A simple and flexible correlation for predicting the viscosity of crude oils

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A simple and flexible correlation for predicting the viscosity of crude oils

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A B S T R A C T

A simple and flexible correlation has been developed for the prediction of the kinematic viscosity of crude oils based on the Walther equation (1931). The correlation was developed in order to improve on the accuracy of existing correlations and to assist oil spill weathering models, reservoir models and other models where the knowledge of viscosity of uncharacterised crude oils is required. The data used to build the correlation consist of measurements from 137 crude oils from various locations around the world, with 254 viscosity measurements taken at two temperatures, 0 °C and 15 °C, resulting in kinematic viscosity values ranging from 2 cSt to 9,000,000 cSt. The correlation can be used in three different ways depending on the available data. When a single viscosity measurement is available, it can predict the viscosity at a different temperature with an average absolute deviation (AAD) of 13.7% for the studied temperature range. When the specific gravity (SG) and the 50% mass boiling point is available it can give a viscosity estimation with an AAD of 52.9%, which is an improvement in accuracy by a factor of two compared to previously published correlations with the same inputs. Finally, when only the 50% mass boiling point is available, we have improved the accuracy of the Mehrotra (1995) correlation by 10%.

1. Introduction

The viscosity of a crude oil is the property that describes its resistance to movement, mainly due to collisions, electrostatic forces and hydrogen bonding between its molecules that are moving in different velocities and directions (Mehrotra et al., 1996). It is an important property used in reservoir modelling (Orbey and Sandler, 1993), design of production and transportation equipment, as well as in the design of processing facilities in crude oil refineries (Mendes et al., 2005). Additionally, in the case of an accidental oil spill, viscosity plays an important role on the impact and the fate of the oil spill and consequently on the response measures. Modelling the weathering processes of an oil spill requires knowledge of the viscosity of the fluid. As weathering progresses, the composition as well as the physical properties of the fluid, including viscosity, are changing constantly and a method is required to estimate viscosity since initial values are no longer valid. The incentive of this study was to develop an accurate method of estimating crude oil viscosity that would provide the necessary input to such oil spill weathering model. Crude oils are complex mixtures consisting of thousands of components of different type such as aliphatic, naphthenic and aromatic hydrocarbons, as well as other polar compounds containing heteroatoms (nitrogen, sulphur and oxygen). Therefore a purely theoretical prediction of crude oil viscosity is extremely difficult.

There are numerous published predictive models and correlations for the estimation of the liquid (kinematic) viscosity of crude oils and hydrocarbon mixtures. Typically they can be categorised into two groups, theoretical models or empirical correlations (Mehrotra et al., 1996). Theoretical models are usually more accurate but they are more complex and require a better characterisation of the oil including parameters such as critical properties, chemical composition, acentric factor or other similar properties which in many cases are not available. Empirical correlations are typically simpler and only require a few basic parameters such as mid-boiling point temperature and specific gravity of the oil, or alternatively a single viscosity measurement, in order to extrapolate the viscosity to a different temperature. Most of the empirical correlations are based in some form on one of the three equations developed by Vogel (1921), Walther (1931) or Andrade (1934), respectively. Aboul-Seoud and Moharam (1999) have summarised the most popular correlations with their respective accuracies in predicting the kinematic viscosity of unknown oil fractions, as shown in Table 1.

The comparison made by Aboul-Seoud and Moharam (1999) was based on 45 undefined fractions of 13 crude oils from which 316 viscosity measurements were available at different temperatures. It is evident that the correlations based on the Walther equation (1931) are the best performers, based on their lower average absolute deviation (AAD). The Aboul-Seoud and Moharam (1999) and Mehrotra (1995) correlations are

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Table 1
Comparison of frequently used empirical viscosity correlations (Aboul-Seoud and Moharam, 1999).

Correlation	Equation type	Input data	% AAD
Amin and Maddox (1980)	Andrade (1934)	T_b, SG	37.22
Beg et al. (1988)	Andrade (1934)	T_b, SG	7.40
Dutt (1990)	Vogel (1921)	T_b	6.31
Mehrotra (1995)	Walther (1931)	T_b	5.00
Aboul-Seoud and Moharam (1999)	Walther (1931)	T_b, SG	3.05
Puttagunta et al. (1992)	Walther (1931)	ν @ 37.8 °C	1.59
Aboul-Seoud and Moharam (1999)	Walther (1931)	ν	1.47

Where T_b = 50% mass boiling point, which is the temperature where 50% of the mass of a sample has been distilled; SG = specific gravity; ν = kinematic viscosity; AAD = average absolute deviation.

the best when no viscosity measurement is available, while the Aboul-Seoud and Moharam (1999) correlation is the best when a single viscosity measurement is available. The latter correlation is more flexible since it does not require the viscosity measurement to be taken at a specific temperature, whereas the Puttagunta et al. (1992) correlation requires the viscosity to be measured specifically at 37.8 °C.

While these correlation results are apparently accurate with 5% or less AAD, they are only referring to narrow petroleum fractions of 83 °C–125 °C boiling range (Beg et al., 1988). When these correlations were tested during this study against 254 viscosity measurements from 137 crude oils containing a full range of components from different locations around the world their accuracy was significantly lower, at 101% AAD for the Aboul-Seoud and Moharam (1999) correlation, and 218% AAD for the Mehrotra (1995) correlation. The objective of this study was to develop or improve the existing correlations, without increasing the input parameters, so that a better predictive tool is available, especially for whole crude oils. The Walther equation (1931) was selected as the starting point for the new correlation since it provides the most accurate predictions on distillation fractions (Table 1) and other petroleum products such as lubricants, as was shown in a more recent study by Sánchez-Rubio et al. (2006).

2. Data and methodology

2.1. Data

The data used in this study were obtained from the Environment and Climate Change Canada (Emergencies Science and Technology Division) oil properties database (ETC, 2015). The database contains the properties of around 450 oils and petroleum products.

Due to the requirements of this study only the oils with available distillation, density and viscosity data were used. This amounted to a total of 137 crude oils from multiple locations around the world. Only petroleum crude oils and petroleum products were used in this study, which excluded vegetable, synthetic, catalytic and other types of oils available in the database (ETC, 2015). Crude oils with non-Newtonian fluid behaviour were also excluded from this study since their apparent viscosity is dependent on the applied shear rate, and this would have introduced errors in the correlation. Experimental viscosity data for these oils were available at two temperatures, 0 °C and 15 °C. The range of each property of the qualified set of crude oils is summarised in Table 2.

A subset of 12 crude oils with 20 viscosity measurements, covering

Table 2
Summary of the range of crude oil properties.

Ranges of oil properties	
Number of crude oils	137
50% mass T_b range (K)	358–873
Specific Gravity range	0.806–1.024
Kinematic viscosity range (cSt)	2.5–8,800,000

Where 50% mass T_b range is the temperature where 50% of the mass of the oil has been distilled.

the whole viscosity range for two temperatures (0 °C and 15 °C), was excluded from the development of the correlation so that the subset could be used as unknown crude oil samples for the verification of the efficiency of the new correlation. The methodology that was followed in this study is presented below.

2.2. Viscosity

Puttagunta et al. (1992) and Aboul-Seoud and Moharam (1999) have shown that the effect of temperature on viscosity can be correlated accurately by Walther's equation:

$$\ln[\ln(\nu + 0.8)] = a_1 + a_2\ln(T) \tag{1}$$

where ν = predicted kinematic viscosity (cSt), and T = viscosity prediction temperature (K).

When a single viscosity measurement is available at a specific temperature, the viscosity at a different temperature can be estimated by replacing a_1 with the double logarithm of the known viscosity (Aboul-Seoud and Moharam, 1999). In this case equation (1) becomes:

$$\ln[\ln(\nu + 0.8)] = \ln[\ln(\nu_o + 0.8)] + a_2\ln(T/T_o) \tag{2}$$

where ν = predicted kinematic viscosity (cSt), T = temperature of prediction (K), ν_o = measured kinematic viscosity (cSt) and T_o = temperature of the viscosity measurement (K).

Using a complete set of known viscosities, Aboul-Seoud and Moharam (1999) estimated the value of a_2 to be equal to −3.7, predicted with an AAD equal to 1.47%. This factor has also been confirmed in the current study, with a small adjustment to be equal to −3.682 as discussed below in the results section.

The correlations presented in Table 1 that do not require a viscosity measurement instead require the 50% mass boiling point (T_b) as an input. In this study we have tested both the 50% mass boiling point as well as the weighted average T_b of the crude oil, since the latter describes the skewed distribution of boiling points in a crude oil better than the midpoint. The 50% mass boiling point is the temperature where 50% of the mass of the oil has been distilled. The weighted average T_b is determined by the relationship:

$$\text{weighted average } T_b = \sum_{i=1}^{n} \Delta x_i \times T_{bi} \tag{3}$$

where n = the number of distillation cuts covering the complete range of oil, Δx_i = the mass fraction of the distillation cut i with $\Sigma \Delta x_i = 1$, and T_{bi} = mid boiling point temperature of distillation cut i (K).

The calculation of the weighted average requires the complete distillation range to be known, and preferably the full True Boiling Point (TBP) distillation curve. Since these data are rarely available we describe a method in the Appendix for constructing the complete TBP distillation curve from partial distillation data that are usually available. The 50% mass boiling point can be taken directly from the distillation data. In case of heavy oils where the distillation data stop before the 50% mass boiling point, the same technique can be used to construct the complete distillation curve and find the 50% mass boiling point.

2.3. Specific gravity

The next input parameter in the correlation presented in this study is the specific gravity. Again a different approach have been adopted for this input comparing to the previous developed correlations (Aboul-Seoud and Moharam, 1999; Amin and Maddox, 1980; Mehrotra, 1995). Instead of requiring the measurement of the specific gravity at the temperature of the viscosity prediction, the specific gravity at a constant reference temperature is used, specifically at 15 °C for both oil and water. The reference water density at 15 °C has been taken equal to 0.999099 g/cm^3. This approach has been selected because the role of specific gravity

is to characterise the oil in combination with the T_b. So for oils with the same average boiling temperature T_b but different amounts of aliphatic, aromatic and polar compounds, the difference in specific gravity is correlated to this difference in composition. If the specific gravity is changed for the same oil at different temperatures then this correlation fails, and the correlation factor cannot be maintained at a constant value. The effect of the temperature is already described in the correlation by the term $a_2\ln(T)$ of equation (1). As discussed in the results section, density change due to a temperature change has a negative effect on the efficiency of the correlation.

The second deviation from previous correlation methods that is related to the way specific gravity is used is in the addition of a separate exponent to describe the degree of influence of the density. Aboul-Seoud and Moharam (1999) correlation added the density as an additional input in order to improve the accuracy of the previous correlations, but kept it under the same exponent as the 50% mass boiling point T_b. Since T_b and specific gravity are two different measures we expect the degree of influence of specific gravity to be significantly different to that of T_b, so an individual exponent was assigned to each input parameter. Although an extra factor was added to the correlation, the total number of factors was kept the same by eliminating an extra factor that existed in both previous Walther-type correlations (Aboul-Seoud and Moharam, 1999; Mehrotra, 1995).

Another point that makes the new correlation described here significantly different is that it has been fitted directly to a large set of viscosity data from whole crude oils. Both the Aboul-Seoud and Moharam (1999) and Mehrotra (1995) correlations that use Walther equation (1931) have been fit to narrow distillation cuts originating from a small number of crude oils, and although they predict very accurately those cuts, when tested on whole oils the variation is much larger. For this study we had access to a large amount of crude oil viscosity data covering a wide range of viscosities, so the new correlation was developed on the actual crude oils.

2.4. Temperature factor a_2

The temperature factor a_2 was determined by fitting equation (2) to the full set of crude oils with viscosity measurements at two different temperatures, 15 °C and 0 °C (Fig. 1). The best fit for the temperature factor a_2 was calc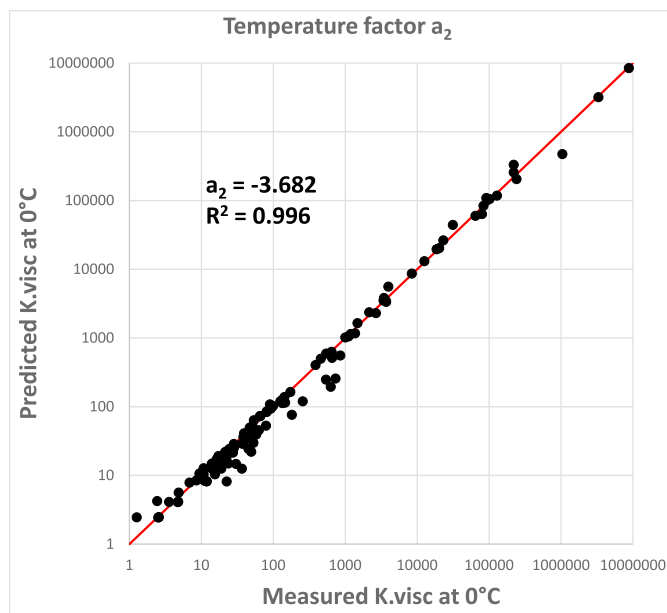ulated by minimising the sum of squared errors with the iterative solver provided in LibreOffice. The temperature factor a_2 was found to be equal to −3.682, with a Pearson's correlation factor (R^2) equal to 0.996.

Finally, complete TBP distillation curves were constructed for the 137 crude oils in the database (ETC, 2015) with available distillation data in order to calculate the weighted average T_b. The Riazi distribution model (1989) (See Appendix) was used to fit the distillation data of all the crude oils and then calculate the complete distillation curve with high degree of accuracy. The average Pearson's R^2 correlation factor was found to be equal to 0.988, and the standard deviation was 0.011 (Table 3).

3. Results and discussion

The Walther equation can be used in three different ways depending on the available input.

3.1. Viscosity as the input

When a single viscosity measurement ν_o is available at temperature T_o, equation (2) can be used with the estimated temperature factor $a_2 = 3.682$ in order to predict the oil viscosity ν at a different temperature T. In this case the viscosity correlation takes the form:

$$\ln[\ln(\nu + 0.8)] = \ln[\ln(\nu_o + 0.8)] + 3.682*\ln(T/T_o) \qquad (4)$$

where ν = predicted kinematic viscosity (cSt), T = temperature of prediction (K), ν_o = measured kinematic viscosity (cSt), and T_o = temperature of the viscosity measurement (K).

We have estimated the error of this method on the full set of crude oils by using the measurement at 0 °C to predict the viscosity at 15 °C and vice versa. When equation (4) is used to estimate the viscosity 15 °C higher, the AAD for the full set of oils was 12.9%, while when it is used to estimate the viscosity 15 °C lower the AAD was 14.7%. The drop of temperature from 15 to 0 °C caused an average viscosity change for the full set of oils equal to 341.3%. These data indicate that equation (4) is capable of predicting the viscosity accurately at any temperature between 0 and 15 °C. The error of prediction between these temperatures is expected to be comparable since interpolating to a closer temperature point will provide a more accurate prediction.

3.2. Specific gravity and 50% mass boiling point as inputs

We examined the effect of having a constant reference temperature for the specific gravity (method 1) in comparison to measuring the specific gravity at the viscosity prediction temperature (method 2). Method 1 provided more accurate results at both temperatures with 5% lower AAD. In addition, method 1 provides more evenly distributed predictions with no tendency to underestimate or overestimate the viscosity. Specifically, at 15 °C the predictions for the full set of oils are on average 2% lower than the measured value. At 0 °C the predicted values are on average 10% lower than the measured value. In contrast, method 2 exhibited a 5% higher AAD, while the predicted values were on average 30% lower at 15 °C and 26% higher at 0 °C. In other words, method 2 tends to always underestimate the viscosity at higher temperatures and overestimate the viscosity at lower temperatures.

Fig. 1. Estimation of the temperature factor a_2 from measured viscosities at two temperatures. The predicted values at 0° were obtained from equation (2) by using the measured viscosity at 15 °C and calculating the value of a_2 that best fits the measured viscosity at 0 °C.

Table 3
Summary of weighted average T_b and 50% mass T_b ranges for the 137 crude oils.

Distillation data details	
Number of crude oils	137
Weighted average T_b range (K)	357–884
50% mass T_b range (K)	358–873
Average distillation curve correlation (R^2)	0.988
Correlation Standard Deviation (SD)	0.0112

Where 50% mass T_b is the temperature where 50% of the mass of the oil has been distilled, and the weighted average T_b range is given by equation (3).

Table 4
Summary of the statistics of the three correlations.

Prediction temperature °C:	15	0	All
Number of Crude Oils:	136	118	254
	% AAD	% AAD	% AAD
Input ν only			
This work	12.9	14.7	13.7
Inputs T_b, SG			
Aboul-Seoud and Moharam (1999)	67.4	141	101
This work	46.6	66.3	52.9
Input T_b only			
Mehrotra (1995)	145	306	218
This work	136	274	199

Where T_b = 50% mass boiling point is the temperature where 50% of the mass of the oil have been distilled; SG = specific gravity; ν = kinematic viscosity; AAD = average absolute deviation.

Next we used the results of the weighted average boiling point (weighted average Tb) as well as the 50% mass boiling point (50% mass Tb) to determine which of the two sets produces the best fit to the 254 viscosity measurements. The hypothesis that the weighted average Tb would better describe the boiling point distribution of the crude oil and hence will provide better viscosity prediction accuracy was not confirmed. The AAD for the weighted average Tb was found equal to 63.8%, in comparison to 52.9% for the 50% mass Tb. It was concluded that parameter a_1 of equation (1) is best described by the combination of 50% mass Tb and constant reference temperature SG.

The final form of Walther's equation (1) that takes as inputs the 50% mass Tb and reference SG at 15 °C is given below by equations (5) and (6).

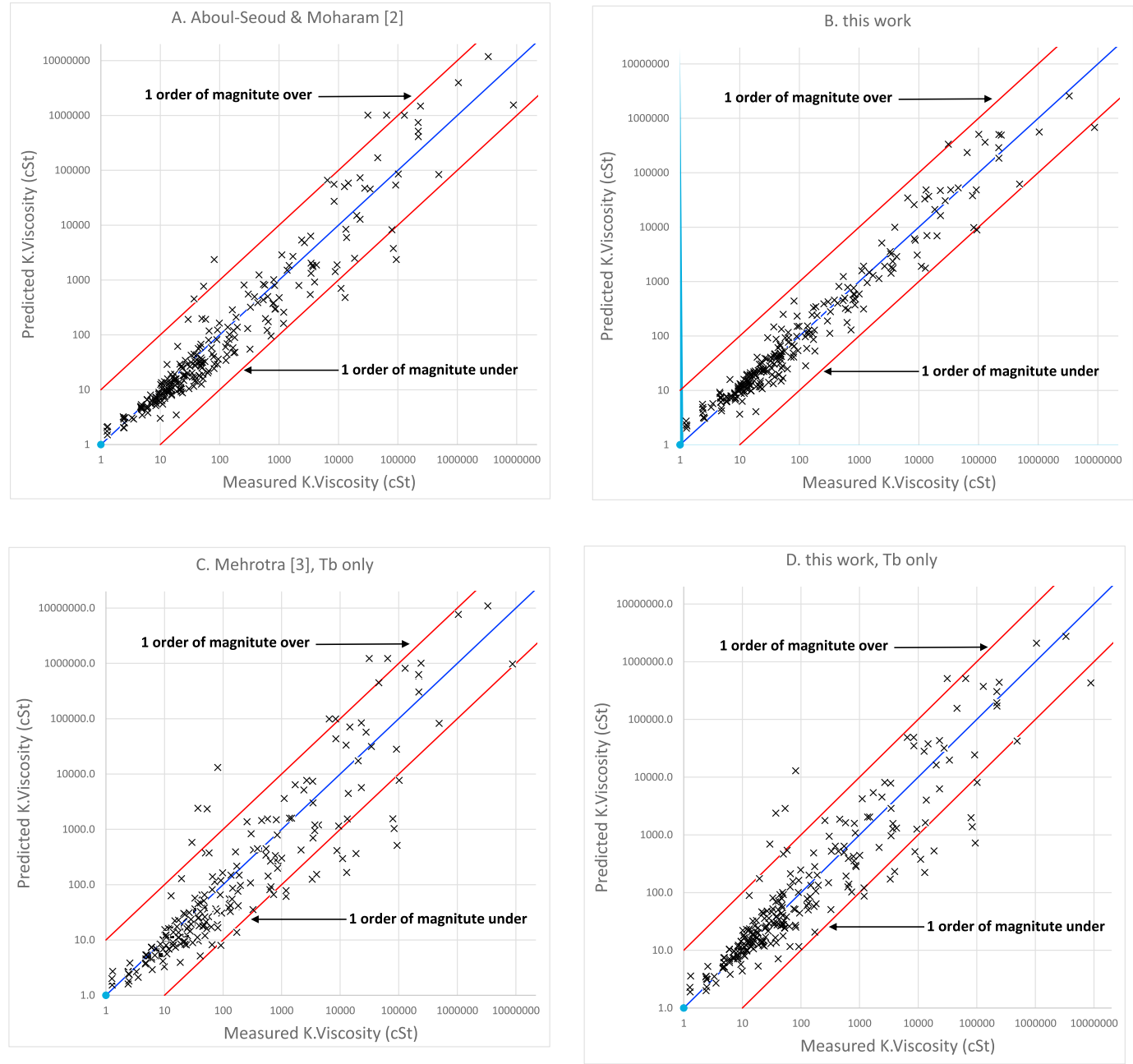The resulting expression for the parameter a_1 becomes:

Fig. 2. Predicted versus measured viscosity, for (A) and (B) correlations with 50% mass T_b and SG inputs, and (C) and (D) correlations with 50% mass Tb only input. The comparison was performed on the complete set of crude oils including viscosity measurements from both 0 and 15 °C. Where (A) Aboul-Seoud and Moharam (1999), (C) Mehrotra (1995), and (B and D) this work.

$$a_1 = 14.69 T_b^{0.0684} SG^{0.267} \tag{5}$$

The complete correlation takes the form:

$$\ln[\ln(v + 0.8)] = 14.69 T_b^{0.0684} SG^{0.267} - 3.682 \ln(T) \tag{6}$$

where v = predicted kinematic viscosity (cSt), T_b = the 50% mass boiling point (K), SG = the specific gravity at 15 °C for both oil and water densities and T = temperature of prediction (K).

When the predictions of equation (6) are compared to those of the Aboul-Seoud and Moharam (1999) correlation that uses the same inputs, we can observe an improvement in accuracy by a factor of 2, with AADs equal to 52.9% and 101% respectively. Although an AAD of 52.9% may seem high for other physical properties of oil, the viscosity of heavier oils can change more than an order of magnitude, or 1000%, in just few degrees Celsius. This correlation thus can give a fast estimate of the viscosity by using readily available data.

3.3. 50% mass boiling point as input

For the case that only the 50% mass T_b is available we tried to improve the fit of Mehrotra (1995) correlation using the complete set of crude oils and succeeded in reducing the AAD by 19% from 218% to 199%, or approximately 10% of the total error as shown on Table 4. In this case Mehrotra (1995) correlation becomes:

$$\ln[\ln(v + 0.8)] = 7.014 T_b^{0.178} - 3.682 \ln(T) \tag{7}$$

where v = predicted kinematic viscosity (cSt), T_b = the 50% mass boiling point (K), and T = temperature of prediction (K).

Fig. 2A and B provide a graphical comparison between the two correlations that require both T_b and specific gravity as inputs. Similarly, Fig. 2C and D are a graphical comparison between the correlations with only T_b as the input. The central blue diagonal line indicates the match between predicted and measured values, and the two red external lines indicate the position of 10 times higher or lower values than the measured one. In both cases the improvement in population density and

distribution around the central line is apparent when comparing the previous correlations (Aboul-Seoud and Moharam, 1999; Mehrotra, 1995) with this work. Especially the comparison of Fig. 2B and D clearly demonstrates the improved accuracy of the prediction when both T_b and specific gravity as used as inputs.

Finally, the predictions of equation (6) are compared with those of the two previous correlations (Aboul-Seoud and Moharam, 1999; Mehrotra, 1995) for a subset of 12 completely unknown crude oils from ETC (2015) with a total of 20 viscosity measurements (Fig. 3). By unknown we mean that these crude oils have not been used during the development of the correlation, and there is no advantage for the correlation of the present study against the other two. The same trend can be observed as for the large set of oils discussed earlier. The predictions of equation (6) are more evenly distributed and closer to the central line, with an AAD of 34.7%, while the Aboul-Seoud and Moharam (1999) and the Mehrotra (1995) correlations have much higher AADs (127% and 214%, respectively). The results for the 12 unknown samples confirm the improvement in accuracy that was observed in the previous large set of crude oils.

4. Conclusions

A new simple correlation for the prediction of crude oil viscosity has been successfully developed. The new correlation is flexible and can be used in three different ways depending on the available data.

When a single viscosity measurement is available, equation (4) can predict the viscosity at a different temperature with an AAD of 13.7% for the range between 0 and 15 °C. This is the most accurate mode of usage and should be preferred if the data is available.

When the specific gravity and the 50% mass boiling point are available, equation (6) provides 4 times better viscosity predictions than the Mehrotra (1995) correlation, and 2 times better viscosity predictions than the Aboul-Seoud and Moharam (1999) correlation, based on 254 viscosity measurements of 137 crude oils from a variety of locations around the world, with viscosities ranging from 2 cSt to 8, 800,000 cSt.

The new correlation has eliminated the need to measure the specific gravity of the crude oil at the temperature of the viscosity prediction, by
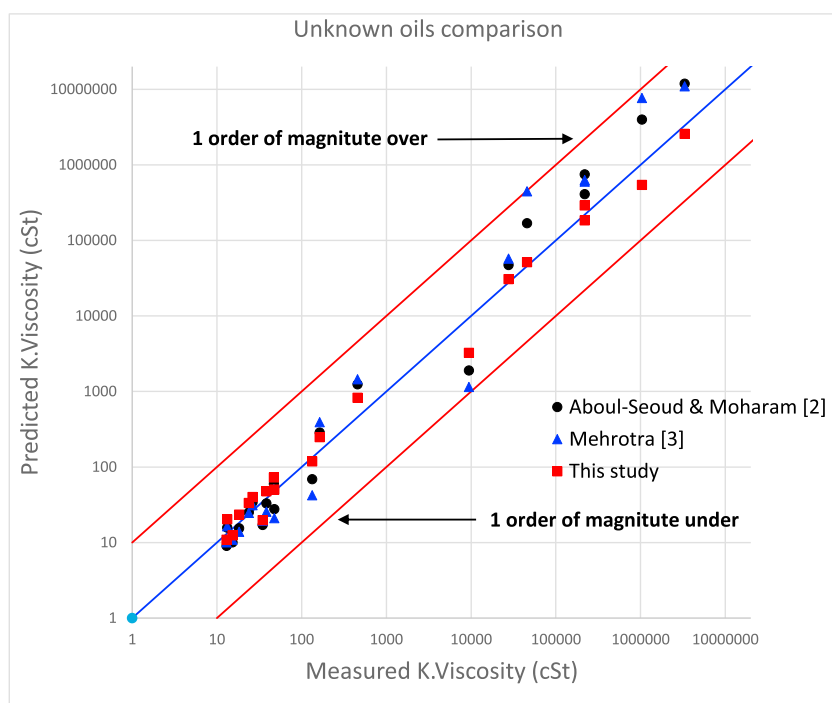

Fig. 3. Comparison of the three correlations on a subset of 12 unknown crude oils. The comparison included a total of 20 viscosity measurements at both 0 and 15 °C. The AADs are equal to 214%, 127% and 34.7% for Mehrotra (1995), Aboul-Seoud and Moharam (1999) and equation (6), respectively.

always using the reference specific gravity of the crude oil at 15 °C that is usually already available.

When only the 50% mass boiling point is available, equation (7) provides 10% better accuracy than that of Mehrotra (1995) while using the same input.

Finally, the present study describes an easy and effective method based on the Riazi distribution model (1989) to estimate the 50% mass boiling point when only incomplete distillation data are available.

APPENDIX

The Riazi distribution model

Riazi (1989, 1997) developed a distribution model for multiple properties of the uncharacterised fractions of oils. He tested the new model on 68 fully characterised oils from various locations and found the AAD for the boiling point prediction equal to 0.56%. Nedelchev et al. (2011) verified the Riazi distribution model for the boiling point properties of 33 crude oils where detailed true boiling point distillation data were available and found an excellent fit to the experimental data, with a correlation factor R^2 over 0.99. The Riazi distribution model is described by equations (A.1) and (A.2):

$$\frac{T_i - T_o}{T_o} = \left(\frac{A}{B} \ ln \left(\frac{1}{1 - x_i} \right) \right)^{1/B} \tag{A.1}$$

The linear form of equation (A.1) is:

$$Y = C_1 + C_2 X \quad \text{where} \quad Y = \ln \left(\frac{T_i - T_o}{T_o} \right) \quad \text{and} \quad X = \ln \ln \left(\frac{1}{1 - x_i} \right) \tag{A.2}$$

Where:

T_o = initial boiling point in Kelvin;
T_i = temperature at which i percent is distilled;
x_i = volume or weight fraction of distillate;
A, B = constants to be determined from experimental data;
C_1 = intercept;
C_2 = slope.

By plotting the terms Y and X from available partial distillation data the slope (C_2) and the intercept (C_1) can be determined.
The constants A and B are calculated from equation (A.3):

$$B = \frac{1}{C_2} \quad A = Be^{C_1 B} \tag{A.3}$$

Then equation (A.1) can be solved to produce the complete distillation curve from which the missing weighted average Tb of the distillation residue can be calculated. An example of the procedure showing the partial distillation data for the Sockeye crude oil, which is available on the Environment and Climate Change Canada oil properties database (ETC, 2015), is given in Table A.1.

Table A.1
Experimental distillation data for the Sockeye crude oil (ETC, 2015).

Temperature °C	wt% distillate
40	0.4
60	0.4
80	1.5
100	3.3
120	5.1
140	7.1
160	9.2
180	11.3
200	13.2
250	18.9
300	24.7
350	30.9
400	36.8
450	43.3
500	50.1
550	57.3
600	63.5
650	69.2

In Fig. A.1 the terms Y and X from equation (A.2) are cross-plotted for the Sockeye crude oil (ETC, 2015), and the slope $C_2 = 0.6061$ and intercept $C_1 = 0.7001$ were established by the least sum of squared errors.
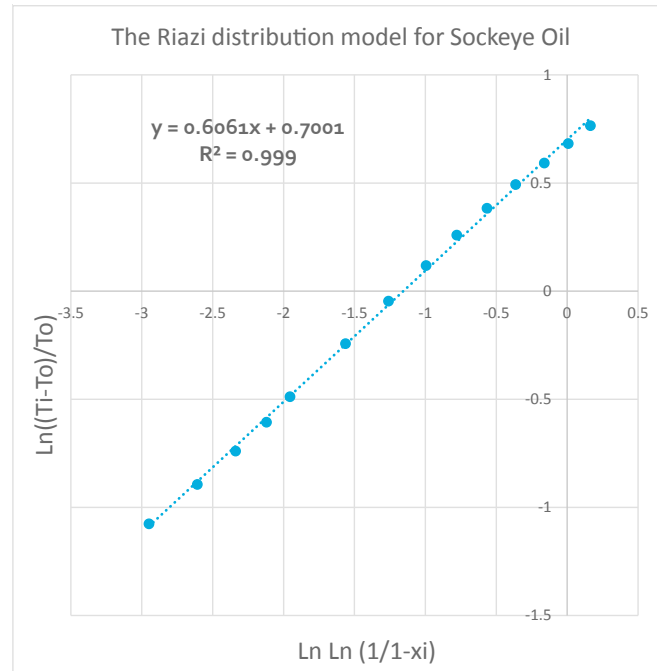

Fig. A.1. Boiling point distribution of Sockeye crude oil, fitted using the Riazi distribution model with a correlation factor of $R^2 = 0.999$.

The complete distillation curve can then be constructed by solving equations (A.3) and (A.1) for the full boiling range of Sockeye crude oil, as shown on Fig. A.2, and the weighted average T_b is determined from the relationship:

$$T_{b\ average} = \sum_{i=1}^{n} \Delta x_i \times T_{bi} \tag{A.4}$$

$$\Delta x_i = x_i - x_{i-1} \tag{A.5}$$

Where:

T_{bi} = mid-point boiling temperature of mass fraction Δx_i. For example, if $\Delta x_{90} = x_{90} - x_{89}$, then T_{bi} is the result of equation (A.1) for $x_{89.5} = 0.895$;
x_i = cumulative mass fraction i;
n = number of mass fractions.

By splitting the whole oil into 100 equal mass fractions of 1% increment, each Δx_i is equal to 1%. In order to calculate the average boiling point of, for example, the Sockeye crude oil distillation residue where the distillation is stopped at 69% mass or x = 0.69, we need to solve for:

$$T_{b\ residue} = \frac{\sum_{i=69}^{100} \Delta x_i \times T_{bi}}{\sum_{i=69}^{100} \Delta x_i} \tag{A.6}$$

Finally the average of the whole crude oil will be given by the equation:

$$T_{b\ average} = x_{dist} T_{b\ dist} + x_{residue} T_{b\ residue}$$

$$e.g. \quad 0.69 T_{b\ dist} + 0.31 T_{b\ residue} \quad \text{for Sockeye crude oil} \tag{A.7}$$

Where:

x_{dist} = the mass fraction of the distillate;
$T_{b\ dist}$ = the average boiling point of the distillate, calculated with equation (A.6) for x_i up to 0.69 directly from the distillation data. Naturally, there will not be 69 Δx fractions, but as many as the distillation data provide.
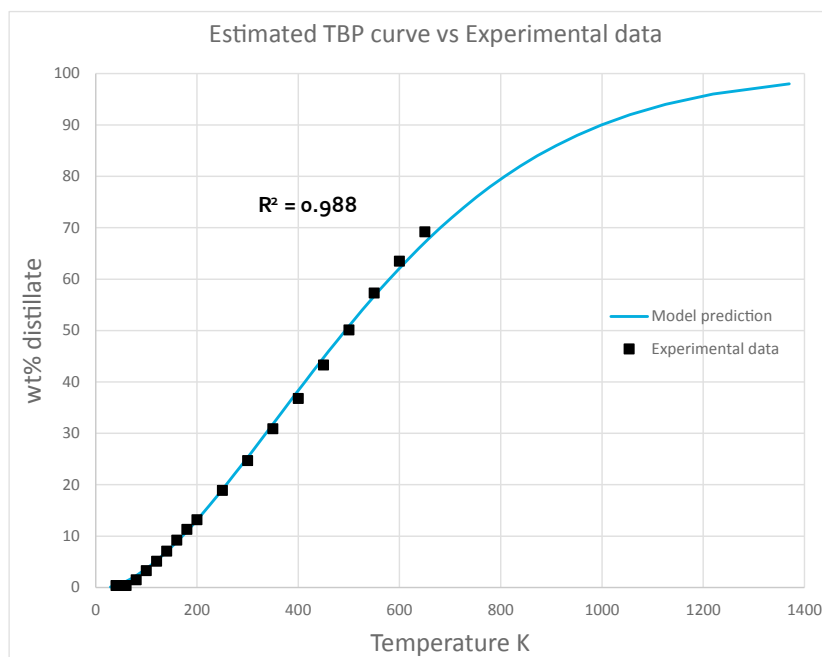
Fig. A.2. The complete TBP distillation curve of Sockeye crude oil, estimated using the Riazi distribution model.

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