Mathematical Systems, Direct Proofs and Counter Examples

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Discrete Mathematics 2

1   Introduction

In this paper we will explore and expound through ,mathematical proof-ing, several fundamental theorems and inequalities in Discrete Math. These proving challenges will task us to revisit and refine our basic understanding of Algebra, Number Theory and most of all writing proofs.

This paper requires the knowledge and skill of writing cohesive mathematical proofs by establishing a domain, hypothesis and a conclusion. Relying heavily on establishing statements that are supported by mathematical laws, theorems and lemmas. Using these fundamental concepts, we are able to structure and justify our arguments or counter arguments Our goal is to prove, disprove our hypothesis and possible create a better and more logically sound counter argument

2   Euler's Number Proof

Task: Prove that e, Euler's number is not a rational number

$$e = \sum_{k=0}^{\infty} \frac{1}{k!} \notin \mathbb{Q}$$

Figure 1.0

Recalling the definition of a rational number, denoted as $\mathbb{Q}$,

$$\frac{a}{b} = \mathbb{Q}$$

Figure 1.1

Let us create a hypothesis, assume that euler's number is a rational number.

hypothesis:

$$e \in \mathbb{Q}$$

Suppose that the summation from k = 0 to ∞ can be expressed in a more simpler form

$$\sum_{k=0}^{\infty} \frac{1}{k!} = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} ... + \frac{1}{k!} = \frac{a}{b}$$

<div align="right">Figure 1.2</div>

$$e = \frac{a}{b}$$

<div align="right">Figure 1.2</div>

According to our hypothesis where $e \in \mathbb{Q}$, the reciprocal of eulers number should also be rational

$$e = \frac{a}{b} \quad = \quad e^x = \frac{b}{a}$$

s.t. x = -1

<div align="right">Figure 1.3</div>

$$e^{-1} = \frac{1}{e} = \frac{b}{a} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!}$$

s.t. $a, b \in \mathbb{Z}$

<div align="right">Figure 1.4</div>

Using a, let us group the summation of the infinite series into two terms. The first term will be the summation from 0 to a and the second term will be the summation from a to infinity.

$$e^{-1} = \sum_{k=0}^{a} \frac{(-1)^k}{k!} + \sum_{k=a+1}^{\infty} \frac{(-1)^k}{k!}$$

<div align="right">Figure 1.5</div>