5a. Lista de Exercícios de MAT0206 e MAP0216

1^o. semestre de 2021

1. Decida se cada uma das séries abaixo é convergente. Se possível, calcule sua soma.

   1) $\displaystyle\sum_{n=0}^{\infty}\left(\frac{1}{10^n}+2^n\right)$ 2) $\displaystyle\sum_{k=0}^{\infty}(-1)^k\, t^{\frac{k}{2}}$ para $0<t<1$ 3) $\displaystyle\sum_{n=0}^{\infty}u^n(1+u^n)$ para $|u|<1$

   4) $\displaystyle\sum_{n=0}^{\infty}x^n\cos\left(\frac{n\pi}{2}\right)$ para $|x|<1$ 5) $\displaystyle\sum_{n=0}^{\infty}\sin^{2n}x$ para $|x|<\frac{\pi}{2}$ 6) $\displaystyle\sum_{n=1}^{\infty}\left(\sum_{j=1}^{n}\frac{n}{j}\right)$

   7) $\displaystyle\sum_{n=1}^{\infty}\cos\left(\frac{n\pi}{2}\right)$ 8) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{\sqrt{n+1}+\sqrt{n}}$ 9) $\displaystyle\sum_{k=1}^{\infty}\frac{k}{\sin k}$

   10) $\displaystyle\sum_{s=1}^{\infty}\cos\left(\frac{1}{s}\right)$ 11) $\displaystyle\sum_{k=1}^{\infty}\frac{2+\cos\ k}{k}$

2. É convergente ou divergente? Justifique.

   1) $\displaystyle\sum_{n=3}^{\infty}\frac{1}{\sqrt{n^2-4}}$ 2) $\displaystyle\sum_{n=2}^{\infty}\frac{\text{arctg}\ n}{n^2}$ 3) $\displaystyle\sum_{n=1}^{\infty}\frac{e^{\frac{1}{n}}}{n^2}$ 4) $\displaystyle\sum_{n=1}^{\infty}\frac{2^n}{(n!)^{\lambda}}$, $\lambda>0$

   5) $\displaystyle\sum_{n=1}^{\infty}\frac{(2n)!}{(n!)^2}$ 6) $\displaystyle\sum_{n=2}^{\infty}\frac{\ln n}{n}$ 7) $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n^{\ln n}}$ 8) $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt[3]{n+2}}{\sqrt[4]{n^3+3}\ \sqrt[5]{n^3+5}}$

   9) $\displaystyle\sum_{n=1}^{\infty}\left(1-\cos\frac{1}{n}\right)$ 10) $\displaystyle\sum_{n=2}^{\infty}\frac{1}{(\ln n)^n}$ 11) $\displaystyle\sum_{n=2}^{\infty}\frac{\ln n}{n^2}$ 12) $\displaystyle\sum_{n=2}^{\infty}\frac{\ln n}{n^p}$, $p>0$

   13) $\displaystyle\sum_{n=2}^{\infty}\ln\left(1+\frac{1}{n^p}\right)$, $p>0$ 14) $\displaystyle\sum_{n=2}^{\infty}\sqrt{n}\ \ln\left(\frac{n+1}{n}\right)$ 15) $\displaystyle\sum_{n=1}^{\infty}\frac{n!3^n}{n^n}$ 16) $\displaystyle\sum_{n=1}^{\infty}\frac{n!e^n}{n^n}$

   17) $\displaystyle\sum_{k=1}^{\infty}\frac{e^{\frac{1}{k}}}{k^k}$ 18) $\displaystyle\sum_{n=1}^{\infty}3^n\left(\frac{n}{n+1}\right)^{n^2}$ 19) $\displaystyle\sum_{n=1}^{\infty}\frac{n^3}{(\ln 2)^n}$

3. Decidir se a série converge absolutamente, condicionalmente ou diverge.

   1) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt{n}}$ 2) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^{\frac{3}{2}}}$ 3) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{2n^2+1}{n^3+3}$ 4) $\displaystyle\sum_{n=2}^{\infty}(-1)^n\frac{1}{\ln n}$

   5) $\displaystyle\sum_{n=2}^{\infty}(-1)^n\frac{\ln n}{n}$ 6) $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n}{n(\ln n)^2}$ 7) $\displaystyle\sum_{n=2}^{\infty}(-1)^n\frac{\sqrt{n}}{\ln n}$ 8) $\displaystyle\sum_{n=1}^{\infty}(-1)^{2n+1}\frac{1}{\sqrt{n}}$

   9) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\sin\frac{1}{n^p}$, $p>0$ 10) $\displaystyle\sum_{n=2}^{\infty}\frac{\ln n}{n^2}$ 11) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{\ln n}{\sqrt{n}}$

4. Verifique as relações 1) e 2) abaixo e use-as para calcular as somas 3) - 7):

   1) $\sum_{n=1}^{\infty}[f(n+1)-f(n)]=\lim\limits_{n\to\infty}f(n)-f(1)$, se o limite existir.

   2) $\sum_{n=1}^{\infty}[f(n+1)-f(n-1)]=\lim\limits_{n\to\infty}[f(n)+f(n+1)]-f(0)-f(1)$, se o limite existir.

3) $\sum\limits_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right)$ 4) $\sum\limits_{n=1}^{\infty} (-1)^n \ln\left(\frac{n}{n+2}\right)$ 5) $\sum\limits_{n=1}^{\infty} \left[\sin\left(\frac{1}{n}\right) - \sin\left(\frac{1}{n+1}\right)\right]$

6) $\sum\limits_{n=1}^{\infty} \frac{n}{(n+1)!}$ 7) $\sum\limits_{n=1}^{\infty} \frac{1}{(n+1)\ ...\ (n+k)}$, $(k \geq 2)$

5. Determine os valores de $x \in \mathbb{R}$ para os quais as séries convergem.

   1) $\sum\limits_{n=1}^{\infty} x^n(1+x^n)$ 2) $\sum\limits_{n=1}^{\infty} x^n \cos\left(\frac{n\pi}{2}\right)$ 3) $\sum\limits_{n=2}^{\infty} (-1)^{n+1} \frac{1}{n^{\ln x}}$ 4) $\sum\limits_{n=1}^{\infty} n! x^n$

   5) $\sum\limits_{n=1}^{\infty} \left(x^n + \frac{1}{2^n x^n}\right)$ 6) $\sum\limits_{n=0}^{\infty} (-1)^{n+1} e^{-n \sin x}$ 7) $\sum\limits_{n=0}^{\infty} \frac{2n+1}{(n+1)^5} x^{2n}$

6. Seja (a_n) uma sequência qualquer dos dígitos 0,1,2,...,9. Mostre que a série

$$\frac{a_1}{10} + \frac{a_2}{100} + ... + \frac{a_n}{10^n} + ...$$

   é convergente.

7. Seja (a_n) uma sequência de números positivos. Mostre que $\sum \frac{a_n}{1+a_n}$ converge se e só se $\sum a_n$ converge.

8. Se $\sum\limits_{n=1}^{\infty} a_n = s$, calcule $\sum\limits_{n=1}^{\infty} (a_n + a_{n+1})$.

9. Mostre que, se $\sum (a_n)^2$ converge então $\sum \frac{a_n}{n}$ também converge.

10. Mostre que, se (a_n) é sequência decrescente e $\sum a_n$ converge então $n \cdot a_n \to 0$.

11. Seja (a_n) uma sequência decrescente, com $\lim_{n\to\infty} a_n = 0$. Mostre que a série $\sum a_n$ converge se e somente se $\sum 2^n \cdot a_n$ converge (Teste de Condensação de Cauchy). Use este resultado para discutir a convergência da série $\frac{1}{n^p \ln n}$.