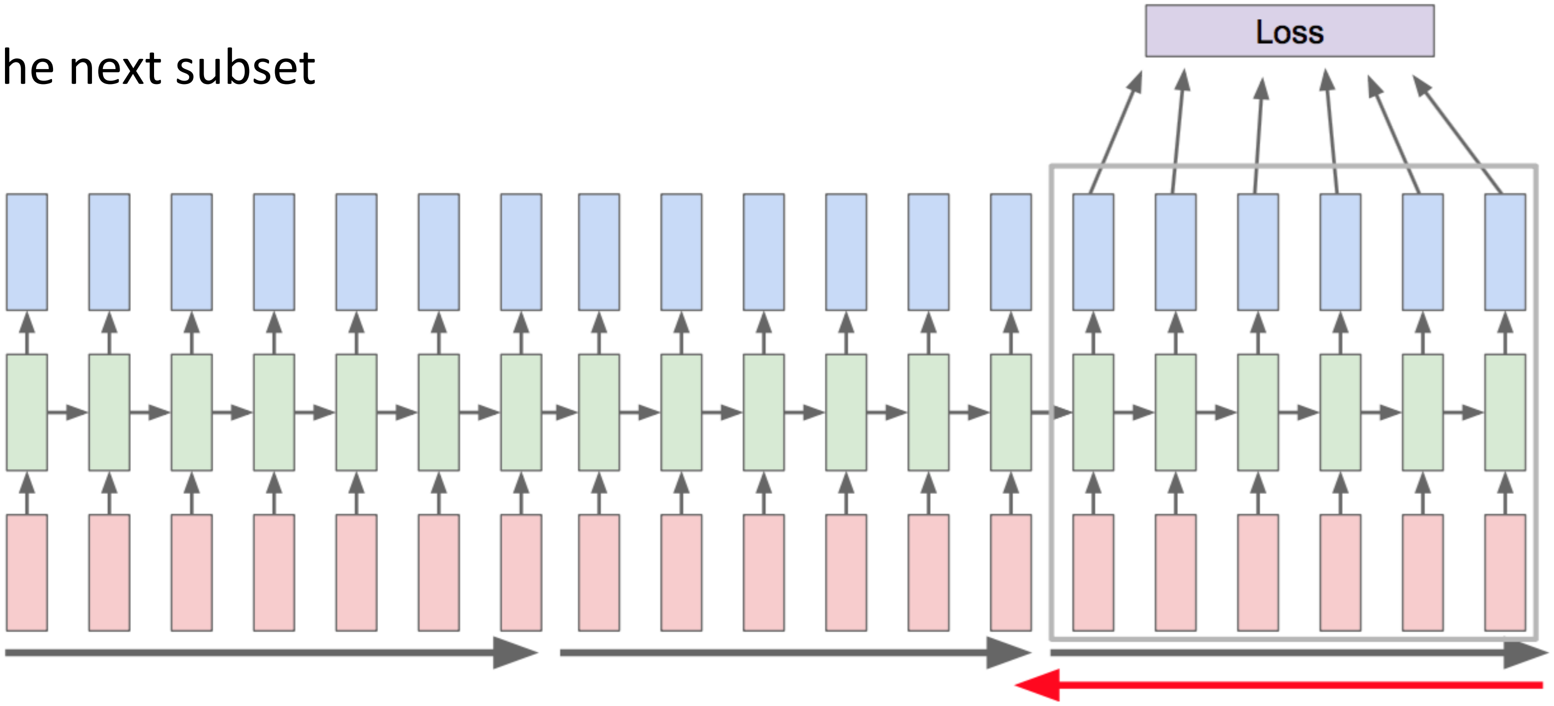
$$W = W - \eta \nabla C$$

- **Problem**: backpropagating through the whole sequence the parameter update is computational expensive
- **Solution**: truncated backpropagation through time (TBPTT)

Truncated Backpropagation Through Time

MLTS Exercise 10

- Instead of passing through the complete sequence, forward pass through a subset
- Backpropagate through the subset
- Continue with the next subset



Adapted from https://tjmachinelearning.com/lectures/guest/rnnadv/rnnadv.html

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Long Short-Term Memory Network

The Problem With Long-Term Dependency

MLTS Exercise 10

Backward pass is completely linear:

- If weights are large (> 1), the gradient grows exponentially
- If weights are small (< 1), the gradients shrink exponentially

If we train on long sequences (> 100 time steps) the gradient may easily *explode* or *vanish*

The cat, which already ate ..., was/were full.

The Problem With Long-Term Dependency

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- If weights are large (> 1), the gradient grows exponentially
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If we train on long sequences (> 100 time steps) the gradient may easily *explode* or *vanish*

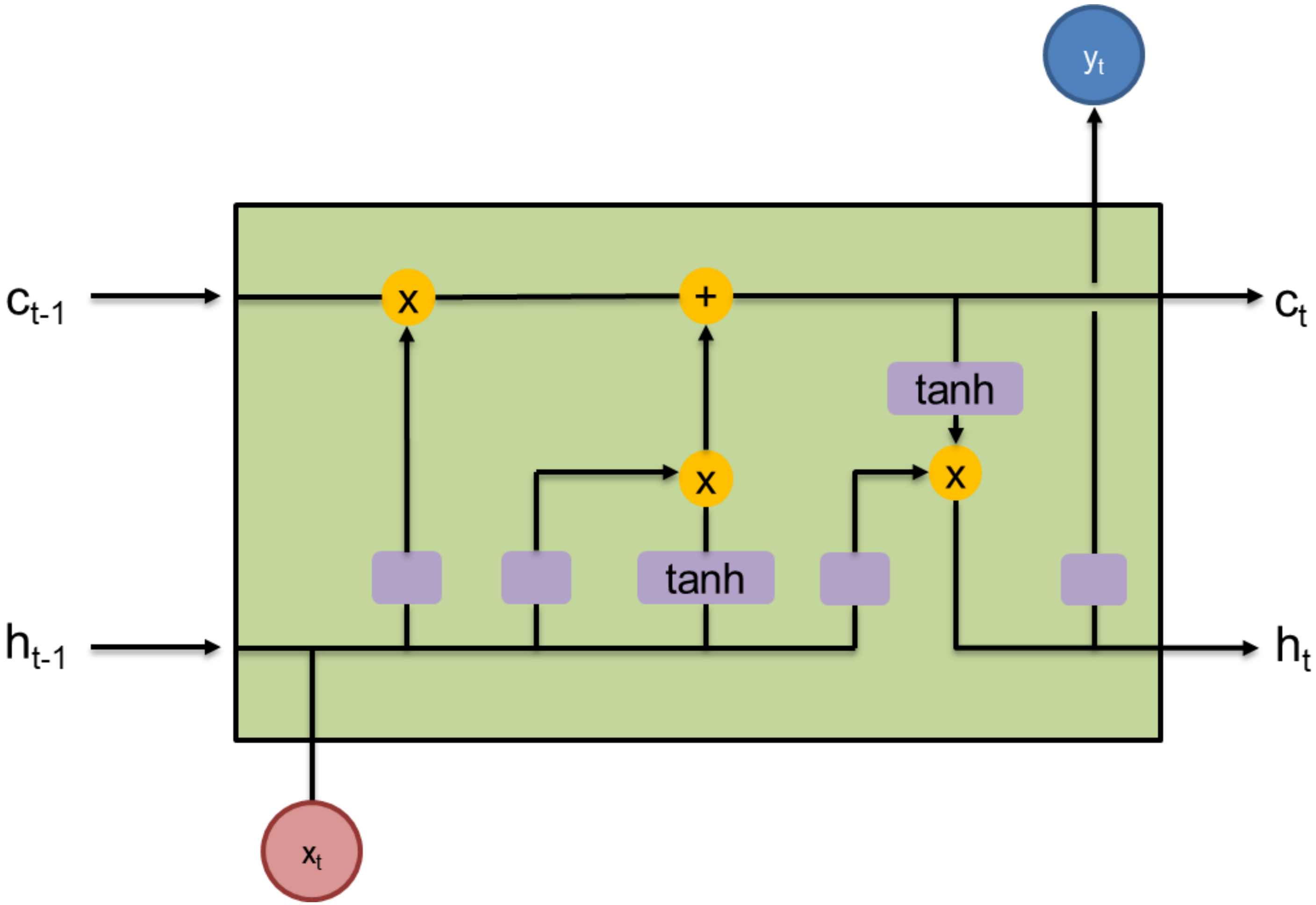
Solution: Change RNN architecture

- e.g.: Long Short Term Memory Networks (LSTMs)

Long Short-Term Memory Network

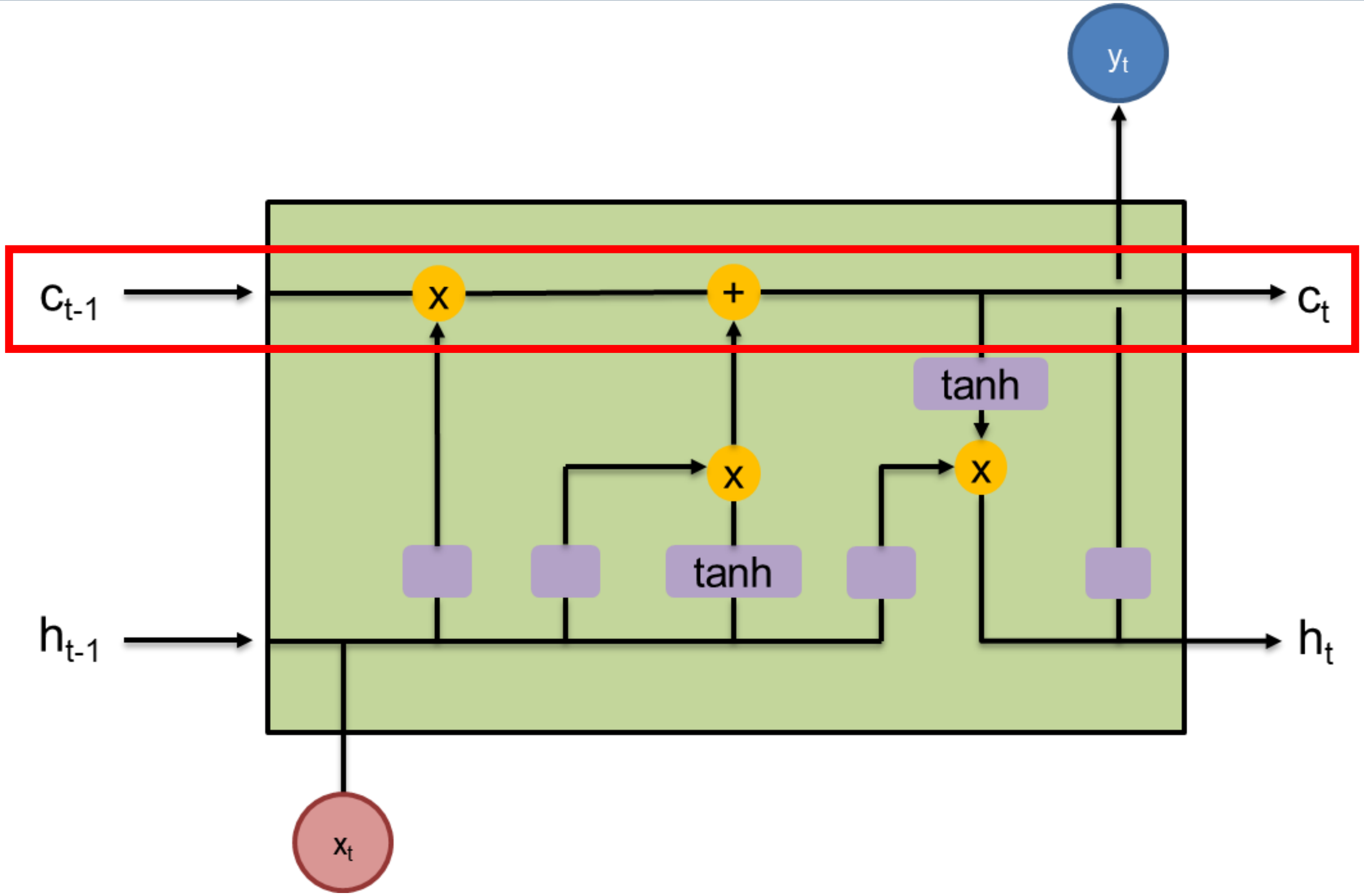
MLTS Exercise 10

LSTM unit:



Long Short-Term Memory Network

MLTS Exercise 10

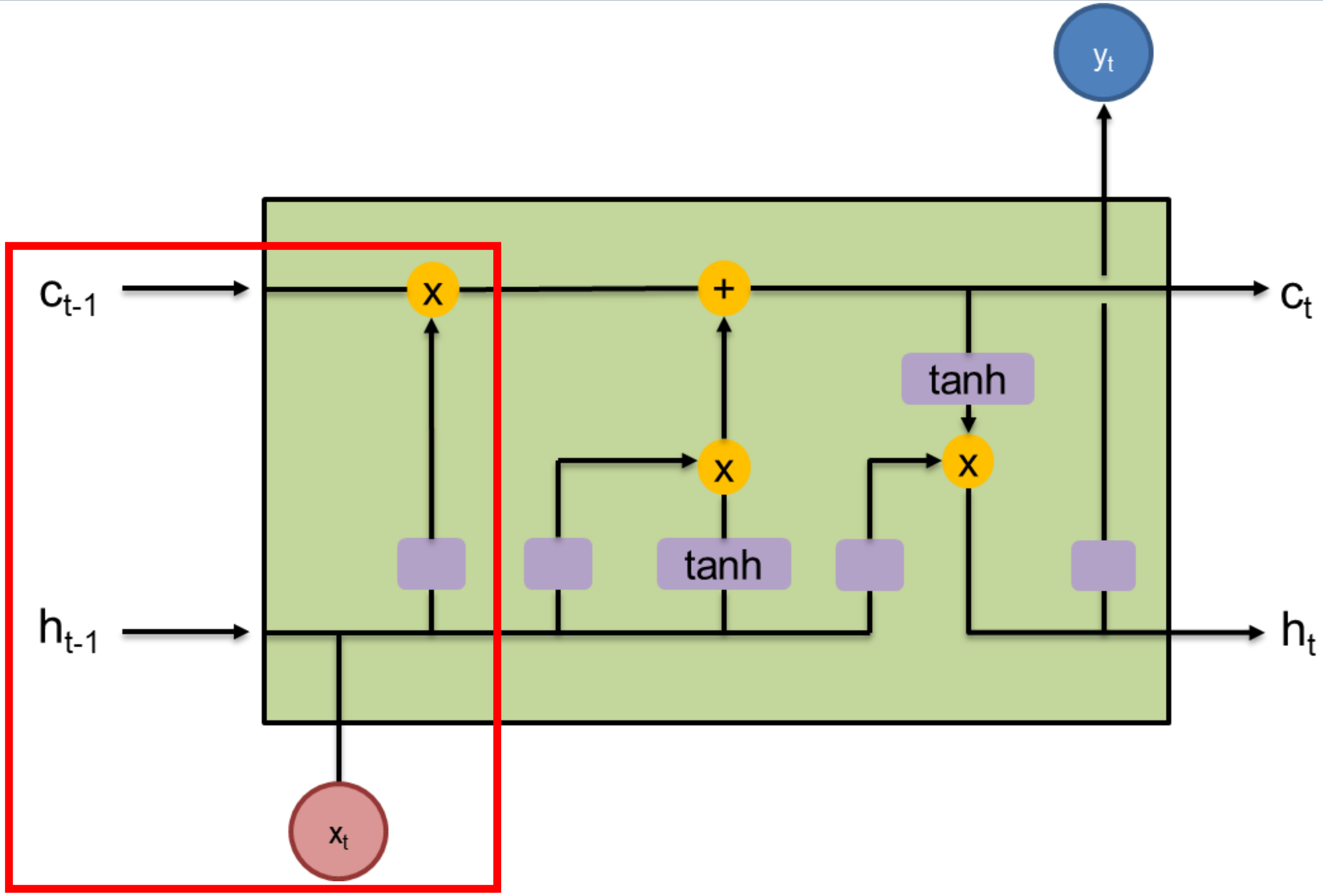


Cell state:

- Only minor linear interactions
- Easy flow of information, relatively unchanged
- Adding or removing information to the cell state is regulated by gates

Long Short-Term Memory Network

MLTS Exercise 10



Forget gate:

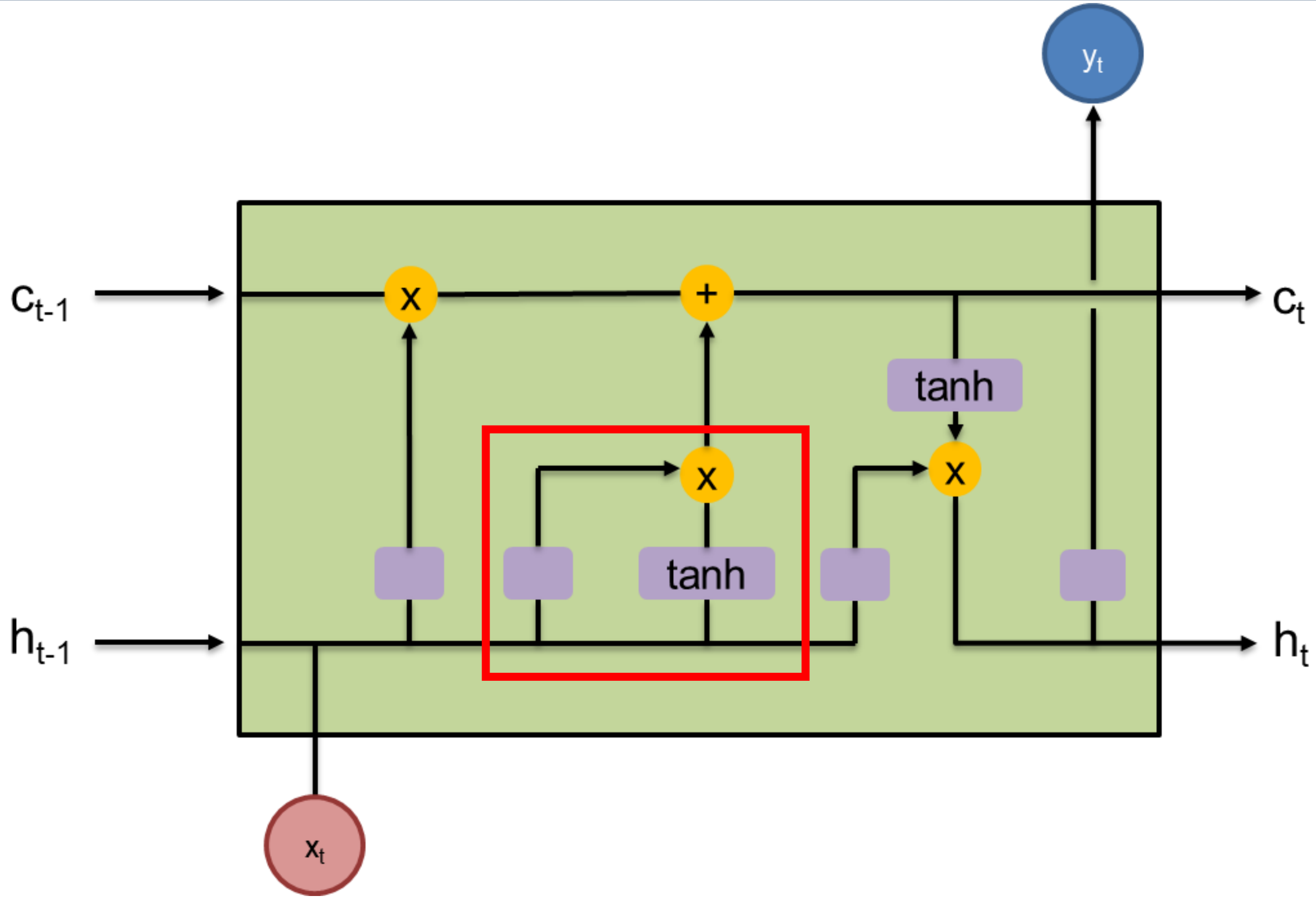
Decide how much of the previous cell state will be forgotten

Sigmoid layer squashes h_{t-1} and x_t between 0 and 1

$$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f)$$

Long Short-Term Memory Network

MLTS Exercise 10



Input gate:

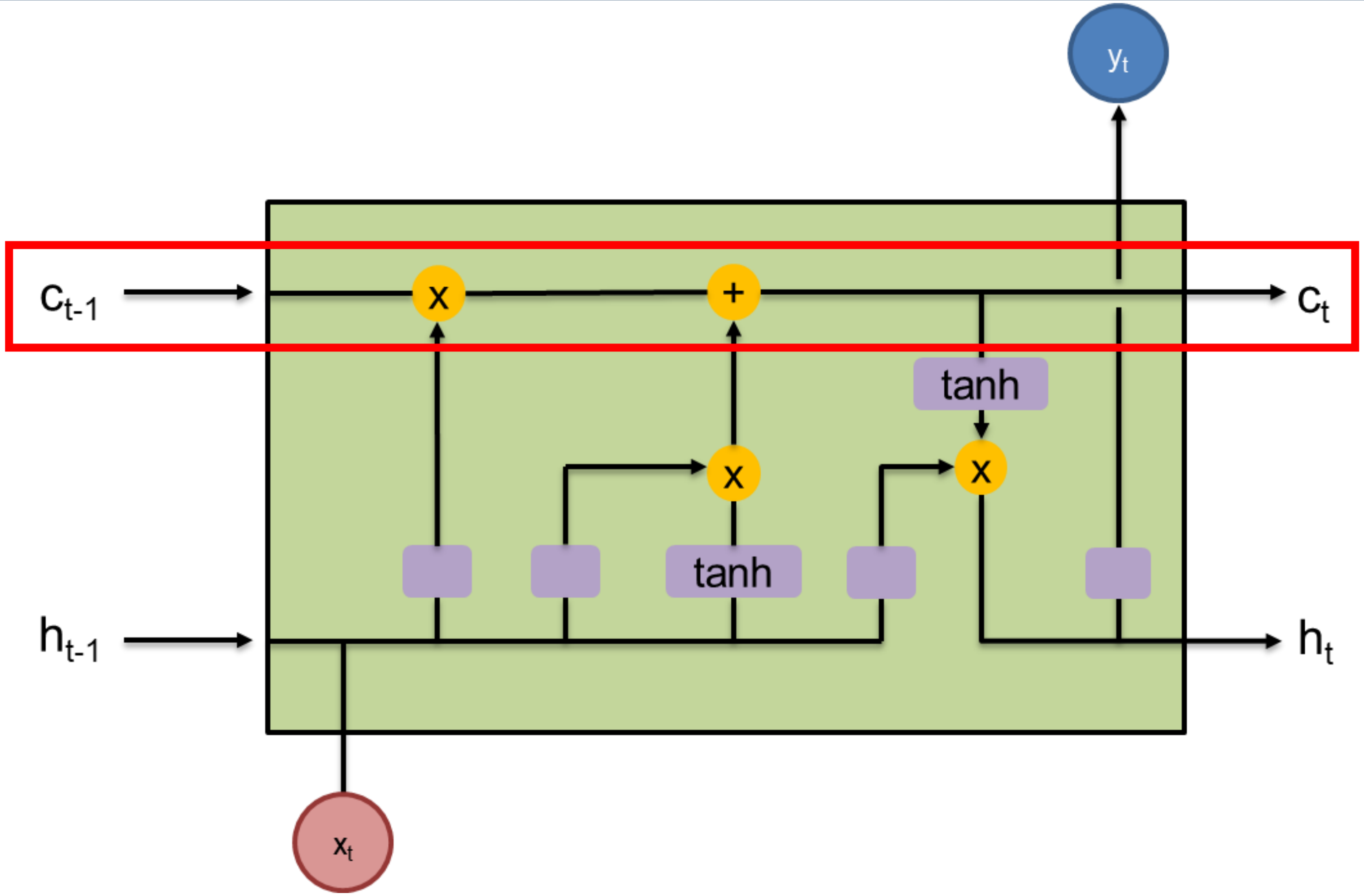
Decide what information we are going to store in the cell state

$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i)$$

$$g_t = tanh(W_c \cdot [h_{t-1}, x_t] + b_c)$$

Long Short-Term Memory Network

MLTS Exercise 10



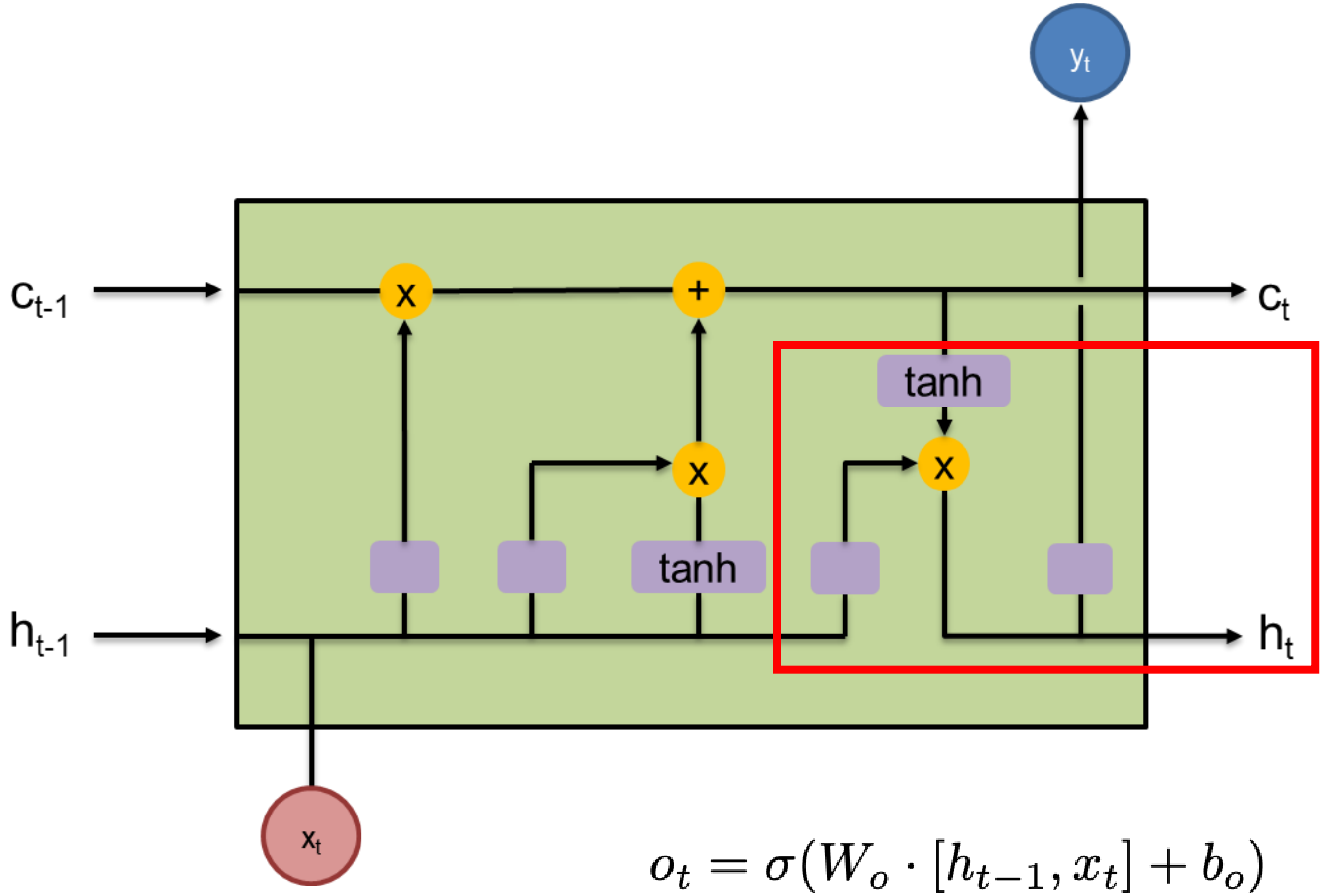
Combining values:

Update the old cell state c_{t-1} into c_t

$$c_t = f_t \odot c_{t-1} + i_t \odot g_t$$

Long Short-Term Memory Network

MLTS Exercise 10



Output gate:

Define output based on the cell state

$$o_t = \sigma(W_o \cdot [h_{t-1}, x_t] + b_o)$$
$$h_t = o_t \odot tanh(c_t)$$
$$y_t = \sigma(W_y \cdot h_t + b_y)$$

Gated Recurrent Units

MLTS Exercise 10

LSTMs are great in order to avoid vanishing gradients

BUT: Loads of weights and biases which need to be optimized → difficult and slow training

Gated Recurrent Units (GRU) reduce the number of parameters to simplify training

Basically, hidden state and cell state are combined into a single parameter

- Forget and input gate are merged into an update gate

Concept was first presented by Cho et al. in 2014