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Homework 2

1. When the min player plays sub-optimally a game will be driven to nodes with higher scores for the max player.

   To say that a min player is playing suboptimally is to say that at a given depth, the min player selects some move m, where theres exists a node n where $h(m) < h(n)$, which means that the heuristic is saying that node n is a better move then node m. Since at each min-level minimax assumes that the min player chooses the best strategy for them, there can be no situation where a suboptimal move by the min-player will result in the game reaching a better outcome for them. If there was a better outcome, then the minimax algorithm would assume that the min player is going to choose it.

2. Nodes are pruned with Alpha-Beta pruning only in relationship to the neighbor nodes. Just because we have arrived a node that has previously been pruned, does not neccessarily mean we can automatically prune that node again.

3. (a) For Player 1, Strategy Z Weakly Dominates Strategy Y, and Strategy X Weakly Dominates Strategy Y
   For Player 2, Strategy C Weakly Dominates Strategy A
   (b) 1. Remove Strategy Y
   2. Strategy B now weakly dominates strategies A and C. Remove Strategy C 3. Strategy Z weakly dominates strategy X. Remove strategy X. 4. Strategy B dominates strategy A. Remove Strategy A. 5. (B, Z) remains with payoff (3,3)
   (c) 1. Remote Strategy A 2. X Dominates Y and Z, remove Z 3. C Dominates B, Remove B 4. X Dominates Y, Remove Y 5. (X, C) remains with payoff (1,1)
   (d) The Nash Equilibria are (X,B) and (Y,B)

4. (a) Not all Damsels are in distress
   (b) All knights have a horse
   (c) There exists a horse owned by all knights
   (d) There is a knight who owns one and only one strong horse
   (e) There does not exist a knight who has saved all damsels

5. (a) $\neg \exists x Damsel(x) \wedge (\exists y Knight(y) \Rightarrow HasSaved(x,y))$
   (b) $\exists x, y Knight(x) \wedge Horse(y) \wedge Owns(x,y) \wedge HasSaved(y,x)$
   (c) $\forall x, y Damsel(x) \wedge Knight(y) \wedge HasSaved(x,y) \Rightarrow (Strong(x))$
   (d) $\exists x, y \forall z, w Knight(x) \wedge Horse(y) \wedge HasSaved(x,y) \wedge Knight(z) \wedge Horse(w) \wedge HasSaved(z,w) \Rightarrow (x = z \wedge y = w)$
   (e) $(\forall x Damsel(x) \wedge InDistress(x) \Rightarrow \neg \exists y HasSaved(y,x)) \wedge (\forall z Damsel(z) \wedge \neg InDistress(z) \Rightarrow \exists w HasSaved(w,z))$

6. From the statement of the problem we can establish some first-order clauses
   1. $\forall x, y, z UnregisteredGun(y) \wedge Sells(x,y,z) \Rightarrow Criminal(x)$
   2. $\exists x Owns(Reese, x) \wedge UnregisteredGun(x)$
   3. $\forall x UnregisterGun(x) \wedge Owns(Reese, x) \Rightarrow Sells(Laslo, x, Reese)$

   We can use existential elimination to turn $\exists x Owns(Reese,x) \wedge UnregisteredGun(x)$ into $Owns(Reese, UG_1)$ and $UnregisteredGun(UG_1)$

   Then we can use Generalized Modus Ponens to replace statement 3 X/UG_1 $UnregisterGun(UG_1) \wedge Owns(Reese, UG_1) \Rightarrow Sells(Laslo, UG_1, Reese)$

   Again we can use Generalized Modus Ponent to replace x,y and z in 1 to yield $UnregisteredGun(UG_1) \wedge Sells(Laslo, UG_1, Reese) \Rightarrow Criminal(Laslo)$

   Therefor Laslo is a criminal.

7. $Unify[Parent(x,y) \wedge CPA(x1) \wedge Senator(y1), Senator(Jim)] = (y1/Jim)$

   $Unify[Parent(x,y) \wedge CPA(x1), CPA(Amy) \wedge Senator(Jim)] = (x1/Amy)$

   $Unify[Parent(x,y), Parent(amy, jim) \wedge CPA(Amy) \wedge Senator(Jim)] = (x/amy), jim/y)$

   $Unify[Answer(x,y), Parent(amy, jim) \wedge CPA(Amy) \wedge Senator(Jim)] = (x/amy), jim/y)$

   Answer = (Amy, Jim)

8. Note: These calculations were done in base 10, not in base 2.

   (a) $I(p,n) = -\frac{4}{9}lg(\frac{4}{9}) - \frac{5}{9}lg(\frac{5}{9})$ = 0.157 - (-0.142) = .299
   (b) $I_{a_1=T} = -\frac{3}{4}lg(\frac{3}{4}) - \frac{1}{4}lg(\frac{1}{4}) = 0.094 - (-.151) = 0.245$
   $I_{a_1=F} = -\frac{1}{5}lg(\frac{1}{5}) - \frac{4}{5}lg(\frac{4}{5}) = 0.134 - (-.078) = .212$
   $E(a_1) = \frac{4}{9}0.245 + \frac{5}{9}0.212 = .128$

$gain(a_1) = .299 - .128 = .171$

$I_{a_2=T} = -\frac{2}{5}lg(\frac{2}{5}) - \frac{3}{5}lg(\frac{3}{5}) = 0.159 - (-.133) = 0.289$
$I_{a_2=F} = -\frac{2}{4}lg(\frac{2}{4}) - \frac{3}{4}lg(\frac{2}{4}) = 0.151 - (-.151) = 0.301$
$E(a_2) = \frac{5}{9}0.289 + \frac{4}{9}0.301 = .294$
$gain(a_2) = .299 - .294 = .005$

(c) This table contains the expected values for all possible splits

Split	Total Values	Values in Split	Positive	negative	I(split)	E(Split)
<=1	9	1	1	0	0	0.25539
>1	9	8	3	5	0.287313	
<=3	9	2	1	1	0.30103	0.297571
>3	9	7	3	4	0.296583	
<=4	9	3	2	1	0.276435	0.276435
>4	9	6	2	4	0.276435	
<=5	9	5	2	3	0.292285	0.296172
>5	9	4	2	2	0.30103	
<=6	9	6	3	3	0.30103	0.292832
>6	9	3	1	2	0.276435	
<=7	9	8	4	4	0.30103	0.267582
>7	9	1	0	1	0	

The best split then occurs at 1, which has the lowest E value, splitting between <=1 and >1.

9. This problem I'm a little unsure about but I'll take a stab. When considering entropy at a given node we arrive an equation that has the form

$$-\frac{Success}{total}lg\frac{Success}{total} - \frac{Failure}{total}lg\frac{Failure}{total}$$

Where $\frac{Success}{total} + \frac{Failure}{total} = 1$ So if we then split a node into smaller successor nodes, the number of successes and failures remains the same, but we break them up in different ways.

$$\frac{Success_1}{Total_1}I(1) + \frac{Failure_1}{Failure_1}I(1)$$

And since I is of the form $\sum_{k=1}^{d} log z_k$, which means that we can then use the hint to say that the sum entropy for the split children node will always be less then the entropy of the parent node.

10. B is better then A at Labor, Cleve
    A is better then B at Vehicle, Anneal, Tic-Tac-Toe, Auto, Australia, Credit, Glass, Ionosphere, Horse, and Sonar

    C is better then A at Tic-Tac-Toe and Cleve, Wine, Labor, Lymphography, and Diabetes
    A is better then C at Auto and Anneal

    C is better then B at Tic-Tac-Toe, Vehicle, Anneal, Australia, Credit, Auto, Ionosphere, Glass, and Sonar.
    B is not better then C at anything.

    B appears to be inferior when compared to A and C in the majority of tests. For example the average Z value over all tests for AB is 1.6 and for CB is 2.55. C appears to be the best classifier when compared to A and B. The average Z value for CA is .96.

	n	acc(a)	acc(b)	acc(c)	Zab	Zac	Abc
Anneal	898	92.09	79.62	87.19	7.582377	3.407128	-4.31156
Australia	690	85.51	76.81	84.78	4.132543	0.381256	-3.7581
Auto	205	81.95	58.05	70.73	5.280198	2.67283	-2.68093
Breadt	699	95.14	95.99	96.42	-0.77187	-1.19025	-0.42071
Cleve	303	76.24	83.5	84.49	-2.22859	-2.5563	-0.33234
Credit	690	85.8	77.54	85.07	3.965309	0.384379	-3.58743
Diabetes	768	72.4	75.91	76.82	-1.57114	-1.99002	-0.41974
German	1000	70.9	74.7	74.4	-1.90949	-1.75573	0.154006
Glass	214	67.29	48.59	59.81	3.918408	1.607633	-2.32944
Heart	270	80	78.8	83.7	0.34475	-1.11538	-1.45865
Hepatitis	155	81.94	83.23	87.1	-0.29945	-1.25584	-0.95849
Horse	368	85.33	78.8	82.61	2.308832	1.005655	-1.30967
Ionosphere	351	89.17	82.34	88.89	2.588799	0.118693	-2.47258
Iris	150	94.67	95.33	96	-0.26226	-0.54617	-0.28493
Labor	57	78.95	94.74	92.98	-2.49394	-2.15632	0.391391
Led7	3200	73.34	73.16	73.56	0.162655	-0.19928	-0.36193
Lymphography	148	77.03	83.11	86.49	-1.30928	-2.10729	-0.80987
Pima	768	74.35	76.04	76.59	-0.76681	-1.02018	-0.25351
Sonar	208	78.85	69.71	76.92	2.132512	0.474246	-1.66235
Tic-Tac-Toe	958	83.72	70.04	98.33	7.101552	-11.1872	-16.9687
Vehical	846	71.04	45.04	74.94	10.83583	-1.80651	-12.5521
Wine	178	94.38	96.63	98.88	-1.02447	-2.35254	-1.43285
Zoo	101	93.07	93.07	96.04	0	-0.93017	-0.93017