600.363/463 Solution to Homework 8
Posted: 11/5/03
Due: Wednesday, 11/12/03

26.1-6

$f_1 + f_2$ satisfies skew symmetry and flow conservation, but might violate the capacity constraint.

26.1-7

Prove it by verifying it satisfies all the three properties:

- Capacity: $\alpha f_1(u,v)+(1-\alpha)f_2(u,v) \leq \alpha \cdot c(u,v)+(1-\alpha)c(u,v) = c(u,v)$
- Symmetry: $(\alpha f_1+(1-\alpha)f_2)(u,v) = -\alpha f_1(v,u)-(1-\alpha)f_2(v,u) = -(\alpha f_1 + (1-\alpha)f_2)(u,v)$
- Conservation: $\sum(\alpha f_1 + (1-\alpha)f2)(u,v) = \alpha \sum f_1(u,v) + (1-\alpha)\sum f_2(u,v) = 0$

26.2-9

For any two vertices u and v in G, you can define a flow network G_{uv} consisting of the directed version of G with all edge capacities 1, $s = u$, and $t = v$. G_{uv} has $O(V)$ vertices and $O(E)$ edges, as required. Let f_{uv} denote a maximum flow in G_{uv}. We claim for any $u \in V$, the edge connectivity $k = \min_{v \in V-\{u\}} |f_{uv}|$ (this van be proved). Thus we can find k as follows:
EDGE-CONNECTIVITY(G)
  Select any vertex $u \in V$
  for each vertex $v \in V - u$
    do set up the f;pw metwprl G_{uv} as described above
      find the maximum flow f_{uv} on G_{uv}.
  return the minimum of the $|V|-1$ max-flow values: $\min_{v \in V-\{u\}} |f_{uv}$

The claim follows from the max-flow min-cut theorem and the fact that we chse capacities so that the capacity of a cut is the number of edges crossing it.

26.2-10

From the time (u, v) is a critical edge until it is again a critical edge, $\delta(s, u)$ increases by at least 2, as shown in Theorem 26.9. Similarly, you can show that $\delta(v, t)$ also increases by at least 2. Thus the length of the augmenting path $s \leadsto u \rightarrow v \leadsto t$ increases by at least 4 between times (u, v) is critical. Since the length of an augmenting path cannot exceed $|V| - 1$, (u, v) can be critical $< |V|/4$ times. Edmonds-Karp terminates when there are no more augmenting paths, so it must terminate when there are no more critical edges, which akes at most (# edges)·(max # times each edge critical)¡$|E_f|(V|/4)$ iterations. In general, $|E_f| \leq 2|E|$, so the number of iterations is at most (actually, fewer than) $|E||V|/2$. But if we assume that G always has edges in both directions (i.e., $(u, v) \in E$) iff$(v, u) \in E$), then $|E_f| \leq |E|$, and the number of iterations is at most $|E||V|/4$.