MAE0221

PROBABILIDADE I

PEDRO GIGECK FREIRE 10737136 26/05/2021

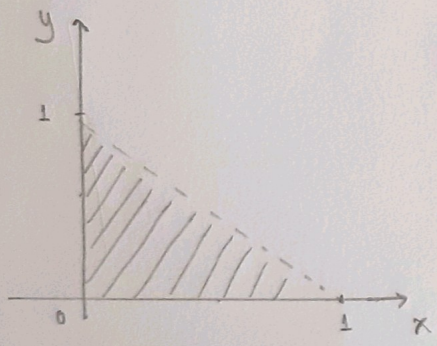
PROVINHA 05

Considere $(X,Y)$ com f.d.p conjunta

$$f(x,y) = 3(x+y) \quad \text{para } x \in (0,1),\ y \in (0,1),\ 0 < x+y < 1$$

a) Desenho a região do suporte conjunto de $(X,Y)$

Vamos desenhar a região no plano cartesiano




(Se $Y = y$, então $X = x < 1 - y$)

b) Encontre a f.d.p de $X$

$$f_X(x) = \int_{y=-\infty}^{\infty} f(x,y)\,dy = 0 + \int_{y=0}^{\infty} f(x,y)\,dy$$

(para $0 < x < 1$)

$$= \int_{y=0}^{\infty} 3(x+y)\,\mathbb{1}_{\{x+y<1\}}(y)\,dy$$

(portanto a função indicadora $\mathbb{1}_{\{x+y<1\}}(y) = 1$ se e só se $y < 1-x$)

$$= \int_{y=0}^{1-x} 3(x+y)\,dy + \int_{y=1-x}^{\infty} 0\,dy$$

$$= 3\left[ \int_{y=0}^{1-x} x\,dy + \int_{y=0}^{1-x} y\,dy \right]$$

$$= 3\left[ x\left( y\Big|_0^{1-x} \right) + \frac{y^2}{2}\Big|_0^{1-x} \right]$$

$$= 3\left[ x(1-x-0) + \frac{(1-x)^2}{2} - 0 \right]$$

$$= 3\left( x - x^2 + \frac{x^2 - 2x + 1}{2} \right) = \frac{3}{2}\left( 2x - 2x^2 + x^2 - 2x + 1 \right)$$

$$= \frac{3}{2}\left(1 - x^2\right) \quad , \text{ para } \quad 0 < x < 1 .$$