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1 Sequences

1.1 Definition of Sequences

A sequence is an ordered list of numbers written in the form

$$a_1, a_2, a_3, \ldots, a_n, \quad \textbf{or} \quad a_n \quad \textbf{or} \quad \{a_n\}_{n=1}^{\infty}$$

It is usually represented as a function whose domain is the set of positive integers:

$$a_n = f(n)$$

1.2 a_n as $n \to \infty$

The **limit of a sequence** as $n \to \infty$ describes the long-term behavior of the sequence:

$$\lim_{n\to\infty} a_n = L$$

means that the terms of the sequence get arbitrarily close to L as n becomes large. If such a number L exists, we say the sequence **converges** to L. Otherwise, it **diverges**.

1.3 Convergence/Divergence

A sequence $\{a_n\}$ **converges** to $L \in \mathbb{R}$ if

$$\lim_{n\to\infty} a_n = L \quad \Longleftrightarrow \quad \forall\varepsilon > 0,\ \exists N \in \mathbb{N},\ \forall n > N,\ |a_n - L| < \varepsilon.$$

Otherwise, the sequence **diverges**.

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5.1 Taylor Series

A Taylor series is an infinite sum that represents a function as a power series centered at a point a. If a function $f(x)$ is infinitely differentiable at $x = a$, then its Taylor series is given by:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n$$

This expansion approximates the function near $x = a$

5.2 Maclaurin Series

A Maclaurin Series is a special case of Taylor Series centered at $x = 0$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}x^n$$