UNIVERSITY OF BIRMINGHAM

Artificial Intelligence 1
Informed Search

Dr Leonardo Stella

School of Computer Science

2023/2024 - Week 9

Aims of the Session

This session aims to help you:

- Understand the concept of a heuristic function
- Explain the steps involved in A*
- Analyse the performance of A* and apply the algorithm to solve search problems

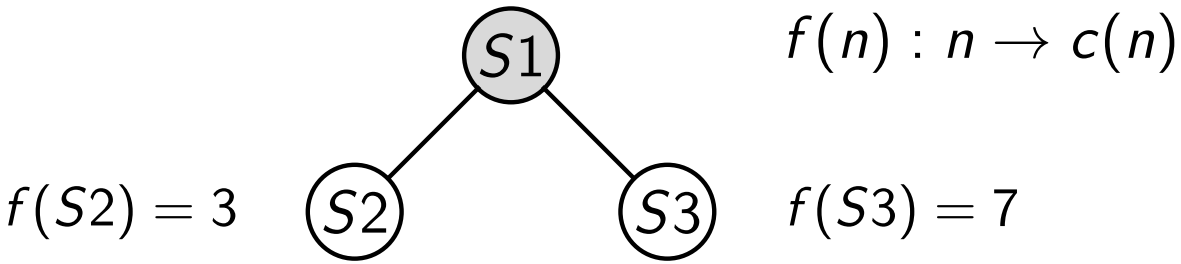
Outline

Outline

Outline

Informed Search

- **Informed search** strategies use problem-specific knowledge beyond the definition of the problem itself
- In general, informed search strategies can find solutions more efficiently compared to uninformed search
- The general approach, called best-first search, is to determine which node to expand based on an evaluation function $f(n)$



- This function acts as a cost estimate: the node with the lowest cost is the one that is expanded next

Heuristics

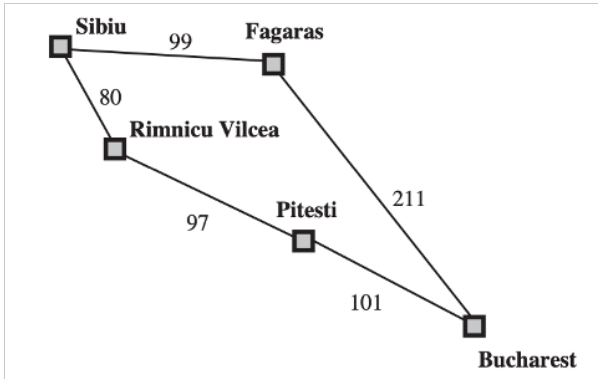
- The evaluation function $f(n)$ for most best-first search algorithms includes a heuristic function as a component:
  $h(n)$: estimated cost of the cheapest path from node n to a goal node
- Heuristic functions are the most common form in which new knowledge is given to the search algorithm. If n is a goal node, then

$$h(n) = 0$$

- A heuristic can be a rule of thumb, common knowledge; it is quick to compute, but not guaranteed to work (nor to yield optimal solutions)

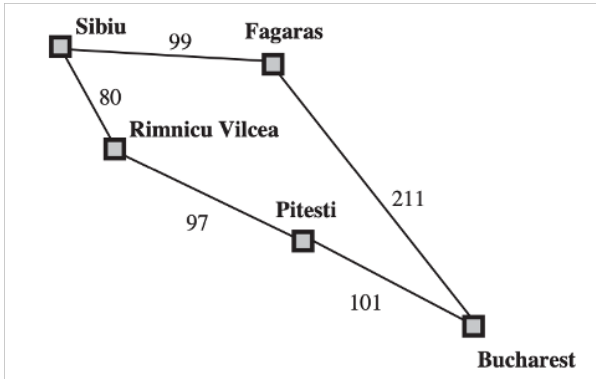
Heuristics

- Consider the following problem, where we want to find the shortest path to Bucharest (from, e.g., Sibiu) in Romania
- A possible heuristic is the straight-line distance heuristic, h_{SLD}
- This is a useful heuristic as it is correlated with actual road distances



Heuristics

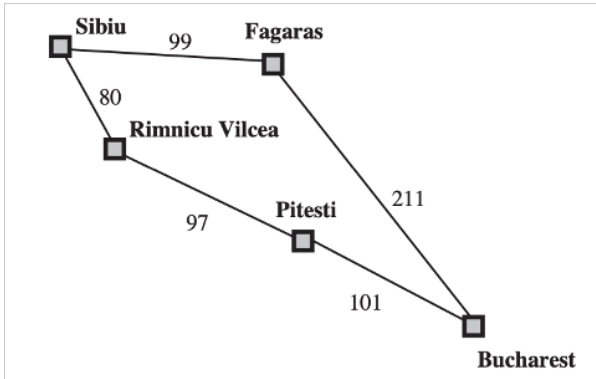
- Consider the following problem, where we want to find the shortest path to Bucharest (from, e.g., Sibiu) in Romania
- The straight-line distances $h_{SLD}(n)$ are shown in the table below
- For example, the SLD from Sibiu would be $h_{SLD}(Sibiu) = 253$



Arad	366	**Mehadia**	241
Bucharest	0	**Neamt**	234
Craiova	160	**Oradea**	380
Drobeta	242	**Pitesti**	100
Eforie	161	**Rimnicu Vilcea**	193
Fagaras	176	**Sibiu**	253
Giurgiu	77	**Timisoara**	329
Hirsova	151	**Urziceni**	80
Iasi	226	**Vaslui**	199
Lugoj	244	**Zerind**	374

Greedy Best-First Search

- Consider the following problem, where we want to find the shortest path to Bucharest (from, e.g., Sibiu) in Romania
- If we use $f(n) = h_{SLD}(n)$, then from Sibiu we expand Fagaras
- This is because $h_{SLD}(Fagaras) = 176 < h_{SLD}(Rimnicu\ Vilcea) = 193$
- When $f(n) = h(n)$, we call this strategy **greedy best-first search**



Arad	366	**Mehadia**	241
Bucharest	0	**Neamt**	234
Craiova	160	**Oradea**	380
Drobeta	242	**Pitesti**	100
Eforie	161	**Rimnicu Vilcea**	193
Fagaras	176	**Sibiu**	253
Giurgiu	77	**Timisoara**	329
Hirsova	151	**Urziceni**	80
Iasi	226	**Vaslui**	199
Lugoj	244	**Zerind**	374

A* Search

- The most widely known informed search strategy is **A***
- This search strategy evaluates nodes using the following evaluation function

$$f(n) = g(n) + h(n)$$

  where $g(n)$ is the cost to reach node n and $h(n)$ is the heuristic from node n to the goal
- This is equivalent to the cost of the cheapest solution through node n

A* Search

- Steps of the A* algorithm:
  - **Expand** the node in the frontier with smallest $f(n) = g(n) + h(n)$
  - **Repeated states and loopy paths**. If a node is in the list of visited nodes, **do not add** it to the frontier.
    If the state of a given child is in the frontier:
    - If the frontier node has a larger $g(n)$, place the child into the frontier and remove the node with larger $g(n)$ from the frontier
  - **Stop** when a goal node is visited

Performance of A*

- A* is complete and optimal if $h(n)$ is **consistent**

Definition: A heuristic is said to be **consistent** (or **monotone**), if the estimate is always no greater than the estimated distance from any neighbouring node n' to the goal, plus the cost of reaching that neighbour:

$$h(n) \leq cost(n, n') + h(n')$$

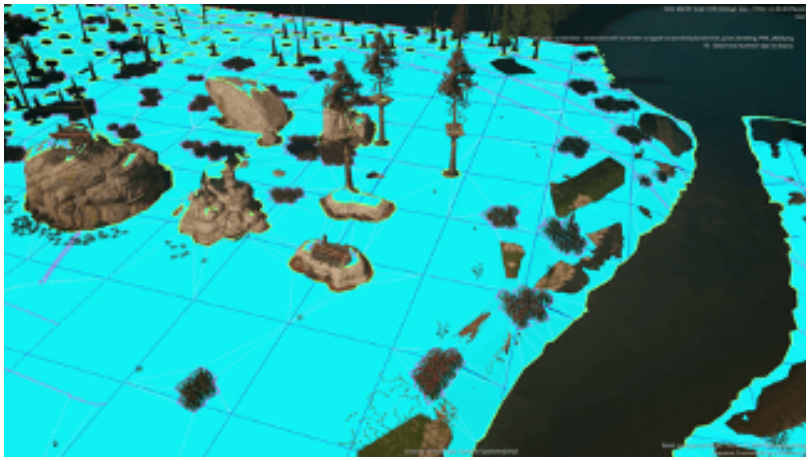
- The number of states generated by A* is exponential in the length of the solution, namely for constant step costs: $\mathcal{O}(b^{\epsilon d})$
- If h^* is the actual cost from root node to goal node, $\epsilon = \frac{h^* - h}{h^*}$ is the relative error
- Space is the main issue with A*, as it keeps all generated nodes in memory, not suitable for large-scale problems

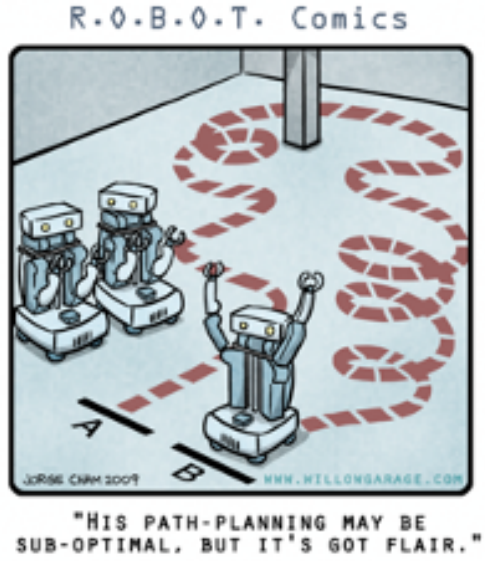
Performance of A*

- Let us summarise the performance of A*.
  - **Completeness**: if the heuristic $h(n)$ is consistent, then A* is complete
  - **Optimality**: if the heuristic $h(n)$ is consistent, then A* is optimal
  - **Time complexity**: $\mathcal{O}(b^{\epsilon d})$, where ϵ is the relative error of the heuristic
  - **Space complexity**: $\mathcal{O}(b^d)$, since we keep in memory all expanded nodes and all nodes in the frontier

Applications of A*

- A* is one of the most widely adopted search algorithms
- The most common settings are in games and in robotics





References

Russell, A. S., and Norvig, P., *Artificial Intelligence A Modern Approach*, 4^{th} Edition. Prentice Hall.

- Chapter 3 – "Solving Problems by Searching", Section 3.5 "Informed (Heuristic) Search Strategies" up to and including Section 3.6.2 "Generating Admissible Heuristics From Relaxed Problems".

Aims of the Session

You should now be able to:

- Understand the concept of a heuristic function
- Explain the steps involved in A*
- Analyse the performance of A* and apply the algorithm to solve search problems

Thank you!