***Continuous time signals**: Linearity and Time Invariance*

Time invariant: if *x(t)* is the input and *y(t)* the output, and $y(t)=T\left[x(t)\right]$ then: $y(t)\Big\|_{(t-t_o)} = y(t)\Big\|_{x(t-t_o)}$	*Linear system:* if *x(t)* is the input, *y(t)* the output, and $y(t)=T\left[x(t)\right]$ then: $T\left\{\alpha_1 x_1(t)+\alpha_2 x_2(t)\right\}=\alpha_1 y_1(t)+\alpha_2 y_2(t)$ where α_1 and α_2 are constants.

Singularity Functions** and **Convolution of two continuous time signals

*Unit-step function(**u(t)**):*	*Dirac delta-function (**δ(t)**):*	$\delta(t)=\dfrac{du(t)}{dt}$	$\int_{-\infty}^{\infty} f(t)\cdot\delta(t-t_o)\cdot dt = f(t_o)$
$u(t-t_o)=\begin{cases}1 & \text{if } t\geq t_o \\ 0 & \text{if } t<t_o\end{cases}$	$\delta(t)=\begin{cases}0 & \text{if } t\neq 0 \\ \infty & \text{if } t=0\end{cases}$; $\int_{-\infty}^{\infty}\delta(t)dt=1$	$\delta(t)=\delta(-t)$	$f(t)\cdot\delta(t-t_o)=f(t_o)\cdot\delta(t-t_o)$

<u>Convolution</u>: $y(t)=x(t)\otimes h(t)=\int_{-\infty}^{\infty}x(\tau)h(t-\tau)d\tau$ and $y(t)=x(t)\otimes h(t)=h(t)\otimes x(t)$

TRIGONOMETRIC Fourier Series

Periodic function: $f(t)=f(t+nT)$, where ***n*** is an integer and ***T*** is the fundamental period, and $\omega_o = 2\pi/T$

$$f(t)=a_o+\sum_{n=1}^{\infty}\left[a_n\cos\left(n\omega_o t\right)+b_n\sin\left(n\omega_o t\right)\right]$$

$$a_o=\frac{1}{T}\int_0^T f(t)dt \quad ; \quad a_n=\frac{2}{T}\int_0^T f(t)\cdot\cos(n\omega_o t)dt \quad ; \quad b_n=\frac{2}{T}\int_0^T f(t)\cdot\sin(n\omega_o t)dt$$

***Even** function: f(t)=f(-t)*	***Odd** function: f(t)= -f(-t)*
$a_o=\frac{2}{T}\int_0^{T/2} f(t)dt$; $a_n=\frac{4}{T}\int_0^{T/2} f(t)\cdot\cos(n\omega_o t)dt$; $b_n=0$	$a_o=0$; $a_n=0$; $b_n=\frac{4}{T}\int_0^{T/2} f(t)\cdot\sin(n\omega_o t)dt$

EXPONENTIAL Fourier Series

$$f(t)=\sum_{n=-\infty}^{\infty}c_n e^{jn\omega_o t} \quad \text{with} \quad c_n=\frac{1}{T}\int_0^T f(t)\,e^{-jn\omega_o t}dt \quad \text{and} \quad c_0=a_0 \quad \text{and} \quad c_n=\frac{a_n - jb_n}{2}$$

AMPLITUDE-PHASE Fourier Series

$$f(t)=a_o+\sum_{n=1}^{\infty}A_n\cdot\cos\left(n\omega_o t+\phi_n\right) \quad \text{with} \quad A_n=\sqrt{a_n^2+b_n^2} \quad \text{and} \quad \phi_n = \text{-atan}\left(\frac{b_n}{a_n}\right)$$

Continuous time Fourier Transform Pairs

$\left(n, t, \omega, a, \omega_o, t_o\right.$ are Real numbers ; $j =$ complex number ; $n =$ integer ; $\omega =$ angular frequency$\left.\right)$

Time Domain	Fourier Transform		Time Domain	Fourier Transform
$f(t)$	$\int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$		$\frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega) e^{j\omega t} d\omega$	$F(\omega)$
$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(\omega) + a_2 F_2(\omega)$		$f(a \cdot t)$ $a \neq 0$	$\frac{1}{\lvert a \rvert} F\left(\frac{\omega}{a}\right)$
$e^{j\omega_o t} f(t)$	$F(\omega - \omega_o)$		$f(t - t_o)$	$e^{-j\omega t_o} F(\omega)$
$\frac{d^n\left[f(t)\right]}{dt^n}$ $(n>0)$	$\left(j\omega\right)^n F(\omega)$		$\left(-jt\right)^n f(t)$ $(n>0)$	$\frac{d^n\left[F(\omega)\right]}{d\omega^n}$
$e^{-at} u(t)$ $a > 0$	$\frac{1}{a + j\omega}$		$e^{at} u(-t)$ $a > 0$	$\frac{1}{a - j\omega}$
$\cos\left(\omega_o t\right)$	$\pi\left[\delta(\omega - \omega_o) + \delta(\omega + \omega_o)\right]$		$\sin\left(\omega_o t\right)$	$\frac{\pi}{j}\left[\delta(\omega - \omega_o) - \delta(\omega + \omega_o)\right]$
$\delta(t)$	1		$\delta(t - t_o)$	$e^{-j\omega t_o}$
1	$2\pi \cdot \delta(\omega)$		$e^{j\omega_o t}$	$2\pi \cdot \delta(\omega - \omega_o)$
$u(t)$	$\pi \cdot \delta(\omega) + \frac{1}{j\omega}$		$u(-t)$	$\pi \cdot \delta(\omega) - \frac{1}{j\omega}$
$u(t) - u(-t)$	$\frac{2}{j\omega}$		$u(t) + u(-t)$	$2\pi\delta(\omega)$
$t^{(n-1)} \cdot e^{-a \cdot t} \cdot u(t)$ $a > 0$; $n > 0$	$\frac{(n-1)!}{(a + j\omega)^n}$		$t^{(n-1)} \cdot e^{a \cdot t} \cdot u(-t)$ $a > 0$; $n > 0$	$\frac{(n-1)!}{(a - j\omega)^n} \cdot (-1)^{n-1}$
$\cos\left(\omega_o t\right) \cdot f(t)$	$\frac{1}{2}\left[F(\omega - \omega_o) + F(\omega + \omega_o)\right]$		$\sin\left(\omega_o t\right) \cdot f(t)$	$\frac{1}{2j}\left[F(\omega - \omega_o) - F(\omega + \omega_o)\right]$

***Impedances**: frequency domain*		
$Z_R = R$	$Z_L = j\omega L$	$Z_C = \frac{1}{j\omega C}$

Transfer function [H(ω)]
Cutoff frequency (ω_c) : $\lvert H(\omega_c) \rvert = \frac{H_{\max}}{\sqrt{2}}$

Discrete time signals (n and n_o are integer numbers)		
Impulse function: $\delta[n-n_o]=\begin{cases}1, & n=n_o\\ 0, & n\neq n_o\end{cases}$		*Unit-step function*: $u[n-n_o]=\begin{cases}1, & n\geq n_o\\ 0, & n<n_o\end{cases}$
Convolution	$y[n]=x[n]\otimes h[n]=\sum_{k=-\infty}^{\infty}x[k]\cdot h[n-k]$	$\delta[n]\otimes f[n-n_o]=\delta[n-n_o]\otimes f[n]\equiv f[n-n_o]$
Periodic signals	$x[n]=x[n+N]$, where $\boldsymbol{N}$ is the period of the discrete time signal. If a periodic signal $x(t)$ with period $\boldsymbol{T_o}$ is sampled with a period $\boldsymbol{T}$, and $x[n]=x[n+N]=x(n\cdot T)$, then $\frac{k}{N}=\frac{T}{T_o}$,where k is an integer If $x[n]=e^{j\cdot\Omega_o\cdot n}$ and $x[n]=x[n+N]$ then $\frac{\Omega_o}{2\pi}=\frac{m}{N}$, where m is an integer	
Sum of periodic signals	If $x_1[n]$, $x_2[n]$, $x_3[n]$,… are periodic signals with periods N_1, N_2, N_3,…, respectively, then the sum $x[n]=x[n+N]=x_1[n]+x_2[n]+x_3[n]+\ldots$ is periodic if: $m\cdot N_1=k\cdot N_2=l\cdot N_3=\ldots.=N$, where m, k, l,… are integer numbers.	

<u>Discrete time Fourier Transform Pairs</u>

$f[n]$	$F(\Omega)$		$f[n]$	$F(\Omega)$
$x[n]$	$X(\Omega)=\sum_{n=-\infty}^{\infty}x[n]\cdot e^{-jn\Omega}$		$\delta[n]$	1
$x[n]-x[n-1]$	$\left(1-e^{-j\Omega}\right)\cdot X(\Omega)$		1	$2\pi\cdot\sum_{k=-\infty}^{\infty}\delta(\Omega-2\pi k)$
$x[n-n_o]$	$e^{-jn_o\Omega}\cdot X(\Omega)$		$u[n]$	$\frac{1}{1-e^{-j\Omega}}+\sum_{k=-\infty}^{\infty}\pi\cdot\delta(\Omega-2\pi k)$
$e^{jn\Omega_o}\cdot x[n]$	$X(\Omega-\Omega_o)$		$a^n\cdot u[n]$ $(\lvert a\rvert<1)$	$\frac{1}{1-a\cdot e^{-j\Omega}}$
$x[-n]$	$X(-\Omega)$		$n\cdot a^n\cdot u[n]$ $(\lvert a\rvert<1)$	$\frac{a\cdot e^{j\Omega}}{\left(e^{j\Omega}-a\right)^2}$
$x[n]\otimes y[n]$	$X(\Omega)\cdot Y(\Omega)$		$a^{-n}\cdot u[-n-1]$ $(\lvert a\rvert<1)$	$\frac{a\cdot e^{j\Omega}}{(1-a\cdot e^{j\Omega})}$
$x[n]\cdot y[n]$	$\frac{1}{2\pi}X(\Omega)\otimes Y(\Omega)$		$n\cdot x[n]$	$j\frac{dX(\Omega)}{d\Omega}$
$\cos(\Omega_o n)$	$\pi\cdot[\delta(\Omega-\Omega_o)+\delta(\Omega+\Omega_o)]$		$\sin(\Omega_o n)$	$\frac{\pi}{j}\cdot[\delta(\Omega-\Omega_o)-\delta(\Omega+\Omega_o)]$

z-Transform Pairs

$f[n]$	$F(z)$		$f[n]$	$F(z)$
$x[n]$	$X(z)=\sum_{n=0}^{\infty} x[n]\cdot z^{-n}$		$a^n \cdot u[n]$	$\frac{z}{z-a}$, $\lvert z\rvert>\lvert a\rvert$
$u[n]$	$\frac{z}{z-1}$, $\lvert z\rvert>1$		$n\cdot a^n \cdot u[n]$	$\frac{a\cdot z}{(z-a)^2}$, $\lvert z\rvert>\lvert a\rvert$
$-u[-n-1]$	$\frac{z}{z-1}$, $\lvert z\rvert<1$		$(n+1)\cdot a^n \cdot u[n]$	$\frac{z^2}{(z-a)^2}$, $\lvert z\rvert>\lvert a\rvert$
$x[n-n_o]$	$z^{-n_o}\cdot X(z)$, $\lvert z\rvert>0$		$n\cdot x[n]$	$-z\cdot\frac{d[X(z)]}{dz}$
$x[-n]$	$X\left(\frac{1}{z}\right)$		$e^{j\Omega_o n}\cdot x[n]$	$X\left(e^{-j\Omega_o}\cdot z\right)$
$\delta[n-n_o]$	z^{-n_o} , $z\neq 0$		$x_1[n]\otimes x_2[n]$	$X_1(z)\cdot X_2(z)$
$\delta[n]$	1		$n\cdot u[n]$	$\frac{z}{(z-1)^2}$, $\lvert z\rvert>1$
$\cos(b\cdot n)\cdot u[n]$	$\frac{z\cdot(z-\cos(b))}{z^2-2z\cdot\cos(b)+1}$, $\lvert z\rvert>1$		$\sin(b\cdot n)\cdot u[n]$	$\frac{z\cdot\sin(b)}{z^2-2z\cdot\cos(b)+1}$, $\lvert z\rvert>1$
$a^n\cdot\cos(b\cdot n)\cdot u[n]$	$\frac{z\cdot(z-a\cdot\cos(b))}{z^2-2a\cdot z\cdot\cos(b)+a^2}$, $\lvert z\rvert>\lvert a\rvert$		$a^n\cdot\sin(b\cdot n)\cdot u[n]$	$\frac{a\cdot z\cdot\sin(b)}{z^2-2a\cdot z\cdot\cos(b)+a^2}$, $\lvert z\rvert>\lvert a\rvert$

Mathematical identities

$e^{\pm jx} = \cos(x) \pm j \cdot \sin(x)$; $\cos(x) = \dfrac{e^{jx} + e^{-jx}}{2}$; $\sin(x) = \dfrac{e^{jx} - e^{-jx}}{2j}$

$\sin^2(x) + \cos^2(x) = 1$; $\cos\left(x \pm \dfrac{\pi}{2}\right) = \mp\sin(x)$; $\sin(x \pm \pi) = -\sin(x)$; $\cos(x) = \cos(-x)$; $\sin(x) = -\sin(-x)$

$\cos(A \pm B) = \cos(A)\cdot\cos(B) \mp \sin(A)\cdot\sin(B)$; $\sin(A \pm B) = \sin(A)\cdot\cos(B) \pm \sin(B)\cdot\cos(A)$

$\cos(A)\cdot\cos(B) = \dfrac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$; $\sin(A)\cdot\sin(B) = \dfrac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$

$\cos(A)\cdot\sin(B) = \dfrac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$; $\sin(A)\cdot\cos(B) = \dfrac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$

if $n \equiv 0, \pm1, \pm2, \ldots \rightarrow$ $\sin(n\pi) = 0$; $\cos(2n\pi) = 1$; $\cos(n\pi) = (-1)^n$

$j = \sqrt{-1}$; $z = a + jb \rightarrow z = |z|\underline{\angle\theta}$ where $|z| = \sqrt{a^2 + b^2}$ and $\theta = \operatorname{atan}(b/a)$

$\int \cos(ax)dx = \dfrac{\sin(ax)}{a}$; $\int \sin(ax)dx = \dfrac{-\cos(ax)}{a}$; $\int e^{ax}dx = \dfrac{e^{ax}}{a}$; $\int x \cdot e^{ax}dx = \dfrac{x \cdot e^{ax}}{a} - \dfrac{e^{ax}}{a^2}$

$\int x \cdot \cos(ax)dx = \dfrac{\cos(ax) + ax\sin(ax)}{a^2}$; $\int x \cdot \sin(ax)dx = \dfrac{\sin(ax) - ax\cos(ax)}{a^2}$

Quadratic Equation: $ax^2 + bx + c = 0$ $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\frac{d}{dx}\left(\frac{1}{a \pm b \cdot x}\right) = \frac{\mp b}{(a \pm b \cdot x)^2}$$

Partial fraction(distinct poles) : $F(s) = \dfrac{N(s)}{(s+p_1)\cdot(s+p_2)\cdots(s+p_n)} = \dfrac{K_1}{(s+p_1)} + \dfrac{K_2}{(s+p_2)} + \ldots + \dfrac{K_n}{(s+p_n)}$

$$\Rightarrow K_i = (s+p_i)\cdot F(s)\Big|_{s=-p_i}$$

Integration by parts : $\int u \cdot dv = u \cdot v - \int v \cdot du$

$\sum_{k=0}^{\infty} a^k = \dfrac{1}{1-a}, \quad |a| < 1$; $\sum_{k=0}^{N} a^k = \dfrac{1-a^{N+1}}{1-a}$; $\sum_{k=N_1}^{N_2} a^k = \dfrac{a^{N_1} - a^{N_2+1}}{1-a}$, $N_2 > N_1$; $\sum_{k=N}^{\infty} a^k = \dfrac{a^N}{1-a}, \quad |a| < 1$