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1 Logic and Proofs

1.1 Propositional Logic

Proposition is a statement that is **either** true or false, but not both at the same time. We usually represent it with variables like p, q, and r.
e.g. "The sky is blue." is a proposition, but "Listen to me" is not.

Logical Connectives

- Negation: $\neg p$. It is not the case that p.
- Conjunction: $p \wedge q$. "and"
- Disjunction: $p \vee q$. "or"
- Implication: $p \to q$. If p then q, q if p, q is a consequence of p, p only if q
- biconditional: $p \leftrightarrow q$. $(p \to q) \wedge (q \to p)$, p if and only if q

Truth Table

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Example

Find the truth value of $\boldsymbol{(p \vee q) \to \neg r}$

p	q	r	$p \vee q$	$\neg r$	$(p \vee q) \to \neg r$
T	T	T	T	F	F
T	T	F	T	T	T
T	F	T	T	F	F
T	F	F	T	T	T
F	T	T	T	F	F
F	T	F	T	T	T
F	F	T	F	F	T
F	F	F	F	T	T

Logical Equivalence (L.E.)

Two statements are logically equivalent if they always have the same truth value in every possible scenario.
e.g. p and q are biconditional, i.e. $p \leftrightarrow q$, means that p and q are logically equivalent.
There are two ways to prove L.E., one is by truth table, the other is by applying logical laws.

Show that $p \to q$ is logically equivalent to $\neg q \to \neg p$

p	q	$p \to q$	$\neg q \to \neg p$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

1.2 Application of Propositional Logic