

## 1 Floating source height

Imagine that the source's height would be "floating", just like the other vertices. ( Which means its height is not fixed to  $|V|$ .) However, we set a "cap" of  $|V|$  on the height of the source and cap of  $2|V|$  on the height of other vertices. The height of  $t$  is fixed as 0. In this case, we can solve the max flow problem by adding an imaginary source  $S$  before the original source  $s$ , and make the edge  $(S, s)$  has a capacity that is the sum of the capacities of all edges that comes from  $s$ .

a. Is it still true that source  $s$  does not have augmenting path to the sink  $t$ ? (prove or give counter-example)

**Answer:**

No. Do the following operations to the example graph:  $\text{push}(s, a)$ ,  $\text{push}(s, b)$ ,  $\text{relabel}(a)$ ,  $\text{push}(a, b)$ ,  $\text{relabel}(b)$ ,  $\text{push}(b, t)$ ,  $\text{relabel}(b)$ ,  $\text{push}(b, s)$ .

b. Is it still true that each vertex with excess has an augmenting path back to the source?

**Answer:**

Yes, the proof would be similar to the proof of Lemma 26.20.

## 2 . Exhaustive relabeling

Imagine that the algorithm is modified so that relabeling of all vertices is done first, and pushes occur only when no relabelling are possible.

a. Is it true now that height of each vertex is upper bounded by  $V$ ?

**Answer:**

False. No matter how, if the max flow is less than the flow from  $s$  initially, there must be some flow to come back to  $s$  via some vertex. This means that vertex's height must be  $V+1$ .

b. is the algorithm identical to edmonds-karp ? If not, what is the difference?

**Answer:**

they are different. Edmonds-karp uses BFS and only augment the shortest path. while preflow/push flood all the path.

c. Analyze the algorithm's cost.

**Answer:**

The complexity is the same as the original algorithm.

### 3. 26.4-3

Suppose that a maximum flow has been found in a flow network  $G = (V, E)$  using a push-relabel algorithm. Give a fast algorithm to find a minimum cut in  $G$ .

**Answer:**

Since there are  $n = |V|$  nodes in the network, among the  $n + 1$  integers  $0, \dots, n$ , there is at least one integer  $k$  which is not used in the labelling. Let  $S = \{v | h(v) > k\}$ , and  $T = V - S$ . So  $s \in S$ , and  $t \in T$ . Also, for any edge  $(u, v)$  with  $u \in S$  and  $v \in T$ , we have:  $h(u) > k > h(v)$ . So:  $h(u) > h(v) + 1$ . So the edge  $(u, v)$  must be saturated.

So all the edges connecting  $S$  to  $T$  is saturated. So this is a minimum cut.