

# Contents

<b>1</b>	<b>Vector and Linear Equations</b>	<b>1</b>
1.1	System of Linear Equations . . . . .	1
1.2	Row Reduction and Echelon Forms . . . . .	1
1.3	Linear Independence, Span, and Dimensions . . . . .	1
<b>2</b>	<b>Matrix Algebra and Linear Transformation</b>	<b>1</b>
2.1	Matrix Operations . . . . .	1
2.2	Properties of Matrices . . . . .	1
2.3	Inverse and Elementary Matrices . . . . .	1
2.4	Permutations and Transposes . . . . .	1
2.5	Derivatives and Finite Difference Matrices . . . . .	1
2.6	Linear Transformations and Matrix Representation . . . . .	1
<b>3</b>	<b>The Four Fundamental Subspaces</b>	<b>1</b>
3.1	Column Space . . . . .	1
3.2	Null Space . . . . .	1
3.3	Row Space . . . . .	1
3.4	Left Null Space . . . . .	1
3.5	Rank-Nullity Theorem . . . . .	1
3.6	Orthogonality Relations Between Subspaces . . . . .	1
<b>4</b>	<b>Determinants</b>	<b>1</b>
4.1	Introduction to Determinants . . . . .	1
4.2	Properties of Determinants . . . . .	1
4.3	Cramer's Rule, Area, Volume, and Linear Transformations . . . . .	1
<b>5</b>	<b>Orthogonality and Projections</b>	<b>1</b>
5.1	Inner Product, Length, and Distance . . . . .	1
5.2	Orthogonal and Orthonormal Sets . . . . .	1
5.3	Gram-Schmidt Process . . . . .	1
5.4	Least-Square Solutions: $Ax \approx b$ . . . . .	1
5.5	Inner Product Spaces and Generalizations . . . . .	1
<b>6</b>	<b>Eigenvalues and Eigenvectors</b>	<b>1</b>
6.1	Eigenvalues and Eigenvectors . . . . .	1
6.2	The Characteristic Equation . . . . .	1
6.3	Diagonalization . . . . .	1
6.4	Complex Numbers, Vectors, and Matrices . . . . .	1
6.5	Solving Linear Differential Equations . . . . .	1
<b>7</b>	<b>Symmetric Matrices and Quadratic Forms</b>	<b>1</b>
7.1	Spectral Theorem . . . . .	1
7.2	Quadratic Forms and Diagonalization . . . . .	1
7.3	Positive/Negative Definite Forms . . . . .	1
7.4	Constrained Optimization via Quadratic Forms . . . . .	1

<b>8</b>	<b>Matrix Factorizations</b>	<b>1</b>
8.1	LU Decomposition . . . . .	1
8.2	QR Decomposition . . . . .	1
<b>9</b>	<b>Linear Algebra in Optimization</b>	<b>1</b>
9.1	Singular Value Decomposition (SVD) . . . . .	1
9.2	Principal Component Analysis (PCA by SVD) . . . . .	1
9.3	Minimizing a Multivariable Function . . . . .	1
9.4	Backpropagation and Stochastic Gradient Descent . . . . .	1
9.5	Constraints, Lagrange Multipliers, Minimum Norms . . . . .	1
9.6	Linear Programming, Game Theory, and Duality . . . . .	1

## 1 Vector and Linear Equations

### 1.1 System of Linear Equations

### 1.2 Row Reduction and Echelon Forms

### 1.3 Linear Independence, Span, and Dimensions

## 2 Matrix Algebra and Linear Transformation

### 2.1 Matrix Operations

### 2.2 Properties of Matrices

### 2.3 Inverse and Elementary Matrices

### 2.4 Permutations and Transposes

### 2.5 Derivatives and Finite Difference Matrices

### 2.6 Linear Transformations and Matrix Representation

## 3 The Four Fundamental Subspaces

### 3.1 Column Space

### 3.2 Null Space

### 3.3 Row Space

### 3.4 Left Null Space

### 3.5 Rank-Nullity Theorem

### 3.6 Orthogonality Relations Between Subspaces

## 4 Determinants

### 4.1 Introduction to Determinants

### 4.2 Properties of Determinants

### 4.3 Cramer's Rule, Area, Volume, and Linear Transformations

## 5 Orthogonality and Projections

### 5.1 Inner Product, Length, and Distance

### 5.2 Orthogonal and Orthonormal Sets

### 5.3 Gram-Schmidt Process

### 5.4 Least-Square Solutions: $Ax \approx b$

### 5.5 Inner Product Spaces and Generalizations

## 6 Eigenvalues and Eigenvectors

### 6.1 Eigenvalues and Eigenvectors

### 6.2 The Characteristic Equation

### 6.3 Diagonalization