Volatility and Multivariate Analysis

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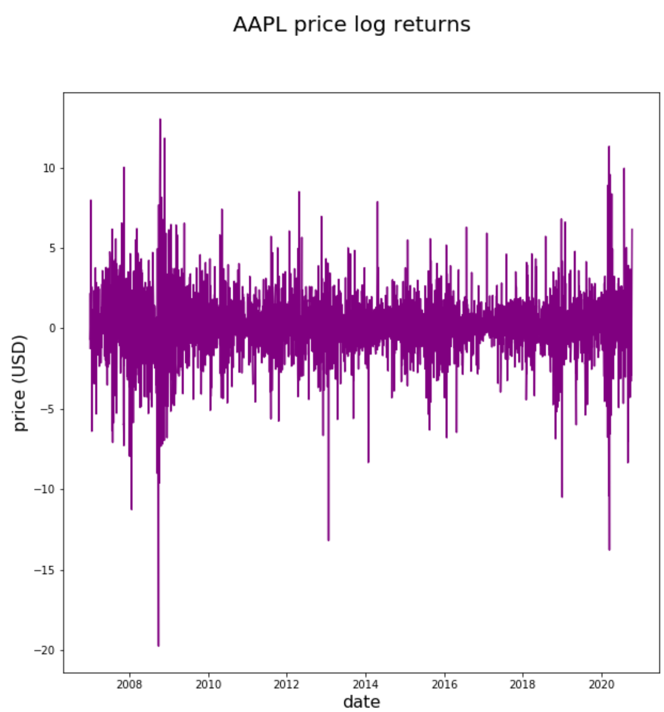
Volatility Analysis

The first section of this report describes our attempts to model the returns of Apple's stock from the period of January 2007 to present day. This period was chosen specifically because it could be argued that monetary policy was consistent over this period (specifically, the use of quantitative easing and increased intervention by central banks).



The time series is mean reverting with volatility clusters. Interestingly, on larger time scales, the volatility spikes tend to coincide with major financial crises – for example the 1987 stock market crash, the dotcom crash (2000) and the subprime mortgage crisis (2008). In general, the spikes tend to be unsymmetrical with more severe drawdowns. Due to the lack of memory effects and asymmetry of the volatility, ARCH and IGARCH-type models shouldn't be used. Although TGARCH and EGARCH models could be used, a first approximation GARCH model will be used instead.

The original time series was confirmed to be stationary according to both the ADF (H_0 rejected) and KPSS tests (H_0 not rejected). Next, we attempted to fit an ARMA model to the time series in order to understand the properties of the conditional expectation. Using *auto_arima* from the *pmdarima* package, an MA(1) model was obtained with the following statistics:

```
                               SARIMAX Results
==============================================================================
Dep. Variable:                      y   No. Observations:                 3469
Model:               SARIMAX(0, 0, 1)   Log Likelihood               -7419.425
Date:                Tue, 13 Oct 2020   AIC                          14844.850
Time:                        14:28:14   BIC                          14863.305
Sample:                             0   HQIC                         14851.440
                               - 3469
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
intercept      0.1112      0.034      3.247      0.001       0.044       0.178
ma.L1         -0.0352      0.010     -3.467      0.001      -0.055      -0.015
sigma2         4.2194      0.048     87.944      0.000       4.125       4.313
===================================================================================
Ljung-Box (Q):                       90.84   Jarque-Bera (JB):              7471.44
Prob(Q):                              0.00   Prob(JB):                         0.00
Heteroskedasticity (H):               0.58   Skew:                            -0.46
Prob(H) (two-sided):                  0.00   Kurtosis:                        10.13
===================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
```

Although the model parameter ma.L1 is statistically significant, the Ljung-Box test shows statistically significant serial correlations in the model residuals. Therefore, the model is not a good fit for the conditional expectation and this is because the residuals follow a leptokurtic distribution (see the results of the Jarque-Bera test). The H test shows that the residuals exhibit heteroscedasticity. Despite the inadequacy of MA(1) for describing the conditional expectation, we will proceed with the fitting of a GARCH(p,q) process on the model's residuals.

As there is no package that automatically determine the parameters p and q for a GARCH model in python, we instead performed a grid search, minimising the AIC. The lowest AIC of 13928 gave $p = 1$ and $q = 1$ and so a GARCH(1,1) process was fit to the residuals of the previously discussed MA(1) model.

```
                     Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:                   None   R-squared:                      -0.002
Mean Model:             Constant Mean   Adj. R-squared:                 -0.002
Vol Model:                      GARCH   Log-Likelihood:               -6960.07
Distribution:                  Normal   AIC:                           13928.1
Method:            Maximum Likelihood   BIC:                           13952.8
                                        No. Observations:                 3469
Date:                Tue, Oct 13 2020   Df Residuals:                     3465
Time:                        14:28:36   Df Model:                            4
                                 Mean Model
==========================================================================
                 coef    std err          t      P>|t|    95.0% Conf. Int.
--------------------------------------------------------------------------
mu             0.0824  2.992e-02      2.755  5.862e-03 [2.380e-02,  0.141]
                              Volatility Model
==========================================================================
                 coef    std err          t      P>|t|    95.0% Conf. Int.
--------------------------------------------------------------------------
omega          0.1444  3.711e-02      3.892  9.962e-05 [7.167e-02,  0.217]
alpha[1]       0.1190  2.029e-02      5.865  4.499e-09 [7.922e-02,  0.159]
beta[1]        0.8477  2.290e-02     37.022 5.147e-300   [  0.803,  0.893]
==========================================================================

Covariance estimator: robust
```

All individual parameters are statistically significant and furthermore, $\alpha + \beta = 0.97 < 1$ indicating model stability. As expected, this was not an appropriate fit to the MA(1) process's residuals since the ACF of the GARCH(1,1) squared residuals showed serial correlation. Despite our best attempts, the shortcomings of our models are best explained by the fact that both of these models require the conditional variance to be time independent: an assumption that is very likely to be violated in the real world. According to our model, forecasting one period forward $t + 1$ suggests that the return of Apple stock should be 8.5% tomorrow.

Multivariate Analysis

1. **Describe the economic theories and models used to calculate equilibrium FX**
   Early theories revolve around the so-called "law of one price". The Purchasing Power Parity (PPP) theory argues that the exchange rate is determined by the ratio of two countries' relative price levels in the long run (Krugman and Obstfeld, 2009)[3]. PPP is simple to calculate and provides a pretty good estimate of an exchange rate between two countries, but there are some drawbacks to using PPP. Many countries import goods that aren't available locally, which means that these goods/services will sell at higher prices. In addition, the government could place tariffs on imported goods further increasing the price of the good/service. Furthermore, some companies might have a competitive advantage in certain countries, so they may charge higher prices for a good/service. This can greatly inflate the exchange rate when using PPP, and would lead to inaccurate results. However, PPP is still a good metric for estimating what the exchange rate between two countries might be.

The capital market analog to PPP is the Interest Rate Parity (IRP) which states that the forward exchange rate is governed by the difference in interest rates of two countries. One can combine PPP and IRP to form a behavioral equilibrium exchange rate (BEER) model (Stephens, 2004 [7]).

The Behavioral Equilibrium Exchange Rate (BEER) model attempts to calculate an equilibrium exchange rate for a pair of currencies by looking at various economic fundamentals. After Johansen (1988) [2] introduces the Vector Error Correction Model, researchers develop a family econometric method, Behavioral Equilibrium Exchange Rate (BEER), to estimate the equilibrium exchange rate. Early pioneers include Clark and MacDonald (1999) [1] and Maeso–Fernandez et al. (2002) [6]. The method is more econometrical than economical and is based on a cointegrating relationship between the select macroeconomic variables that are expected to have effects on the exchange rate. The most common macroeconomic variables that are employed in the BEER model include inflation, nominal interest rates, productivity differential to allow the Balassa-Samuelson effect to be captured in the model, and relative fiscal policy stance such as government debt, terms of trade. The advantage of the BEER approach is that it can decompose the exchange rate into the permanent and the cyclical component. For instance, using the HP filter researchers can obtain the permanent equilibrium exchange rate and cyclical shocks of the exchange rate. Also, instead of estimating a normative exchange rate, the method can produce a measure of exchange rate misalignment. To compute the misalignment, we can follow the steps below (MacDonald and Dias, 2007) [5]:

(1) Using the VECM method to estimate the cointegrating relationship between the real exchange rate and select macroeconomic variables that can influence the exchange rate.

(2) Computing the current misalignment by taking the difference between the actual real exchange rate and the fitted value produced by the estimated model.

(3) Decomposing the fundamentals into permanent and cyclical components by using HP filter.

(4) The total alignment is calculated by computing the permanent component of the equilibrium exchange rate by plugging permanent of fundamentals into the estimated model. Then take the difference between the fitted value and actual value of the exchange rate.

Another economic theory used to calculate the equilibrium exchange rate, is the Fundamental Equilibrium Exchange Rate (FEER) model, which is introduced by Williamson (1994) [8]. This model looks at the Real Effective Exchange Rate and determines if it's compatible with the macroeconomic equilibrium, meaning it looks at internal and external macroeconomic factors to determine if the exchange rate is fundamentally correct. The model assumes that inflation rates are low and that there is full employment in the economy. It aims to find imbalances or misalignment in the current real exchange rate. When looking at internal balances, the model is assessed based on full-employment and low inflation rates. However when looking at external balances, the model is assessed based on desired net cash flows and net savings which are determined by factors like consumption smoothing and demographic factors. Essentially, the model aims to find an equilibrium exchange rate under ideal economic conditions.

To estimate the FEER exchange rate, there are two approaches. Firstly, we can estimate a full-blown macroeconomic model and solve for the real exchange rate by imposing the internal and external balance constraint to the estimated model. Secondly, one can use the current account equation and make is equal to the capital account (MacDonald, 2007) [4]. We can see that compared to BEER, FEER may require estimating an open economy macroeconomic model which can be cumbersome (MacDonald and Dias, 2007) [5].

2. Indicate macroeconomic variables used to determine the equilibrium FX

There are many different variables that can go into determining the equilibrium foreign exchange rate. According to the PPP and IRP, we may want to include the price level variables, such as CPI, and interest rate variables, such as real interest rate differential. High inflation rates mean that a country's currency is worth less per unit of currency. This would drive the value of the currency down, causing its equilibrium FX to move with it. When interest rates are higher, foreign investors are drawn to invest in the currency thereby increasing the overall value of the currency. This will change the currency's exchange rate relative to other currencies. By FEER, we would like to include the variables associated with output, such as GDP or output gap. Variables with respect to external balance, such as net capital flow and accumulated current balance, can also be included to determine the exchange rate, depending on the results of ADF tests and data availability.

3. Explain the connection between linear regression and Vector Error Correction (VEC)

We can observe that the Vector Error Correction representation consists of a set of linear equations. This can be easily checked if we look at the *ith* equation in the Vector Error Correction. We regression the difference of *ith* variable on the lags of level of all the variables and the lags of difference of all the variables. The model is still linear in parameters that we need to estimate. In essence, Vector Error Correction is still linear regression. If we re-rewrite the model in Matrix form and expand the parameter and variables matrix, we can still apply OLS to estimate the parameters as the following form: $\beta = (X^T X)^{-1} X y$.

4. Calculate the equilibrium FX using VEC

The foreign exchange rate pair used for calculating the FX equilibrium is the CAD/USD pair using the Behavioral Equilibrium Exchange Rate approach. Python was used to run the calculations. The following annual variables are included in the VEC model:

- Annual Average CAD/USD – This is the average currency pair rate by year for the Canadian dollar based against the United States dollar. Data Source: Federal Reserve Economic Data (FRED) https://fred.stlouisfed.org/
- Crude Oil Prices: West Texas Intermediate (WTI) – the log of the oil index is taken. Canada's economy is heavily reliant on crude oil prices since it is one of their main exports. This is why it was included in the model. Data Source: OECD Data https://data.oecd.org/
- Terms of Trade – the log of the terms of trade are taken. Data Source: OECD Data https://data.oecd.org/
- Inflation (CPI) – this is the difference in CPI between Canada and the United States. Data Source: OECD Data https://data.oecd.org/
- Central government debt, total (% of GDP) – this is the difference between Canada and the United States government debt as a Percentage of GPD. Data Source: Federal Reserve Economic Data (FRED) https://fred.stlouisfed.org/
- Long-term Interest Rates – this is the difference between Canada and the United States long-term interest rates. Data Source: OECD Data https://data.oecd.org/ The data spans from 1989 to 2016.

The data spans from 1989 to 2016. Now that the data is loaded, the lag difference for the VEC model needs to be determined. The main VEC model will have no deterministic value. Using the select_order function from the stats models vecm library and AIC minimization. The lag order for this dataset is 2 based on the output below since the minimum AIC is 2.

```
 VECM Order Selection (* highlights the minimums)
==================================================
      AIC         BIC           FPE         HQIC
--------------------------------------------------
0      -15.87      -13.82   1.407e-07       -15.30
1      -18.65      -14.85   1.559e-08       -17.60
2     -29.73*     -24.17*  2.085e-12*      -28.19*
--------------------------------------------------
nc lags based on aic:  2
```

Next, a trace and maximum eigen Johansen cointegration tests are run to see how many cointegration relationships exist in the variables at 5% significance. The deterministic order is set to no deterministic terms. Below is the python output for both tests:

Johansen cointegration test using trace test statistic with 5% significance level

r_0	r_1	test statistic	critical value
0	6	229.7	83.94
1	6	137.9	60.06
2	6	84.75	40.17
3	6	44.69	24.28
4	6	13.81	12.32
5	6	3.979	4.130

Johansen cointegration test using maximum eigenvalue test statistic with 5% significance level

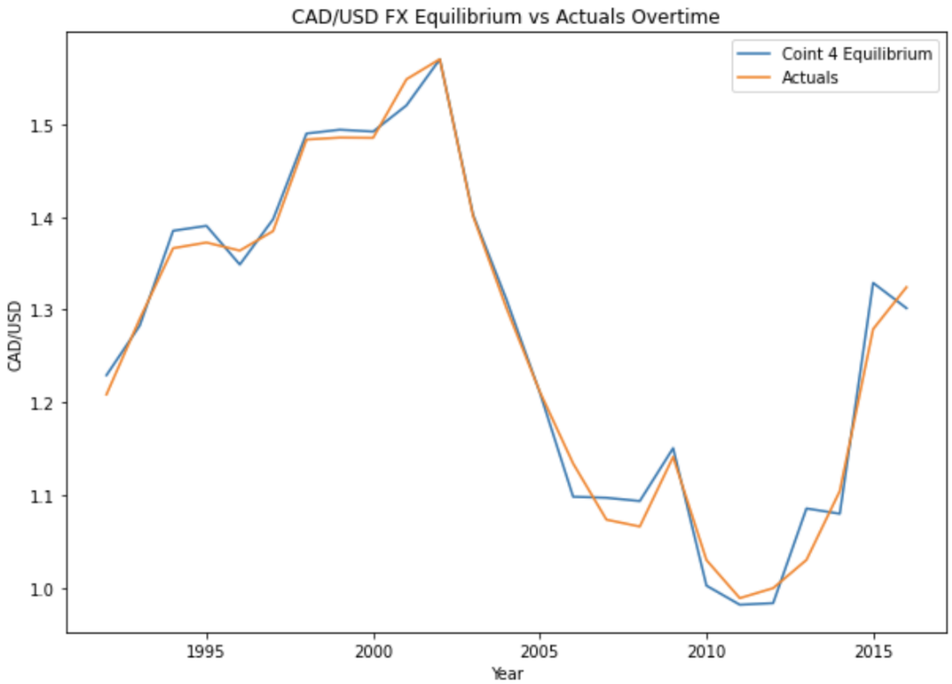
r_0	r_1	test statistic	critical value
0	1	91.87	36.63
1	2	53.11	30.44
2	3	40.06	24.16
3	4	30.88	17.80
4	5	9.827	11.22

The test outputs combined suggest there are 4 or 5 cointegration relationships. We fail to reject the null hypothesis for the trace test at 5 cointegration relationships and we fail to reject the maximum eigenvalue test null hypothesis at 4 cointegration relationships.

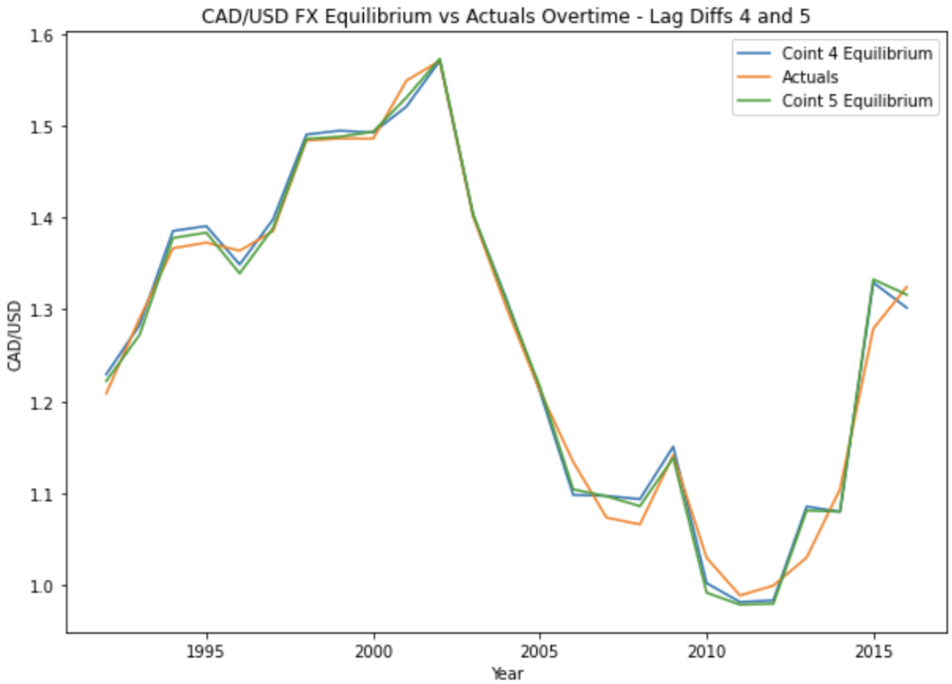
The VEC model then has the following parameters:

- k_ar_diff = 3
- coin_rank = 4
- deterministic = nc (no deterministic terms)

The fitting the VEC model produces the equilibrium of the CAD/USD exchange rate is plotted on the graph below:



The equilibrium exchange rate appears to move a little faster in the reversal of trends than the actual exchange rate. This suggests the actual CAD/USD exchange is undervalued when it is in an upward trend and overvalued when it is in downward trend as compared to the equilibrium value. While this is what the model suggests as an equilibrium, I would actually say the VEC model needs some additional validation and iterative development before accepting the model's equilibrium exchange rate. To support this, VEC model with 5 lag differences was also developed and plotted as an equilibrium:



Both the lag 4 and 5 models display similar behavior, but the magnitude of overvalued and undervalued trends are slightly different when comparing each model to the actuals. This is where experimentation and validation would be helpful to build a robust model. There are also additional economic indicators that could be included in the model but this level of model experimentation is outside of the scope of this analysis.

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