UNIVERSITY OF BIRMINGHAM

Artificial Intelligence 1
Informed Search

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School of Computer Science

2023/2024 - Week 9

Aims of the Session

This session aims to help you:

- Understand the concept of a heuristic function
- Explain the steps involved in A*
- Analyse the performance of A* and apply the algorithm to solve search problems

Outline

1. Recap: Uninformed Search

2. Informed Search

Outline

Recap: Quiz

Q1. What does it mean for an environment to be **known**?

- ☐ There are only finitely many actions at any state
- ☐ Possible to determine which states are reached by which action
- ☐ The agent is able to know the current state
- ☐ Each action has exactly one outcome

Recap: Quiz

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Recap: Quiz

Q2. If we relax the assumption for an environment to be discrete, BFS is still complete.

- ○ True
- ○ False

Recap: Quiz

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- ○ True
- ● False, as b would be infinite

Recap: Quiz

Q3. Which components form the **state space** of a search problem?

- ☐ Transition model
- ☐ Initial state
- ☐ Goal test
- ☐ Path cost function
- ☐ Action set

Recap: Quiz

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- ■ Transition model
- ■ Initial state
- □ Goal test
- □ Path cost function
- ■ Action set

Recap: Quiz

Q4. The time complexity of DFS is $\mathcal{O}(b^m)$.

- ○ True
- ○ False

Recap: Quiz

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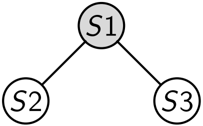
- ● True
- ○ False

Outline

Informed Search

- **Informed search** strategies use problem-specific knowledge beyond the definition of the problem itself
- In general, informed search strategies can find solutions more efficiently compared to uninformed search
- The general approach, called best-first search, is to determine which node to expand based on an evaluation function $f(n)$



$$f(n): n \to c(n)$$

$f(S2) = 3$

$f(S3) = 7$

- This function acts as a cost estimate: the node with the lowest cost is the one that is expanded next

Heuristics

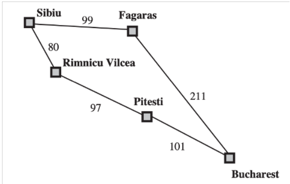
- The evaluation function $f(n)$ for most best-first search algorithms includes a heuristic function as a component:
  $h(n)$: estimated cost of the cheapest path from node n to a goal node
- Heuristic functions are the most common form in which new knowledge is given to the search algorithm. If n is a goal node, then

$$h(n) = 0$$

- A heuristic can be a rule of thumb, common knowledge; it is quick to compute, but not guaranteed to work (nor to yield optimal solutions)

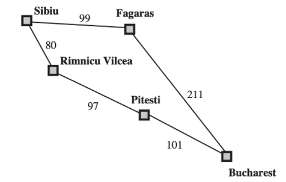
Heuristics

- Consider the following problem, where we want to find the shortest path to Bucharest (from, e.g., Sibiu) in Romania
- A possible heuristic is the straight-line distance heuristic, h_{SLD}
- This is a useful heuristic as it is correlated with actual road distances



Heuristics

- Consider the following problem, where we want to find the shortest path to Bucharest (from, e.g., Sibiu) in Romania
- The straight-line distances $h_{SLD}(n)$ are shown in the table below
- For example, the SLD from Sibiu would be $h_{SLD}(Sibiu) = 253$



Arad	366	**Mehadia**	241
Bucharest	0	**Neamt**	234
Craiova	160	**Oradea**	380
Drobeta	242	**Pitesti**	100
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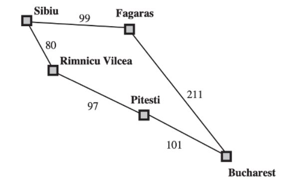
Greedy Best-First Search

- Consider the following problem, where we want to find the shortest path to Bucharest (from, e.g., Sibiu) in Romania
- If we use $f(n) = h_{SLD}(n)$, then from Sibiu we expand Fagaras
- This is because $h_{SLD}(Fagaras) = 176 < h_{SLD}(Rimnicu\ Vilcea) = 193$
- When $f(n) = h(n)$, we call this strategy **greedy best-first search**



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A* Search

- The most widely known informed search strategy is **A***
- This search strategy evaluates nodes using the following evaluation function
$$f(n) = g(n) + h(n)$$
where $g(n)$ is the cost to reach node n and $h(n)$ is the heuristic from node n to the goal
- This is equivalent to the cost of the cheapest solution through node n

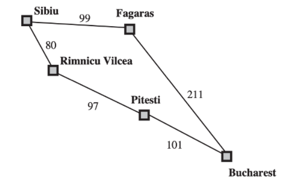
A* Search

- Steps of the A* algorithm:
  - **Expand** the node in the frontier with smallest $f(n) = g(n) + h(n)$
  - **Repeated states and loopy paths**. If a node is in the list of visited nodes, **do not add** it to the frontier.
    If the state of a given child is in the frontier:
    - If the frontier node has a larger $g(n)$, place the child into the frontier and remove the node with larger $g(n)$ from the frontier
  - **Stop** when a goal node is visited

A* Search

Activity. Consider the following problem, where we want to find the shortest path to Bucharest from Sibiu in Romania. In pairs or small groups, calculate $f(n)$ at each step of the A* algorithm.

$$f(Fagaras) = g(Fagaras) + h(Fagaras)$$



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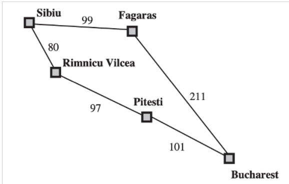
A* Search

Activity. Consider the following problem, where we want to find the shortest path to Bucharest from Sibiu in Romania. In pairs or small groups, calculate $f(n)$ at each step of the A* algorithm.

$$f(Fagaras) = 99 + 176 = 275$$

$$f(Rimnicu\ Vilcea) = g(Rimnicu\ Vilcea) + h(Rimnicu\ Vilcea)$$



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A* Search

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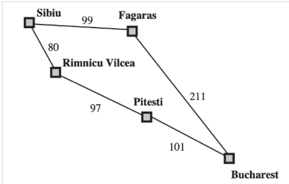
$$f(Rimnicu\ Vilcea) = 80 + 193 = 273$$



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A* Search

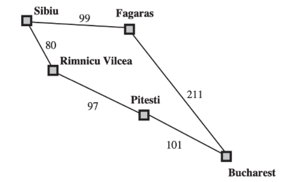
$$f(Fagaras) = 99 + 176 = 275$$

$$f(Rimnicu\ Vilcea) = 80 + 193 = 273$$

$$f(Bucharest) = (99 + 211) + 0 = 310$$

$$f(Pitesti) = (80 + 97) + 100 = 277$$

$$f(Bucharest) = (80 + 97 + 101) + 0 = 278$$



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A⋆ Search

Activity. Consider the following problem, where we want to find the shortest path to Bucharest from Arad in Romania. In pairs or small groups, determine the solution retrieved by A⋆.

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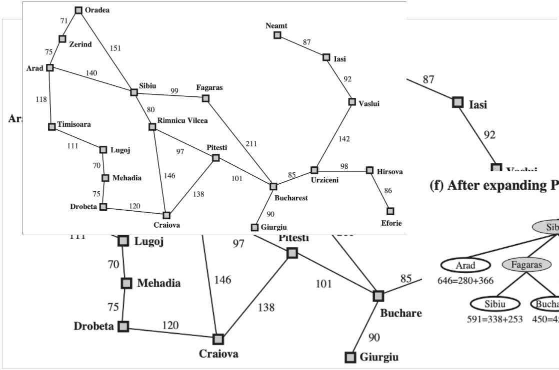
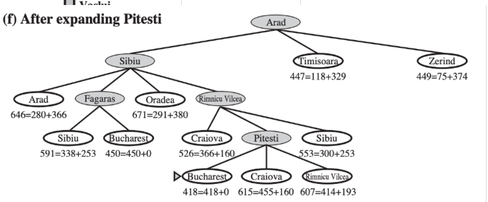




Performance of A*

- A* is complete and optimal if $h(n)$ is **consistent**

Definition: A heuristic is said to be **consistent** (or **monotone**), if the estimate is always no greater than the estimated distance from any neighbouring node n' to the goal, plus the cost of reaching that neighbour:

$$h(n) \leq cost(n, n') + h(n')$$

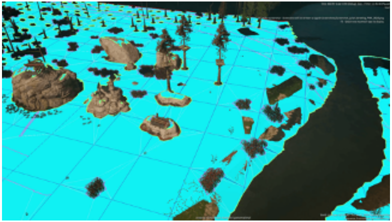
- The number of states generated by A* is exponential in the length of the solution, namely for constant step costs: $\mathcal{O}(b^{\epsilon d})$
- If h^* is the actual cost from root node to goal node, $\epsilon = \frac{h^* - h}{h^*}$ is the relative error
- Space is the main issue with A*, as it keeps all generated nodes in memory, not suitable for large-scale problems

Performance of A*

- Let us summarise the performance of A*.
  - **Completeness**: if the heuristic $h(n)$ is consistent, then A* is complete
  - **Optimality**: if the heuristic $h(n)$ is consistent, then A* is optimal
  - **Time complexity**: $\mathcal{O}(b^{\epsilon d})$, where ϵ is the relative error of the heuristic
  - **Space complexity**: $\mathcal{O}(b^{d})$, since we keep in memory all expanded nodes and all nodes in the frontier

Applications of A*

- A* is one of the most widely adopted search algorithms
- The most common settings are in games and in robotics



References

Russell, A. S., and Norvig, P., *Artificial Intelligence A Modern Approach*, 4^{th} Edition. Prentice Hall.

- Chapter 3 – "Solving Problems by Searching", Section 3.5 "Informed (Heuristic) Search Strategies" up to and including Section 3.6.2 "Generating Admissible Heuristics From Relaxed Problems".

Aims of the Session

You should now be able to:

- Understand the concept of a heuristic function
- Explain the steps involved in A*
- Analyse the performance of A* and apply the algorithm to solve search problems

Thank you!