

Contents

1	Sequences	1
1.1	Definition of Sequences	1
1.2	a_n as $n \rightarrow \infty$	1
1.3	Convergence/Divergence	1
1.4	Monotone and Bounded Sequences	2
2	Series	2
2.1	Notation	2
2.2	Partial Sum	2
2.3	Types of Series	2
2.4	Telescoping	2
3	Convergence Tests	2
3.1	Divergence Test	2
3.2	Comparison	2
3.3	Limit Comparison Test	2
3.4	Ratio Test	2
3.5	Root Test	2
3.6	Integral Test	2
3.7	Alternating Series Test	2
3.8	Absolute vs Conditional Convergence	2
4	Power Series	2
4.1	Definition	2
4.2	Radius and Interval of Convergence	2
4.3	Differentiation and Integration	2
5	Taylor and Maclaurin Series	2
5.1	Taylor Series	2
5.2	Maclaurin Series	2
5.3	Common Expansions	3
5.4	Lagrange Error Bound for Taylor Series	3
5.5	Limits and Approximations	3
6	Applications	3
6.1	Numerical Approximation	3
6.2	Solving ODEs	3
6.3	Non-elementary Integrals	3

1 Sequences

1.1 Definition of Sequences

A sequence is an ordered list of numbers written in the form

$$a_1, a_2, a_3, \dots, a_n, \quad \text{or} \quad a_n \quad \text{or} \quad \{a_n\}_{n=1}^{\infty}$$

It is usually represented as a function whose domain is the set of positive integers:

$$a_n = f(n)$$

1.2 a_n as $n \rightarrow \infty$

The **limit of a sequence** as $n \rightarrow \infty$ describes the long-term behavior of the sequence:

$$\lim_{n \rightarrow \infty} a_n = L$$

means that the terms of the sequence get arbitrarily close to L as n becomes large. If such a number L exists, we say the sequence **converges** to L . Otherwise, it **diverges**.

1.3 Convergence/Divergence

A sequence $\{a_n\}$ **converges** to $L \in \mathbb{R}$ if

$$\lim_{n \rightarrow \infty} a_n = L \quad \Longleftrightarrow \quad \forall \varepsilon > 0, \exists N \in \mathbb{N}, \forall n > N, |a_n - L| < \varepsilon.$$

Otherwise, the sequence **diverges**.

1.4 Monotone and Bounded Sequences

2 Series

2.1 Notation

2.2 Partial Sum

2.3 Types of Series

2.4 Telescoping

3 Convergence Tests

3.1 Divergence Test

3.2 Comparison

3.3 Limit Comparison Test

3.4 Ratio Test

3.5 Root Test

3.6 Integral Test

3.7 Alternating Series Test

3.8 Absolute vs Conditional Convergence

4 Power Series

4.1 Definition

4.2 Radius and Interval of Convergence

4.3 Differentiation and Integration

5 Taylor and Maclaurin Series

5.1 Taylor Series

A Taylor series is an infinite sum that represents a function as a power series centered at a point a . If a function $f(x)$ is infinitely differentiable at $x = a$, then its Taylor series is given by:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n$$

This expansion approximates the function near $x = a$

5.2 Maclaurin Series

A Maclaurin Series is a special case of Taylor Series centered at $x = 0$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

5.3 Common Expansions

5.4 Lagrange Error Bound for Taylor Series

5.5 Limits and Approximations

6 Applications

6.1 Numerical Approximation

6.2 Solving ODEs

6.3 Non-elementary Integrals