Mathematical Systems, Direct Proofs and Counter Examples

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1 Introduction

In this paper we will explore and expound through ,mathematical proofing, several fundamental theorems and inequalities in Discrete Math. These proving challenges will task us to revisit and refine our basic understanding of Algebra, Number Theory and most of all writing proofs.

This paper requires the knowledge and skill of writing cohesive mathematical proofs by establishing a domain, hypothesis and a conclusion. Relying heavily on establishing statements that are supported by mathematical laws, theorems and lemmas. Using these fundamental concepts, we are able to structure and justify our arguments or counter arguments Our goal is to prove, disprove our hypothesis and possible create a better and more logically sound counter argument

2 Euler's Number Proof

Task: Prove that e, Euler's number is not a rational number

$$e = \sum_{k=0}^{\infty} \frac{1}{k!} \notin \mathbb{Q}$$

Figure 1.0

Recalling the definition of a rational number, denoted as $\mathbb{Q}$,

$$\frac{a}{b} = \mathbb{Q}$$

Figure 1.1

Let us create a hypothesis, assume that euler's number is a rational number.

hypothesis:

$$e \in \mathbb{Q}$$

Suppose that the summation from k = 0 to ∞ can be expressed in a more simpler form

$$\sum_{k=0}^{\infty} \frac{1}{k!} = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} ... + \frac{1}{k!} = \frac{a}{b}$$

Figure 1.2

$$e = \frac{a}{b}$$

Figure 1.2

According to our hypothesis where $e \in \mathbb{Q}$, the reciprocal of eulers number should also be rational

$$e = \frac{a}{b} \quad = \quad e^x = \frac{b}{a}$$

s.t. x = -1

Figure 1.3

$$e^{-1} = \frac{1}{e} = \frac{b}{a} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!}$$

s.t. $a, b \in \mathbb{Z}$

Figure 1.4

Using a, let us group the summation of the infinite series into two terms. The first term will be the summation from 0 to a and the second term will be the summation from a to infinity.

$$e^{-1} = \frac{b}{a} = \sum_{k=0}^{a} \frac{(-1)^k}{k!} + \sum_{k=a+1}^{\infty} \frac{(-1)^k}{k!}$$

Figure 1.5

$$\frac{b}{a} - \sum_{k=0}^{a} \frac{(-1)^k}{k!} = \sum_{k=a+1}^{\infty} \frac{(-1)^k}{k!}$$

Figure 1.6

Multiply each term with

$$(-1)^{(a+1)}a!$$

term 1:

$$(-1)^{(a+1)}b(a-1)!$$

this term is an integer since all constants, variables and integers. any operation between integers that integers excluding division and root, will always result an integer

term 2:

$$(-1)^{(a+1)}(\frac{a!}{2!} \cdots + \frac{(-1)^a a!}{a})$$

in this series, every denominator is always less than the numerator, it will always result an integer since the denominator is a factor of the numerator

$$\forall a, b \in \mathbb{Z}, a > b$$

$$\frac{a!}{b!} = c, s.t c \in \mathbb{Z}$$

since term 1 and term 2 is a subtraction between two integers, we can conclude that the left side of the equation is an integer

$$\mathbb{Z} = \sum_{k=a+1}^{\infty} \frac{(-1)^k}{k!}$$

Let us expound on the term on the right side of the equation

$$= \frac{1}{a+1} - \frac{1}{(a+1)(a+2)} + \dots$$

in this expression, an upper and lower bound is defined. The first term is the upper bound. The succeeding terms are the lower bound such that it is lower than the first term as k reaches infinity, the summation of the succeding terms will approach zero. Therefore this this expression will result in a number that is irrational

$$\frac{1}{a+1} - \frac{1}{(a+1)(a+2)} + \dots \notin \mathbb{Z}$$

3 Minskowski's Inequality for Sums

Task: Prove Minskow's Inequality for Sums

$$[\sum_{k=1}^{n}|a_k+b_k|^p]^{\frac{1}{p}} \leq [\sum_{k=1}^{n}|a_k|^p]^{\frac{1}{p}} + [\sum_{k=1}^{n}|b_k|^p]^{\frac{1}{p}}$$

$$\forall a, b, p \in \mathbb{R}$$

$$p > 1, (a_k, b_k) > 0$$

Let us directly prove Minskowski's Inequality for sums by the triangle Inequality theorem

$$\text{Triangle Inequality Theorem}$$

$$|a+b| \leq |a| + |b|$$

Let us first raise terms on both sides to P

$$|a+b|^p \leq |a|^p + |b|^p$$

Do summation notation for all k from 1 to n

$$\sum_{k=1}^{n}|a_k+b_k|^p \leq \sum_{k=1}^{n}|a_k|^p + \sum_{k=1}^{n}|b_k|^p$$

raise to 1 over p by property of absolute value

$$[\sum_{k=1}^{n}|a_k+b_k|^p]^{\frac{1}{p}} \leq [\sum_{k=1}^{n}|a_k|^p]^{\frac{1}{p}} + [\sum_{k=1}^{n}|b_k|^p]^{\frac{1}{p}}$$