a. $N = 3$, $q = 3$

$(0, 0, 3)$
$(0, 1, 2)$
$(0, 3, 0)$
$(0, 2, 1)$
$(1, 0, 2)$
$(1, 2, 0)$
$(1, 1, 1)$
$(2, 0, 1)$
$(2, 1, 0)$
$(3, 0, 0)$

b. Using the formula, we get

$$\Omega(N, q) = \binom{q + N - 1}{q} = \left(\frac{(q + N - 1)!}{q!(N-1)!}\right)$$

$$\rightarrow \frac{5!}{3!\,2!} = 10$$

c. A: $N_A = 2$ $N_B = 2$

$q_A = 4$ $q_B = 2$

(0, 4)	(0, 2)
(1, 3)	(1, 1)
(2, 2)	(2, 0)
(3, 1)	
(4, 0)	

For each of the elements in A, we need to list each element from B.

A	B	A	B	A	B
(0, 4)	(0, 2)	(0, 4)	(1, 1)	(0, 4)	(2, 0)
(1, 3)	(0, 2)	(1, 3)	(1, 1)	(1, 3)	(2, 0)
(2, 2)	(0, 2)	(2, 2)	(1, 1)	(2, 2)	(2, 0)
(3, 1)	(0, 2)	(3, 1)	(1, 1)	(3, 1)	(2, 0)
(4, 0)	(0, 2)	(4, 0)	(1, 1)	(4, 0)	(2, 0)

d. With total energy $q = 6$, knowing $q_A = 6 - b_b$, q_A can have values from 0, 6. In other words, 7 values.

$q_A = \{0, 1, 2, 3, 4, 5, 6\}$

e. See "problem_e" listed below.

f. Number of total microstates are found by multiplying $\Omega_A \cdot \Omega_B$.
For given values of q_A and q_B, we multiply the multiplicities together.
After, we need to iterate through q_A.
This will give alot more states.

g. See "problem_g" listed below.

h. $\ln \Omega(N, q) = \ln\left(\frac{(q+N-1)!}{q!(N-1)!}\right) \simeq \ln\left(\frac{(q+N)!}{q!\,N!}\right)$

$= \ln(q+N)! - \ln q! - \ln N!$

Apply Stirling's approximation on each term:

$\simeq (q+N)\ln(q+N) - (q+N) - [q\ln q - q] - [N\ln N - N]$

$= (q+N)\ln(q+N) \cancel{-q} \cancel{-N} - q\ln q \cancel{+q} - N\ln N \cancel{+N}$

$= (q+N)\underbrace{\ln(q+N)} - q\ln q - N\ln N$

$$\ln\left(q\left(1+\frac{N}{q}\right)\right) = \ln q + \ln\left(\frac{N}{q}+1\right)$$

$$(\text{Taylor}) = \ln q + \frac{N}{q}$$

$\rightarrow (q+N)\left[\ln q + \frac{N}{q}\right] - q\ln q - N\ln N$

$= \cancel{q\ln q} + N + N\ln q + \frac{N^2}{q} \cancel{-q\ln q} - N\ln N$

$= N\left(1 + \cancel{\frac{N}{q}} + \ln\frac{q}{N}\right) \quad \frac{N}{q} \ll 1$

$= N\left(\ln\frac{q}{N} + 1\right)$

i. $S = k \ln \Omega(N, q)$

$$\ln \Omega(N,q) = N\left(\ln \frac{q}{N} + 1\right)$$
$$= N \ln \frac{q}{N} + N$$

$$\rightarrow \Omega(N,q) = e^{N \ln \frac{q}{N} + N}$$
$$= e^{\ln\left(\frac{q}{N}\right)^N} e^N$$
$$= \left(\frac{q}{N}\right)^N e^N$$
$$= \left(\frac{eq}{N}\right)^N$$

Thus we get

$$S = Nk \ln\left(\frac{eq}{N}\right)$$

j. $T = \left(\frac{\partial S}{\partial U}\right)^{-1}$

$$S = Nk\left[\ln \frac{q}{N} + 1\right] = Nk \ln q - Nk \ln N + Nk$$

$$q = \frac{U}{\epsilon} \rightarrow S = Nk \ln \frac{U}{\epsilon} - Nk \ln N + Nk$$

$$\frac{\partial S}{\partial U} = Nk \frac{1}{U/\epsilon} \cdot \frac{1}{\epsilon} = Nk \frac{\cancel{\epsilon}}{U} \frac{1}{\cancel{\epsilon}} = \frac{Nk}{U} = \frac{1}{T}$$

$$\rightarrow T = \frac{U}{Nk} \rightarrow U = NkT$$

k. A spin has two "sides" and is independent. We can therefore write 2^N.

l. Have that $E = -S\mu B = -(S_+ - S_-)\mu B = -2s\mu B$

m. See "problem_m" listed below.

n. If we just insert N, and S_+ into the formula:

$$\Omega(N, S_+) = \frac{N!}{S_+!\underbrace{(N-S_+)!}_{N_-}} = \frac{N!}{S_+!\,S_-!}$$

If S_+ counts number of spins and N is the number of total spins, $N - S_+$ is the remaining spins, S_-.

0. We have that $2s = S_+ - S_-$ and that $N = S_+ + S_-$.

If we combine the two, we find an expression for S_+ and S_- we can use in the formula we found in the last excersice.

$S_- = N - S_+ \rightarrow 2s = S_+ - (N - S_+)$

$= S_+ - N + S_+$

$2s = 2S_+ - N$

$s = S_+ - N/2$

$\rightarrow S_+ = s + N/2$

$S_+ = N - S_- \rightarrow 2s = (N - S_-) - S_-$

$= N - S_- - S_-$

$2s = N - 2S_-$

$s = N/2 - S_-$

$\rightarrow S_- = N/2 - s$

$$\rightarrow \Omega(N, s) = \frac{N!}{(\frac{N}{2} + s)!\,(\frac{N}{2} - s)!}$$

p. Formula 4.90 in the compendium says that

$$P(N, n) = \frac{N!}{(N/2 - n)!\,(N/2 + n)!} \, 2^{-N}$$

which we know can be written as

$$\Omega(N, s) = C(N) e^{-\frac{s^2}{N/2}}$$

$$= C(N) e^{-\frac{2s^2}{N}}$$

where $C(N)$ is a constant.

q. We can see that the analytic curve fits well. However, I enqountered some dificulities getting the hist()- function work as I wanted. (Probably because I use Oqtave, and not MatLab)

r. We know from previous excercises that

$S = k \ln \Omega$ and that $\Omega(N, S_+) = \frac{N!}{S_+!\,(N-S_+)!}$

If we write

$$S = k \ln \left(\frac{N!}{S_+!\,(N - S_+)!} \right)$$

$$= k[\ln N! - (\ln S_+! + \ln (N - S_+)!]$$

$$= k[\ln N! - \ln S_+! - \ln (N - S_+)!]$$

And apply Stirling's approximation.

$$\simeq k[N \ln N - N - (S_+ \ln S_+ - S_+) - ((N-S_+) \ln(N-S_+) - (N-S_+)))$$

$$= k[N \ln N \cancel{-N} - S_+ \ln S_+ \cancel{+S_+} - (N-S_+) \ln(N-S_+) \cancel{+N} \cancel{-S_+}$$

$$= k[N \ln N - S_+ \ln S_+ - (N-S_+) \ln(N-S_+)$$

5. $$\frac{1}{T} = \frac{\partial S}{\partial U} = \frac{\partial S}{\partial U}\frac{\partial S_+}{\partial S_+} = \frac{\partial S}{\partial S_+}\frac{\partial S_+}{\partial U} = \frac{\partial S}{\partial S_+}\left(\frac{\partial U}{\partial S_+}\right)^{-1}$$

From excercise 1., we have that total energy is:

$$U = -2s\mu B \rightarrow -(2S_+ - N)\mu B$$

Which gives

$$\frac{\partial U}{\partial S_+} = \frac{\partial}{\partial S_+}\left(-2S_+\mu B + N\mu B\right) = -2\mu B$$

$$\frac{\partial S}{\partial S_+} = \frac{\partial}{\partial S_+}\left(k[N \ln N - S_+ \ln S_+ - N \ln(N-S_+) + S_+ \ln(N-S_+)]\right)$$

$$= k\left[-\ln S_+ - S_+\frac{1}{S_+} - N\frac{1}{N-S_+}(-1) + \ln(N-S_+) + S_+\frac{1}{N-S_+}(-1)\right]$$

$$= k\left[-\ln S_+ - 1 + \frac{N}{N-S_+} + \ln(N-S_+) - \frac{S_+}{N-S_+}\right]$$

$$= k[\ln\left(\frac{N-S_+}{S_+}\right)$$

$$\frac{1}{T} = \frac{\partial S}{\partial S_+}\left(\frac{\partial U}{\partial S_+}\right)^{-1} = k \ln\left(\frac{N - S_+}{S_+}\right)\frac{1}{-2\mu B}$$

$$\rightarrow T = -\frac{2\mu B}{k}\frac{1}{\ln\left(\frac{N-S_+}{S_+}\right)}$$

1 Code for problem e)

```
% proglem_e.m


% Setup
q = 6;
N_A = 2;
N_B = 2;



% Setup
P = zeros(q+1,1);
a_mult = zeros(q+1,1);
b_mult = zeros(q+1,1);
ab_mult = zeros(q+1,1);

disp('q_value    A_mult   B_mult   AB_mult P(q_A)')

% Calculate
tot_mult = nchoosek( (q+N_A+N_B-1) ,q);
for i=1:(q+1)
    q_A = q-(i-1);
    a_mult = nchoosek( (q_A+N_A-1) , q_A);
    b_mult = nchoosek( (q-q_A+N_B-1) ,(q-q_A));
    ab_mult = a_mult*b_mult;
    P = a_mult*b_mult/tot_mult;
    disp(sprintf('%i         %g       %g       %g       %g', i-1,
    a_mult, b_mult, ab_mult, P));
end


% Output:
% q_value    A_mult   B_mult   AB_mult P(q_A)
% 0          7        1         7       0.0833333
% 1          6        2        12       0.142857
% 2          5        3        15       0.178571
% 3          4        4        16       0.190476
% 4          3        5        15       0.178571
% 5          2        6        12       0.142857
% 6          1        7         7       0.0833333
```

2 Code for problem g)

```
% problem_g

% Setup
N = 101;

N_A = 30;
N_B = 70;

% Setup
q_a = zeros(1,N);
```

```
P = zeros(1,N);

% Make it so
q_b = 0;
for i = 1:N
    q_a(i) = 100 - q_b;
    P(i) = (nchoosek(q_a(i) + N_A - 1, q_a(i))*nchoosek(q_b + N_B - 1, q_b)) / nchoosek((q_a(i) + q_b) + (N_A + N_B) - 1, q_a(i) + q_b);
    q_b++;
end

%q_a
%P
plot(q_a, P);
xlabel('q_a');
ylabel('P(q_a)')


pause()
```

3 Code for problem m)

```
% problem_m

%
N = 60;
n = 50000;
s = zeros(n, 1);

%
for i = 1:n
    A = randi([0,1],N,1);
    S_plus = sum(A);
    S_minus = N - S_plus;
    s(i) = (S_plus-S_minus)/2;
end

%
mean_s = -sum(s)/n;
n_vec = 1:n;
plot(n_vec, -s);
hold on
plot([0 n], [mean_s mean_s], 'r--')
xlabel('Microstate "')
ylabel('Energy')
legend('Total E in microstate', 'Mean energy')
hold off

%
figure()
hist(-s, 50)
xlabel('Energy of the system')
ylabel('Number of occurences')
hold on
plot_vec = linspace(-20,20,n);
```

```
y = 5150*exp(-2*plot_vec.^2/N);
plot(plot_vec, y, 'r--')
legend('Histogram of results','Normal distribution')


pause()
```