MAE0221

PROBABILIDADE I

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EXERCÍCIO 03

Y v.a. discreta, $\lambda > 0$

$$P(Y=k) = p(k) = \frac{e^{-\lambda}\lambda^k}{k!} \mathbb{1}_{\{0,1,2,\dots\}}(k)$$

Calcule $E\left(\frac{1}{Y+1}\right)$

Seja $g(Y) = \frac{1}{Y+1}$

Então $E\left[\frac{1}{Y+1}\right] = E[g(Y)] = \sum_{y=0}^{\infty} g(y)\,p(y)$

$$= \sum_{y=0}^{\infty} \frac{1}{y+1} e^{-\lambda}\frac{\lambda^y}{y!}$$

$$= e^{-\lambda} \sum_{y=0}^{\infty} \frac{1}{y+1}\frac{\lambda^y}{y!}$$

$$= e^{-\lambda} \sum_{y=0}^{\infty} \frac{\lambda^y}{(y+1)!} = e^{-\lambda} \sum_{y=0}^{\infty} \frac{\lambda\lambda^y}{\lambda(y+1)!}$$

$$= \frac{e^{-\lambda}}{\lambda} \sum_{y=0}^{\infty} \frac{\lambda^{y+1}}{(y+1)!}, \text{ seja } x = y+1$$

$$= \frac{e^{-\lambda}}{\lambda} \sum_{x=1}^{\infty} \frac{\lambda^x}{x!} = \frac{e^{-\lambda}}{\lambda}\left(\sum_{x=0}^{\infty}\frac{\lambda^x}{x!} - \frac{\lambda^0}{0!}\right)$$

$$= \frac{e^{-\lambda}}{\lambda}\left(e^{\lambda} - 1\right) = \frac{1 - e^{-\lambda}}{\lambda}$$