# Modified Prisoner's Dilemma: Rock-Paper-Scissors version

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## Abstract

The goal of this project is to analyze the Prisoner's Dilemma under new rules: the players have now three choices that work with the dynamic of Rock-Paper-Scissors. The ties have been modified to introduce cooperative and defective scenarios. We start introducing the Nash Equilibrium and the standard games, then we describe our new version and finally, we simulate experiments with many players with imitation and migration strategies. We compare these simulations with the ones of the standard Prisoner's Dilemma.

## 1 Introduction

We start by giving the definition of Nash Equilibrium and describing two popular game theory games, i.e. Prisoner's Dilemma and Rock-Paper-Scissors. We show in the introduction what is the Nash Equilibrium in

### 1.1 Nash Equilibrium

In game theory, Nash Equilibrium, named after the mathematician John Nash, is a concept that characterizes stable outcomes in a non-cooperative game. In a Nash Equilibrium, each player's strategy optimizes their own benefit, given the other players' strategies. So, a Nash Equilibrium is a set of strategy choices so that no one can increase their own expected payoff just by changing their choice. More formally:

**Definition 1.1** (Nash Equilibrium)**.** Strategy profile $s = (s_i)_{i\in\mathbb{N}}$ is a Nash Equilibrium if, for all $i \in \mathbb{N}$, for all $s_i' \in S_i$:

$$\Pi_i(s_i, s_{i-1}) \geq \Pi_i(s_i', s_{i-1}).$$

A weak Nash Equilibrium is similar to a Nash Equilibrium, but there is more than one strategy to reach the equilibrium:

**Definition 1.2** (Weak Nash Equilibrium)**.** If, for some player, there is exact equality between the strategy in Nash equilibrium and some other strategy that gives exactly the same payoff (i.e. this player is indifferent between switching and not), then the equilibrium is classified as a weak Nash Equilibrium.

## 1.2 Prisoner's Dilemma

The Prisoner's Dilemma is a well-known and extensively studied game that exemplifies the tension between individual rationality and collective welfare. In this game, two individuals are arrested and placed in separate cells, preventing communication between them. They are presented with a choice: to cooperate (C) with each other or to defect (D). The possible outcomes and associated payoffs can vary, we use the following:

| | C | D |
|---|---|---|
| C | -2, -2 | -10, 0 |
| D | 0, -10 | -6, -6 |

Table 1: Standard Prisoner's Dilemma.

Negative values denote punishment or negative outcomes, while zero values represent neutral outcomes.
In the Prisoner's Dilemma, the unique Nash Equilibrium is for both players to betray each other (D, D). In the Prisoner's Dilemma, the Nash Equilibrium of betraying each other arises due to the dominance of self-interest. In this Equilibrium, any deviation by either player leads to a worse outcome for that player. Although both players could gain more by cooperating, the fear of the other player betraying them outweighs the potential benefits of cooperation. Consequently, the Nash Equilibrium of mutual betrayal becomes the rational and stable outcome.

## 1.3 Rock-Paper-Scissors

Rock, paper, scissors is a simple game that involves two players simultaneously selecting one of three options: rock, paper, or scissors. The game follows a set of rules: rock beats scissors, scissors beat paper, and paper beats rock. The payoffs in this game are typically non-numeric, with the aim being to win rather than maximize utility. Anyway, for what we want to do later, we need to introduce numeric payoffs. A standard example is:

| | R | P | S |
|---|---|---|---|
| R | 0, 0 | -1, 1 | 1, -1 |
| P | 1, -1 | 0, 0 | -1, 1 |
| S | -1, 1 | 1, -1 | 0, 0 |

Table 2: Standard Rock-Paper-Scissors.

Again, a player should maximize his positive score.
Regarding the Nash Equilibrium, the Rock-Paper-Scissors game doesn't admit one pure Nash Equilibrium: in fact, there is no choice that will be chosen 100% of the times by one player. For example, if we assume that player 1 always played rock, then player 2 would always win playing paper. On the other hand, if player 2 always played paper, then player 1 could always play scissors to win. The cycle would continue, so, always picking one choice for the whole game doesn't make sense.
Instead, the situation here is a mixed Nash Equilibrium: both players assign a probability to every choice and they have no interest to deviate from the $1/3, 1/3, 1/3$ probability distribution. It has been proved in [1] the following theorem:

**Theorem 1.1** (Rock-Paper-Scissors Equilibrium)**.** *The game of Rock-Paper-Scissors has a unique mixed Nash Equilibrium. In this Equilibrium, both players play the mixed strategy that puts equal probabilities on all three actions.*

## 2 New dilemma

In this section we will discuss a new game, that merges together Prisoner's Dilemma and Rock-Paper-Scissors and that we will call Modified Rock-Paper-Scissors. The main idea to create the new game is to introduce one cooperative tie and two non-cooperative ones, so that the players were biased toward one decision. We decided the Rock to be the cooperative tie. A first version of the table with the new weights could be this:

| | R | P | S |
|---|---|---|---|
| R | -2, -2 | -10, 0 | 0, -10 |
| P | 0, -10 | -6, -6 | -10, 0 |
| S | -10, 0 | 0, -10 | -6, -6 |

Table 3: Modified Rock-Paper-Scissors, first version.

We can observe that also in this new setup, the players maximize their payoff first of all winning against the adversary, but the interesting part is that drawing with two rocks gives them a -2, while drawing in paper or scissors gives them a -6.
We studied the Nash Equilibrium of this game: from the Nash's existence theorem, there is a mixed Nash Equilibrium. Even if we are considering one cooperative tie the strategy to obtain the equilibrium is the same: playing RPS with probabilities $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$: in fact the only asymmetry happens in the diagonal, but when it happens it affects both players the same.
We decided to modify the game further, to explore new situations. This is second version that we study:

We observe that in this version not only there is a cooperative tie, but it modifies a lot Rock-Paper-Scissors, stating that Paper and Scissors don't win

| | R | P | S |
|---|---|---|---|
| R | -2, -2 | -10, 0 | 0, -10 |
| P | 0, -10 | -6, -6 | -6, -6 |
| S | -10, 0 | -6, -6 | -6, -6 |

Table 4: Modified Rock-Paper-Scissors, second version.

against each other, but they always end up in a non-cooperative tie.
The Nash Equilibrium of this game is similar to the Prisoner's Dilemma one: the Nash Equilibrium is reached when both players play Paper. In fact, playing Rock or Scissors, the player could receive the payoff of -10, while this can't happen choosing Paper.
Let's summarize what we did so far. In the first version we introduced asymmetric ties (only one is cooperative) in the Rock-Paper-Scissors game, but the mixed Equilibrium was still obtained by the strategy $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. In the second version we tried to balance the situations where Paper and Scissors are played, making them non-cooperative ties, but we found again the Prisoner's Dilemma Nash Equilibrium. We finally move to the third and last version of Modified Rock-Paper-Scissors, that balances what we did so far:

| | R | P | S |
|---|---|---|---|
| R | -2, -2 | -10, 0 | 0, -10 |
| P | 0, -10 | -6, -6 | -10, -6 |
| S | -10, 0 | -6, -10 | -6, -6 |

Table 5: Modified rock, paper, scissors.

This weights manage to balance this game, reintroducing a payoff of -10 for losing playing Paper against Scissors, that is coherent with the other defeats, but having instead a payoff of only -6 when winning playing Scissors against Paper. Nobody can have a payoff of 0 playing Scissors, but, for example, if player 1 assumes that player 2 is going to play Paper, it can play Scissors to decrease the opponent payoff.
The Nash Equilibrium of this game is a Weak Nash Equilibrium. In fact, the payoff of Paper and Scissors is the same. Choosing Paper brings to the Nash Equilibrium, while choosing Scissors brings to a loop where the opponent has again to choose between Paper and Scissors. We could call it a 'Prosocial Equilibrium', because the Nash Equilibrium is reached if the players - fixing their payoff - take the decision that increases the most the payoff of the opponent, i.e. Paper in this setting.

# 3 Experiments

Having analyzed this new problem setup, we want to run computer simulations of iterated, multi-agent plays of cooperative Rock-Paper-Scissors in a 2D grid.

These experiments have been investigated extensively in the case of the Prisoner's dilemma, leading to intriguing and surprising results [2]. The goal of this section is to examine our newly devised dilemma using the same experimental lens used previously in the field. As such, we use the instructions given by Helbing et al. [2] to run the simulations and examine the results we obtain.

The chosen scheme shares some similarities with the field of cellular automata, where independent agents in a grid governed by simple rules interact with each other and lead to complex outcomes that cannot be explained by simple rules as the ones governing each part of the system. The concept was first discovered by Stanislaw Ulam [3] and John von Neumann [4] while working at Los Alamos National Laboratory and later investigated by notorious scientists such as Stephen Wolfram [5] and Richard Feynman. Phenomena such as emergence and self-organisation have since been thoroughly studied from several different points of view.

In our case, we are defining a new cellular automata in which each cell can have a strategy-state or be blank. By defining the rules that determine winners and losers in the game between cells, and establishing imitation and mobility mechanisms, we expect to find complex properties of the system. In particular, [2] observed that there were outbreaks of cooperative clusters that were robust to exploitative strategies by neighbouring nodes. We expect to find similar situations, and we will study them in detail.

### 3.1 Implementation of the simulation

The problem we study is the iterated play of agents in a 2D grid of size $N \times N$. Every agent $i$ has a fixed strategy $s_i \in \{R, P, S\}$. At every time step, all agents play against their 4 neighbours in the 4 directions of the grid and receive outcomes according to the game's rules.

Every time an agent plays, it performs 3 distinct operations:

1. Play: the agent competes against its 4 neighbours, gathering the corresponding payoffs.

2. Imitate: the agent looks at the strategies its neighbours followed, and imitates the strategy of the neighbour who had the highest payoff.

3. Move: the agent looks at the empty cells in the $(2m+1) \times (2m+1)$ grid around it and computes the expected payoff given its current strategy in those positions. It then decides to move to the empty cell that yields the highest payoff. In the case of a tie, the chosen cell is the closest to the agent.

The order of the agent updates is chosen at random at every time step to make the procedure more akin to a real-life situation (where updates happen independently everywhere).

In order to add variability and test the robustness of the system, we add noise to the imitation process in the following way:

- With probability $1 - r$ imitate the best neighbouring strategy.
- With probability $r$, perform a random strategy update. This is done in the following way: select the cooperative strategy (i.e. rock) with probability $q$, and select one of the other strategies at random with probability $1 - q$.

The simulation parameters we use are: $N = 50$, $m = 15$, $r = 0.05$, $q = 0.05$. And the payoffs for the RPS game are $T = 1.3, R = 1.0, P = 0.1, S = 0.0$, corresponding to the temptation to unilaterally defect, reward for cooperation, punishment for mutual defection and sucker's payoff respectively. All these values were set to be the same as the ones used in [2], but we found out that using different values for the payoffs did not change the results as long as the required Prisoner's dilemma conditions were met (i.e. $T > R > P > S$ and $2R > T + P$).

### 3.2 Results

To start with, we run a sanity check with the Prisoner's dilemma game. The goal is to see if we are able to replicate the results obtained in the original paper [2].

We can observe that the results in figure 1 are very similar to the phenomena reported in the paper. In particular, we can see that there are clusters of cooperators that are robust to the presence of defectors. Whenever a defector gets inside of a cluster of cooperators, the cluster gets broken up into smaller clusters with defectors at the boundary thanks to the success-driven migration mechanisms. This is arguably the most important finding in [2]: migration and imitation together enable the *outbreak of cooperation*, i.e. the formation of clusters of cooperators that are robust to the presence of defectors (and robust to noise in the imitation process too).

We now move on to the modified Rock-Paper-Scissors game. The results are shown in figure 2. We can see that the results are very similar to the ones obtained in the Prisoner's dilemma case: the agents playing scissors quickly disappear, and cooperator clusters are formed, surrounded by cells playing paper. The cause of this is that cooperation is a dominant strategy in the game, as seen before, and agents that choose scissors are likely to quickly lose against the majority of cooperative agents (rock) in the grid. Imitation forces them to switch to rock, further reinforcing these clusters of cooperators.

In step 50 and 99 we can see that there are some scissors agents, but their existence is due to noise in the imitation process. When looking at the sequence of grid configurations, their origin becomes clear, and they are very short-lived, usually disappearing in the next time step.

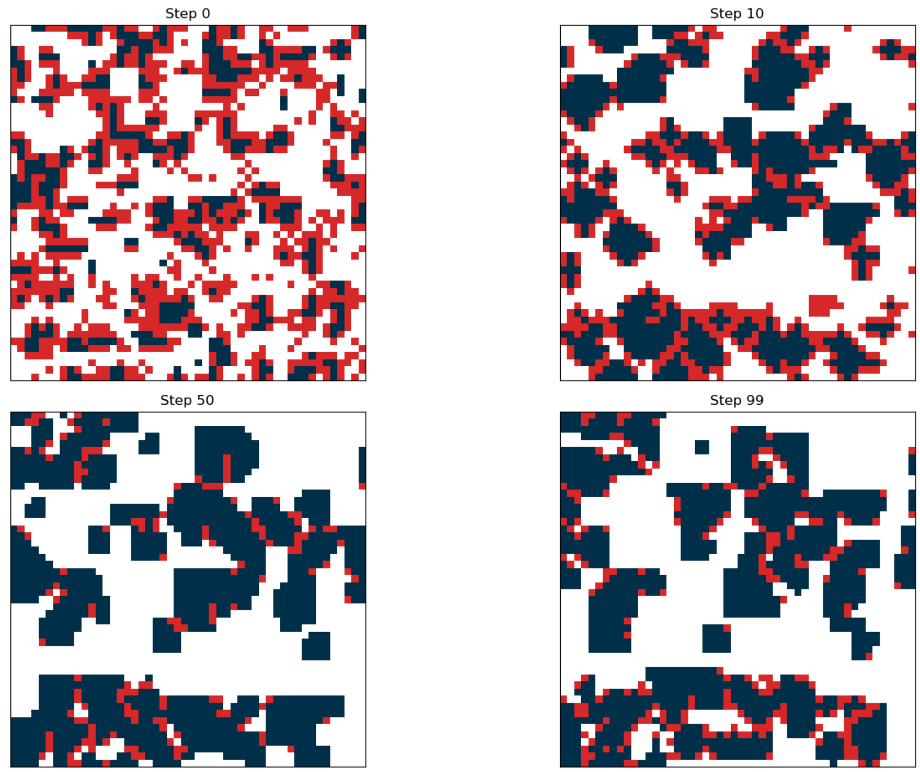


Figure 1: Results of the simulation with the Prisoner's dilemma game. Blue cells are cooperators, red cells are defectors.

# References

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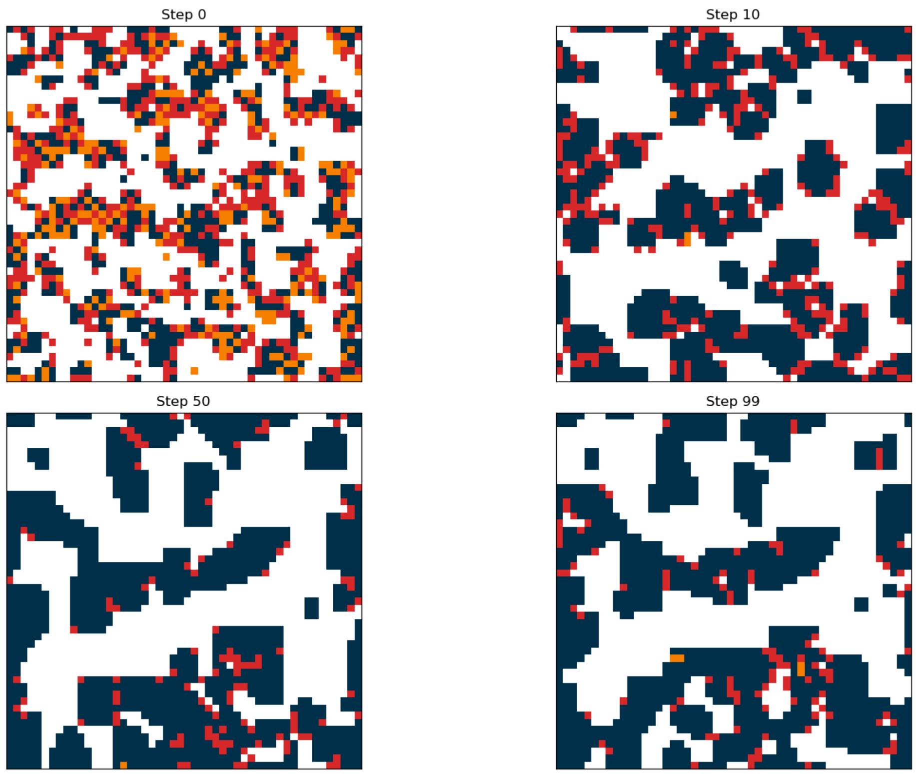


Figure 2: Results of the simulation with the Prisoner's dilemma game. Blue cells are cooperators, red cells are defectors.