Final Exam of Probability Name:

Student No.

1. Let the joint probability density function of random variables X and Y be given by

$$f(x,y)=\begin{cases}2, \text{ if } 0\le y\le x\le 1\\ 0, \text{otherwise.}\end{cases}$$

Are X and Y independent? Why or why not. (10%)

2. A man invites his fiancee to an elegant hotel for a Sunday brunch. They decide to meet in the lobby of the hotel between 11:30 A.M. and 12 noon. If they arrive at random times during this period, what is the probability that the first to arrive has to wait at least 12 minutes? (12%)

3. Let the random variable $X \sim N(\mu,\sigma^2)$. To calculate the density function of the random variable Y= e^X. (12%)

4. The average IQ score on a certain campus is 120. If the variance of these scores is 20, what can be said about the percentage of students with an IQ above 150? (10%)

5. (a) Give the statement of the Central Limit Theorem. (6%)

(b) Using (a) to solve: If 20 random numbers are selected independently from the interval $(0, 1)$, what is the approximate probability that the sum of these numbers is at least eight? (10%)

(Hint: $\sqrt{\frac{20}{12}} \approx 1.29$ or $\sqrt{20}\approx 4.47$, $\sqrt{12}\approx 3.46$)

6. A small college has 1095 students. What is the approximate probability that more than five students were born on Christmas day? Assume that the birthrates are constant throughout the year and that each year has 365 days. (10%)

7. Let the joint probability mass function of random variables X and Y be given by

$$p(x,y)=\begin{cases} k(x^2+y^2), & \text{if } (x,y)=(1,1),(1,3),(2,3) \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the value of the constant *k*. (4%)

(b) Find the marginal probability mass function of X and Y. (6%)

(c) Find E(X). (5%)

8. (10%) A random variable X has the following density function, Calculate $\mathrm{Var}(X)$.

$$f(x)=\frac{1}{2}e^{-|x|}, \text{if } -\infty < x < \infty$$

9. Give the statement of the <u>Week Law of Large Numbers</u>. (5%)