Artificial Intelligence I 2023/2024

Week 9 Tutorial and Additional Exercises

Uninformed Search

School of Computer Science

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In this tutorial. . .

In this tutorial we will be covering

- Search problem formulation and components.
- Breadth-First Search (BFS).
- Depth-First Search (DFS).

Formulating a Search Problem

Definition 1 (Search Problem Formulation and Components)

Formulating a search problem is the process to formally define a search for a solution. A search problem has five components:

- **Initial state**, the state where the agent starts its search;
- **Action set**, the set $\mathcal{A}$ describing the actions that can be executed in any state $s_i \in \mathcal{S}$;
- **Transition model**, a mapping between states and actions, i.e., the states resulting from executing each action $a_i \in \mathcal{A}$ in every state $s_i \in \mathcal{S}$;
- **Goal test**, to determine if a state is a goal state;
- **Path cost function**, which assigns a cost to each path.

- The first three components define the **state space**.
- The state space can take the form of a graph or network.

Solution, Cost and Path

Definition 2 (Solution)

The **solution** of a search problem is the sequence of actions from the initial state to a goal state.

Definition 3 (Cost)

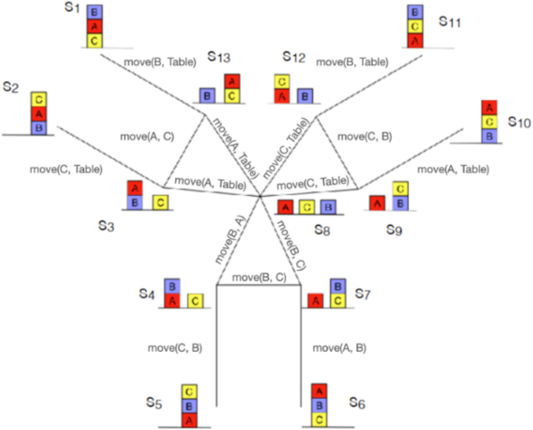
The **cost** of a solution is the sum of the cost of the actions from the initial state to the goal state.

Definition 4 (Path)

A **path** in the state space is a sequence of states connected by a sequence of actions.

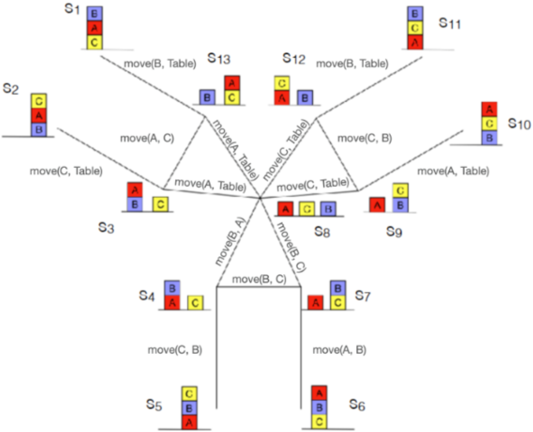
Exercise 1

- Consider the state space graph for the **three-block world**, shown to the right.
- There are three blocks and the goal is to go from one particular configuration to another.
- To do so, blocks can be placed on the table and then moved into different configurations.
- Each move costs 1.



Exercise 1 (continued)

- Provide a search problem formulation for the three-block-world.
- Identify the five components in Definition 1 for this search problem.



Exercise 1: Solution

The five components according to Definition 1 are the following:

- **Initial state**: Any of the 13 states;
- **Action set**: $move(X, Y)$, meaning move block X on top of block Y, where $X, Y \in \{A, B, C, Table\}$, with $X \neq Y$;
- **Transition model**: A single move of a block. Examples are going from $s_3 \rightarrow s_{13}$ via $move(A, C)$, or going from $s_{11} \rightarrow s_{12}$ via $move(B, Table)$;
- **Goal test**: Check if the state is a goal state.
- **Path cost function**: Each step costs 1.

Uninformed Search Strategies

Definition 5 (Uninformed search)

Uninformed Search (also called **blind search**) is a term used to define the set of strategies having no additional information about the state space beyond that provided in the problem formulation.

- Uninformed search strategies can only generate successors and distinguish a goal state from a non-goal state.
- The key difference between two uninformed search strategies is the **order** in which nodes are expanded.

Breadth-First Search

- Recall the definition of Breadth-First Search.

Definition 6 (Breadth-First search)

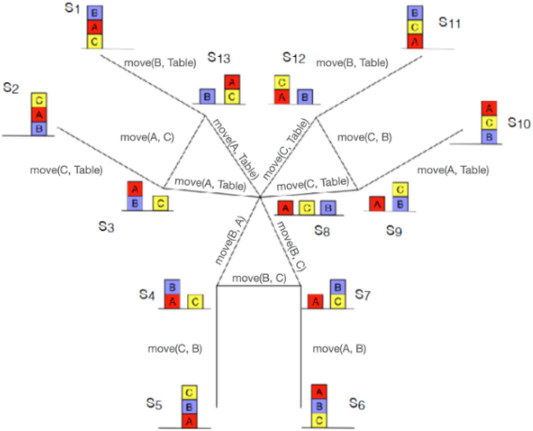
Breadth-First Search (BFS) is an uninformed search strategy in which

- the root node is expanded first;
- then, all the successors of the root node are expanded;
- then, the successors of each of these nodes are expanded;
- until the goal node is placed in the frontier.

- This is equivalent to expanding the shallowest unexpanded node in the frontier.

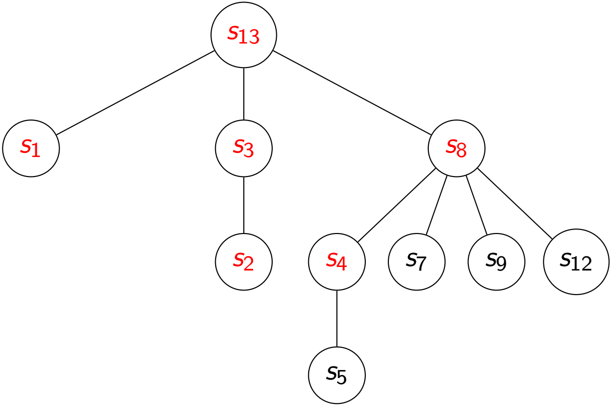
Exercise 2

- Reconsider the three-block world.
- Assume the initial state is s_{13} and the goal state is s_5.
- Use BFS from Definition 6 to solve this search problem.
- Write the BFS tree, and the sequence of nodes, in the order they are expanded.
- When multiple nodes are at the same level, expand the one with the smallest index.



Exercise 2: Solution

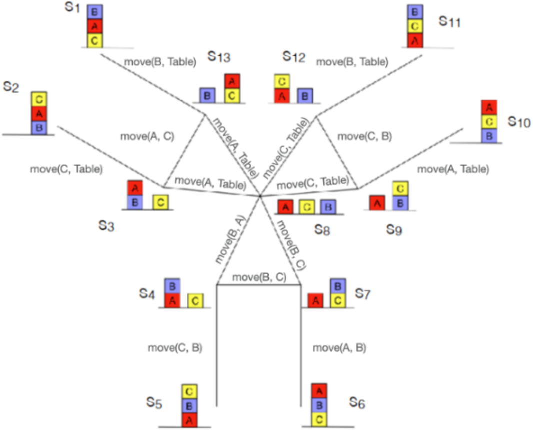
- The BFS tree is the following, where coloured nodes are the ones that were expanded.



- The nodes are expanded in the order: $s_{13}, s_1, s_3, s_8, s_2, s_4$.
- Note that s_7, s_9, s_{12} need not be expanded, because BFS stops when s_5 is placed in the frontier, that is, when s_4 is expanded.

Exercise 3

- Reconsider the three-block world.
- Assume the initial state is s_{13} and the goal state is s_5.
- From what we found in Exercise 2 using BFS, write the solution from Definition 2, and the corresponding cost from Definition 3.
- Also write down the path corresponding to that solution, from Definition 4.



Exercise 3: Solution

- The solution is the sequence of the following actions

  $$move(A, Table), move(B, A), move(C, B).$$

  Since each move costs 1, the total cost of this solution is 3.
- The path corresponding to this solution is the sequence of the following states

  $$s_{13}, s_8, s_4, s_5.$$

Exercise 4

- In this exercise, we consider the well-known **8 queen puzzle**.
- We start with an empty 8×8 chessboard, and the goal is to place 8 queens on the chessboard, so that there are no conflicts (no two queens attack each other).
- Recall that a queen can move any number of squares, either vertically, horizontally, or diagonally, in one move.
- Placing a queen on an empty square costs 1.
- Provide a search problem formulation for the 8 queen puzzle.
- Identify the five components in Definition 1 for this search problem.

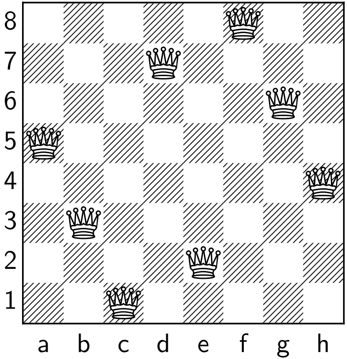
Exercise 4: Solution

The five components according to Definition 1 are the following:

- **Initial state**: An empty 8×8 chessboard;
- **Action set**: $\textit{place}(XY)$, meaning place a queen on the empty square XY, where $X \in \{a, b, \ldots, h\}$, and $Y \in \{1, 2, \ldots, 8\}$;
- **Transition model**: The resulting board after taking an action;
- **Goal test**: All eight queens placed on the board with no conflicts;
- **Path cost function**: Each step costs 1.

Exercise 5

- Reconsider the 8 queen puzzle.
- Discuss how you can apply BFS to find one possible solution.
- Draw the BFS tree for the formulation you propose.
- One such solution is shown below.



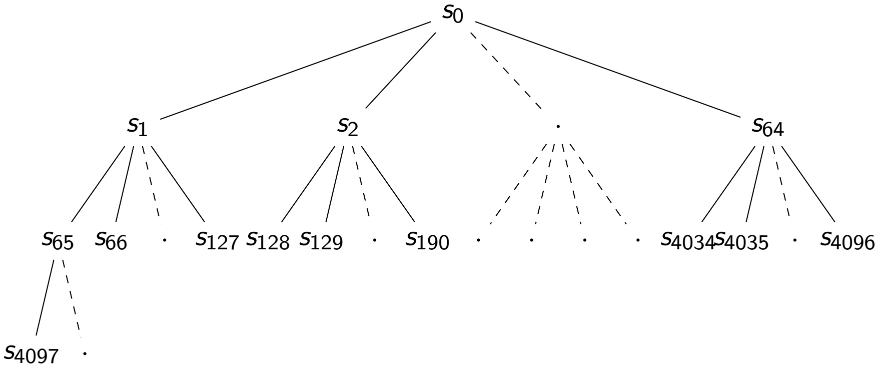
Exercise 5: Solution

We give one possible formulation as follows.

- Define s_0 the initial state with the empty chessboard.
- Define $s_1, s_2, \dots, s_{64}$ the states where the first queen is placed on $a1, a2, \dots, h8$ respectively.
- Define $s_{65}, s_{66}, \dots, s_{127}$ the successors of s_1, that is the states where one queen is already on $a1$, and the second queen is placed on $a2, a3, \dots, h8$ respectively.
- Define $s_{128}, s_{129}, \dots, s_{190}$ the successors of s_2, that is the states where one queen is already on $a2$, and the second queen is placed on $a1, a3, \dots, h8$ respectively.
- Similarly, define the successors of all nodes $s_3, \dots, s_{64}$.
- Repeat this labeling to define the successors for the nodes in the second level, $s_{65}, s_{66}, \dots$, and so on, until the final level has all eight queens placed.

Exercise 5: Solution (continued)

- With this formulation, the BFS tree will be the following (only the first levels are shown).



Measuring Performance

- Recall some quantities for graphs, that are used to measure the performance in a search problem.

Definition 7 (Branching factor)

The **branching factor** of a graph is the maximum number of successors among its nodes.

Definition 8 (Depth)

The **depth** of a graph is the minimum level among its goal nodes.

Definition 9 (Maximum length)

The **maximum length** of a graph is the maximum level among its nodes.

Exercise 6

- Reconsider the 8 queen puzzle, and the graph formulation we gave in Exercise 5.
- Find the branching factor from Definition 7, the depth from Definition 8, and maximum length from Definition 9, of this graph.

Exercise 6: Solution

- The root node has 64 successors, as there are 64 possible choices to place the first queen.
- Each of the 64 nodes on the first level has 63 successors, as there are 63 possible choices to place the second queen.
- Similarly, each node on the second level has 62 successors, and so on, until the seventh level, which they have 56.
- The branching factor equals 64. In fact, the only root that has 64 successors is the root node. Every other node has fewer successors.
- The depth equals 8, as the goal nodes can only be on the eighth level, where all eight queens are placed.
- The maximum length also equals 8, as there is no level past the eighth.

Depth-First Search

- Recall the definition of Depth-First Search.

Definition 10 (Depth-First search)

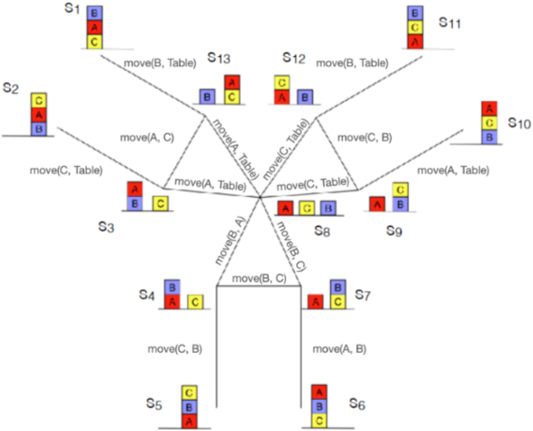
Depth-First Search (DFS) is an uninformed search strategy in which

- the root node is expanded first;
- then, the first (or one at random) successor of the root node is expanded;
- then, the deepest node in the current frontier is expanded;
- until the goal node is visited.

- This is equivalent to expanding the deepest unexpanded node in the frontier.
- Note that for a DFS to terminate, the goal node must be visited, not just appear in the frontier, unlike BFS.

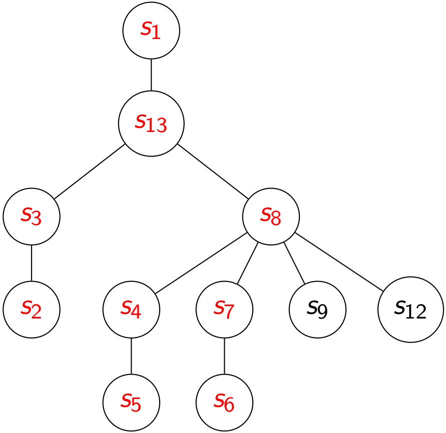
Exercise 7

- Reconsider the three-block world.
- Assume the initial state is s_1 and the goal state is s_6.
- Use DFS from Definition 10 to solve this search problem.
- Write the DFS tree, and the sequence of nodes, in the order they are expanded.
- When multiple nodes are at the same level, expand the one with the smallest index.



Exercise 7: Solution

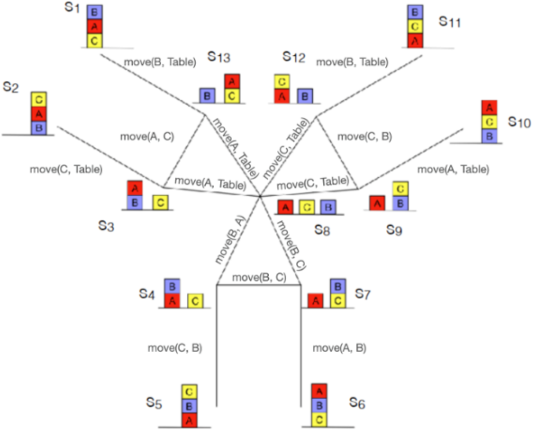
- The DFS tree is the following, where coloured nodes are the ones that were expanded.



- The nodes are expanded in the order: $s_1, s_{13}, s_3, s_2, s_8, s_4, s_5, s_7, s_6$.
- DFS stops when s_6 is visited.

Exercise 8

- Reconsider the three-block world.
- Assume the initial state is s_1 and the goal state is s_{13}.
- From what we found in Exercise 7 using DFS, write the solution from Definition 2, and the corresponding cost from Definition 3.
- Also write down the path corresponding to that solution, from Definition 4.



Exercise 8: Solution

- The solution is the sequence of the following actions

  $$move(B, Table), move(A, Table), move(B, C), move(A, B).$$

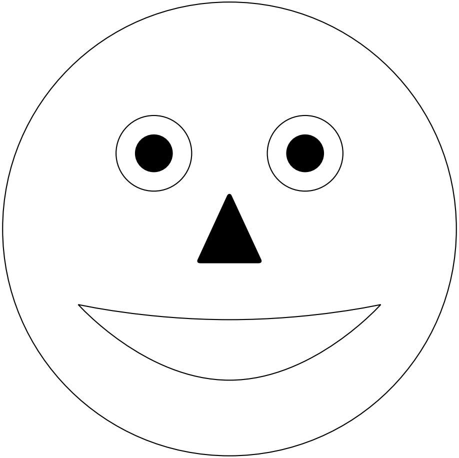
  Since each move costs 1, the total cost of this solution is 4.
- The path corresponding to this solution is the sequence of the following states

  $$s_1, s_{13}, s_8, s_7, s_6.$$

Q&A

Any questions?

Until the next time...



Thank you for your attention!