# FHR Benchmark Equations

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This document contains the assumptions and equations used for the FHR benchmark excel spreadsheet results.

## 1 Quantities of Interest

(a) Effective multiplication factor

(b) Reactivity coefficients ($\beta_{eff}$, fuel Doppler coefficient, FLiBe temperature coefficient, graphite temperature coefficient)

(c) Tabulated fission source distribution, at several levels of granularity (by fuel plate, by fuel stripe, by 1/5-th fuel stripe). Optional: visualized fission density distribution.

(d) Neutron flux averaged over the whole model tabulated in 3 coarse energy groups.

(e) Visualized distribution of the neutron flux distribution, in 3 coarse energy groups

(f) Neutron spectrum, fuel assembly average. Optional: by region.

### 1.1 Effective multiplication factor (a)

Assumptions made:

- No. of CPUs = No. of Nodes × 32 (no. of CPUs in each Blue Waters XE node)
- CPU-time = No. of CPUs × Total time in simulation (in openmc's results file)
- Wall Clock Time = Total time elapsed (in openmc's results file)

### 1.2 Reactivity coefficients (b)

We assume 1 energy group and 6 delayed neutron groups for $\beta_{eff}$.

$$\beta_{eff} = \sum_k \beta_k$$

Doppler reactivity coefficient (fuel):

$$\frac{\Delta\rho}{\Delta T_f} = \frac{\rho_{1150K} - \rho_{1100K}}{1150 - 1100}[\frac{pcm}{K}]$$
$$\delta\frac{\Delta\rho}{\Delta T_f} = \frac{\sqrt{\delta(\rho_{1150K})^2 + (\delta\rho_{1100K})^2}}{1150 - 1100}[\frac{pcm}{K}]$$

Coolant reactivity coefficient (FLiBe):

$$\frac{\Delta\rho}{\Delta T_c} = \frac{\rho_{1150K} - \rho_{1100K}}{1150 - 1100}[\frac{pcm}{K}]$$
$$\delta\frac{\Delta\rho}{\Delta T_c} = \frac{\sqrt{\delta(\rho_{1150K})^2 + (\delta\rho_{1100K})^2}}{1150 - 1100}[\frac{pcm}{K}]$$

Graphite reactivity coefficient (graphite):

$$\frac{\Delta\rho}{\Delta T_g} = \frac{\rho_{1150K} - \rho_{1100K}}{1150 - 1100}[\frac{pcm}{K}]$$
$$\delta\frac{\Delta\rho}{\Delta T_g} = \frac{\sqrt{\delta(\rho_{1150K})^2 + (\delta\rho_{1100K})^2}}{1150 - 1100}[\frac{pcm}{K}]$$

We assumed all graphite.

### 1.3 Fission source distribution (c)

Fission density (FD) is calculated by using openmc's 'fission' score (f) divided by the average of all 'fission' scores:

$$FD_i = \frac{f_i}{f_{ave}}$$

where:

$$f_i = \text{Total fission reaction rate [reactions/src]}$$
$$f_{ave} = \text{average of all } f_i \text{ [reactions/src]}$$

The uncertainty calculations for $f_{ave}$ and $FD_i$:

$$\delta f_{ave} = \frac{1}{N}\sqrt{\sum_i^N f_i^2}$$
$$\delta FD_i = |FD_i|\sqrt{(\frac{\delta f_i}{f_i})^2 + (\frac{\delta f_{ave}}{f_{ave}})^2}$$

where:

$$N = \text{No. of fission score values}$$

## 1.4 Neutron Flux (d, e, f)

Openmc's 'flux' score is given in units of $[\frac{n*cm}{src}]$. For the benchmark, we need to convert it to units of $[\frac{n}{cm^2 s}]$. The conversion:

$$\Phi_c = \frac{N * \Phi_o}{V}$$
$$N = \frac{P * \nu}{Q * k}$$

where:

$$\Phi_c = \text{Converted Flux } [\tfrac{neutrons}{cm^2 s}]$$
$$\Phi_o = \text{Original Flux } [\tfrac{neutrons*cm}{src}]$$
$$N = \text{Normalization factor } [\tfrac{src}{s}]$$
$$V = \text{Volume of fuel assembly } [cm^3]$$
$$P = \text{Power } [\tfrac{J}{s}]$$
$$\nu = \tfrac{\nu_f}{f} \ [\tfrac{neutrons}{fission}]$$
$$Q = \text{Energy produced per fission } [\tfrac{J}{fission}]$$
$$= 3.2044 * 10^{-11} \text{ J per } U_{235} \text{ fission}$$
$$k = k_{eff} \ [\tfrac{neutrons}{src}]$$

Flux standard deviation:

$$\delta\Phi_c = \Phi_c * \sqrt{(\frac{\delta\Phi_o}{\Phi_o})^2 + (\frac{\delta\nu_f}{\nu_f})^2 + (\frac{\delta k}{k})^2 + (\frac{\delta f}{f})^2}$$

Reactor power is calculated based on the given reference specific power ($P_{sp}$) of 200 $\frac{W}{gU}$.

$$P = P_{sp} * V_F * \rho_F * \frac{wt\%_U}{100}$$

where:

$$
\begin{aligned}
P &= \text{Reactor power [W]} \\
V_F &= \text{Volume of fuel } [cm^3] \\
&= \frac{4}{3}\pi r_1^3 * 101 * 210 * 4 * 2 * 6 * 3 \\
\rho_F &= \text{density of fuel } [g/cc] \\
wt\%_U &= \frac{at\%_{U235} * AM_{U235} + at\%_{U238} * AM_{U238}}{\sum(at\%_i * AM_i)} * 100 \\
AM &= \text{atomic mass}
\end{aligned}
$$