Business Process Modeling Report

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Contents

Introduction

This report formalizes and analyzes an internship process within a company, where an intern undergoes a structured workflow involving interactions with both a supervisor and a project leader.

The process begins with an initial interview, followed by the intern's assignment to a work-group. The project leader then evaluates the intern's competencies and assigns suitable tasks with deadlines. Upon task completion, the intern submits a report, which undergoes evaluation. This iterative process continues until the internship period concludes, at which point the supervisor makes a final assessment and determines whether to offer a contract.

A process variant is also introduced, allowing the supervisor to extend the internship instead of terminating it. This modification introduces a feedback loop, increasing workflow complexity and requiring a more sophisticated synchronization and formalization.

This report provides a formalization of the internship process through Business Process Model Notation (BPMN) diagrams and Petri nets, focusing on:

- The **BPMN model** of the standard and extended internship process.
- The **Workflow Nets** for each role: intern, supervisor, and project leader.
- The **Workflow Systems** integrating the roles into a single model.
- A structural and behavioral analysis to verify key correctness properties, including soundness, liveness, and boundedness.

Through these representations, we assess the correctness, efficiency, and adaptability of the internship process. The use of Petri nets enables a rigorous evaluation of workflow properties, ensuring a formally verified and optimized process design.

Soundness of a Workflow Net. A Workflow net is *sound* if it satisfies all of the following conditions:

1. **No dead task:**
$$\forall t \in T.\ \exists M \in [i\rangle.\ M \xrightarrow{t}$$
In other words, every transition can be executed at least once in some reachable marking starting from the initial marking.

2. **Option to complete:**
$$\forall M \in [i\rangle.\ \exists M' \in [M\rangle \text{ such that } M'(o) \geq 1,$$
meaning that at any point in the computation, there is a reachable marking placing at least one token in the final place o, ensuring the process can eventually terminate.

3. **Proper completion:**
$$\forall M \in [i\rangle.\ M(o) > 0 \implies M = o$$
which states that if a reachable marking puts a token in the final place o, then that marking must consist solely of one token in o and no tokens elsewhere, reflecting a clean termination of the process.

These properties ensure that every transition is potentially fireable, the net always has the possibility to reach its final place, and the final marking never leaves residual tokens in other places.

1 BPMN Collaboration Model

The **BPMN model** encapsulates the **Internship Process** by defining three key participants as distinct pools: the **Supervisor**, the **Intern**, and the **Project Leader**. Task dependencies and **message flows** are used to coordinate their interactions, ensuring that responsibilities and deadlines are clearly assigned and managed.

The workflow commences in the **Supervisor** pool with an *Interview User Task* aimed at evaluating the intern's profile. Upon concluding the interview, the supervisor assigns the intern to a work group. This action triggers a *Send Task*, generating a *Message Flow* that activates the intern's pool via a **Message Start Event**.

Once activated, the **Intern** initiates contact with the **Project Leader** through another *Send Task*, resulting in a **Message Flow** to the project leader's pool. A corresponding **Message Start Event** then marks the onset of the project leader's process.

Next, the Project Leader evaluates the intern's competence. This step is modeled with a loop, allowing iterative proposals of different activities:

- If the intern has the necessary skills, the proposed activity is assigned.
- If the intern lacks the required skills, a new activity is proposed until a suitable match is found.

Having determined the most appropriate activity, the project leader uses a *Send Task* to transfer the details to the intern, followed by a second *Send Task* to communicate the completion deadline. The intern acknowledges both tasks as *Receive Tasks*, and a **Signal Intermediate Throw Event** in the project leader's pool synchronizes with a corresponding **Signal Intermediate Catch Event** in the intern's pool to handle the deadline.

Upon completing the assigned activity, the intern sends a **report** back to the project leader via a *Send Task*, which the project leader receives through a **Message Flow**. The project leader then **evaluates** the report and decides how to proceed.

At this juncture, the project leader checks whether the internship is nearing its final phase:

- If sufficient time remains, a new or ongoing activity is assigned via a **Message Flow**, and the intern continues accordingly.
- If the internship is almost over, the project leader issues a *Send Task* informing the intern of the impending conclusion.

When the internship concludes, the project leader sends a **final evaluation** to the **Supervisor** via a **Message Flow**. The supervisor, upon receiving this evaluation through an **Intermediate Message Catch Event**, performs a final assessment:

- If the intern's performance is satisfactory, the supervisor presents a **job offer**.
- If the intern does not meet the required standards, the process ends without an offer.

Overall, this **BPMN collaboration model** delineates a well-structured sequence of interactions. Each participant's *Send* and *Receive Tasks* are linked by **message flows**, and key milestones (deadlines, final evaluations) are managed through **intermediate events**. Figure 1 illustrates the complete workflow, highlighting the sequential nature of the internship process and the communication patterns among the three pools.

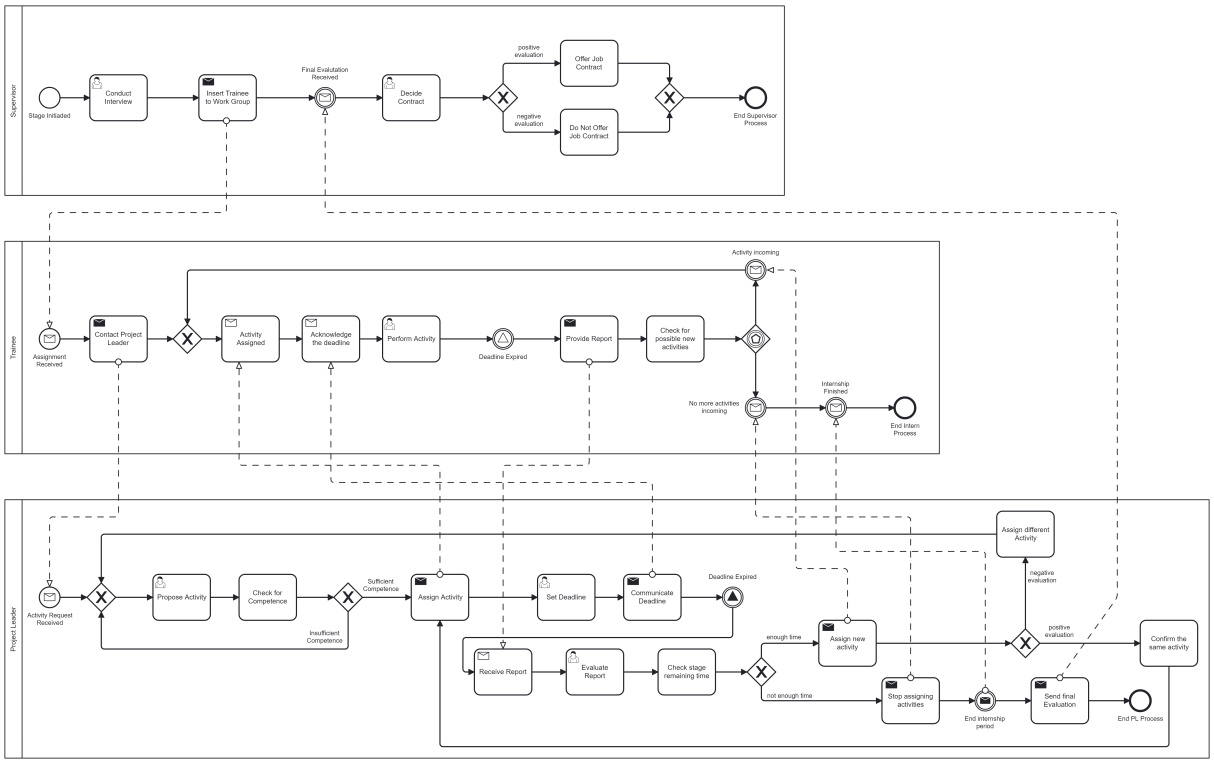


Figure 1: BPMN Diagram of the Internship Process. This diagram represents the structured workflow involving the intern, supervisor, and project leader. The process follows a sequential flow where the intern is assigned tasks, performs activities, and undergoes evaluation until the end of the internship period.

1.1 Variant

This BPMN variant extends the standard internship process by adding the possibility of extending the internship period. The change is introduced in the **Supervisor's** pool, where an *Exclusive Gateway (XOR)* decides whether to prolong or conclude the internship after the intern's final evaluation.

If the internship is extended, a coordinated sequence of **message flows** ensures that all roles remain synchronized. The key steps are as follows:

- The **Supervisor** determines whether to grant the extension based on the intern's final evaluation.
- Upon granting the extension, the supervisor sends a message to both the **Intern** and the **Project Leader** to announce the decision.
- Once notified, each role performs a *Check If Stage Has Been Extended* task to acknowledge the new arrangement.
- Each role then sends a confirmation message back to the supervisor via a **Message End Event**; the supervisor waits for both confirmations using a *Receive Task*.
- Only when these confirmations are received does the supervisor resume the process, and the intern is reassigned to a new work group, officially beginning the extended internship period.

This mechanism ensures that no participant continues the process without collectively confirming the extension. From a verification standpoint, requiring each role's explicit

acknowledgment helps preserve the **soundness** of the workflow and avoids inconsistencies or race conditions once transformed into Petri nets.

As illustrated in Figure 2, this modification integrates a feedback loop, providing additional control over the process while seamlessly building on the original internship workflow.

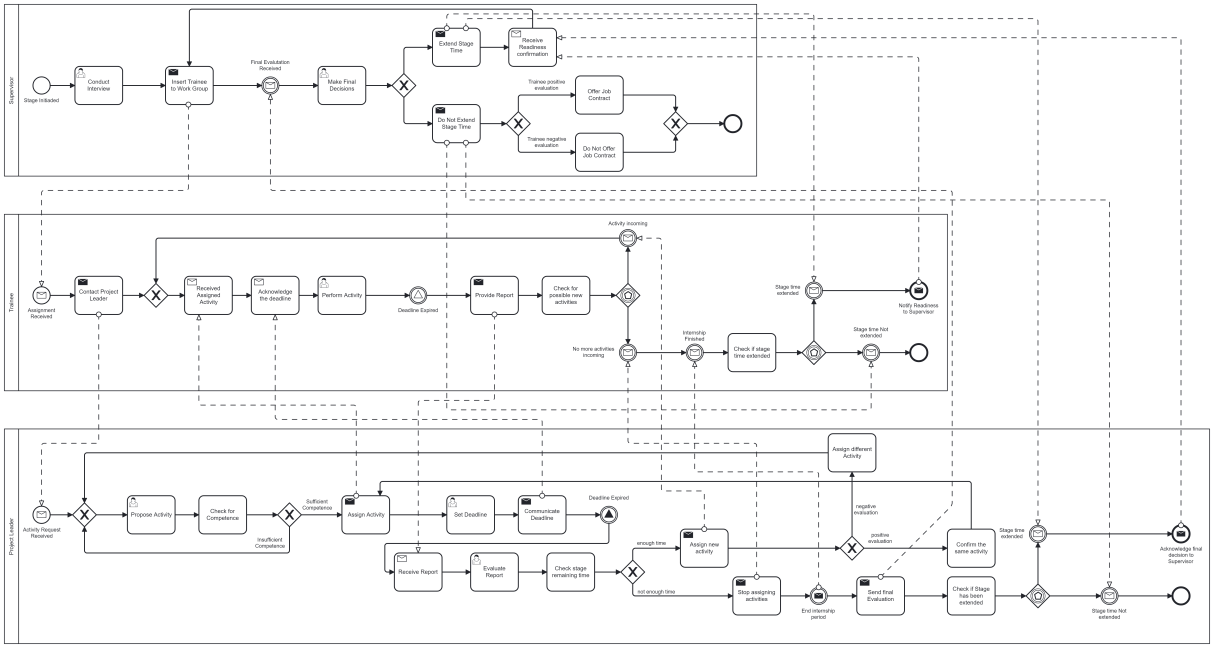


Figure 2: BPMN Diagram of the Internship Process (Variant). This variant extends the standard internship process by introducing an internship extension mechanism. If the extension is approved, the intern is reassigned to a new work group, and the process continues accordingly. This modification introduces a feedback loop, increasing the complexity of the workflow while ensuring formal synchronization.

2 Workflow Nets

2.1 Supervisor

The Petri net representing the process of the **Supervisor** is illustrated in Figure 3a (a) for the base model and in Figure 3a (b) for the variant. The primary structural and behavioral properties of these models are detailed below.

- The base model comprises **6 places**, **6 transitions**, and a total of **12 arcs**. In the variant, these values increase to **9 places**, **10 transitions**, and **20 arcs** due to the introduction of the internship extension mechanism.
- Both models adhere to the definition of **Workflow Nets**, ensuring a structured process with a unique start and end place.
- The nets satisfy **boundedness** and **liveness**, ensuring that execution does not encounter deadlocks and that every transition remains fireable.
- The models belong to the **S-system** class, meaning each transition has exactly one input and one output place. This ensures the presence of a well-defined **S-invariant**, contributing to process consistency.
- Neither of the models belongs to the **T-system** class, as multiple places exhibit more than one transition in their preset or postset.
- Both models uphold **free-choice** properties, ensuring a controlled and structured execution of transitions where preset places are either shared or disjoint.
- The networks exhibit **structural correctness**, evidenced by the absence of **PT-handles** and **TP-handles**.
- The models are **deadlock-free** and **strongly connected**, guaranteeing that every reachable marking can be accessed from any other.
- Both workflow nets are **S-coverable**, with the base model comprising **12 S-components** and the variant containing **19 S-components**, reflecting the increase in structural complexity.
- Since both models are **bounded**, their reachability graph is finite and coincides with the coverability graph, enabling a thorough verification of process execution.

As demonstrated in Figure 4a, the qualitative analysis confirms that both models satisfy the essential correctness properties, validating the reliability and consistency of the workflow execution.

2.1.1 Supervisor Coverability Graph

Base Model. Figure 3c shows the coverability graph for the *Supervisor* Net in its **base** configuration. We observe the following properties:

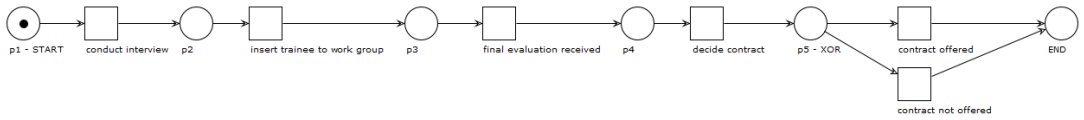
- **Number of vertices:** `6`, **Number of edges:** `5`
- **Presence of cycles:** No. It is strictly acyclic.

Since the net is bounded and free of deadlocks, its reachability graph is finite and coincides with the coverability graph, this applies also for the variant net. Exactly one terminal state exists, demonstrating *soundness*. So we can confirm that the net is *bounded, live* and *sound*.

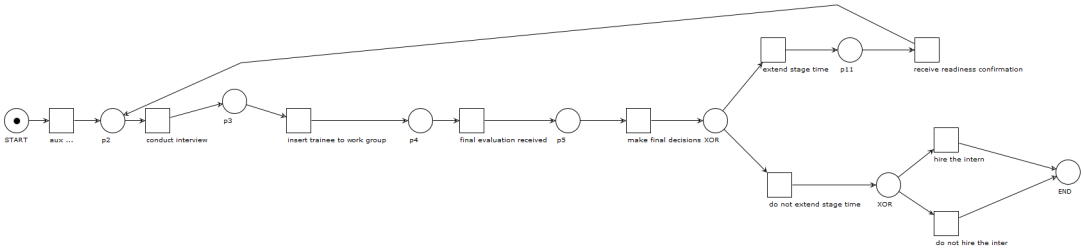
Variant Figure 3d shows the coverability graph for the *Supervisor* Net in its **extended** form. Key metrics include:

- **Number of vertices (states):** `9`, **Number of arcs:** `9`
- **Presence of cycles:** Yes, there is a cycle structure in the net, but it is not an unbounded or endless cycle. Consequently it does not violate *soundness*.

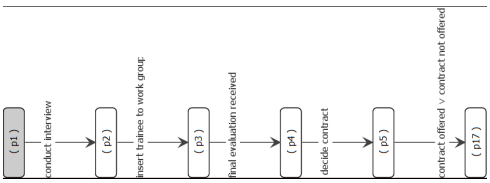
Despite the increased complexity, a single terminal marking still exists, meaning that the variant remains *sound*. It is of course also *bounded* and *live*.



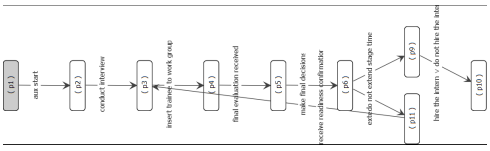
(a) Workflow Net of the Supervisor (Base Model).



(b) Workflow Net of the Supervisor (Variant).

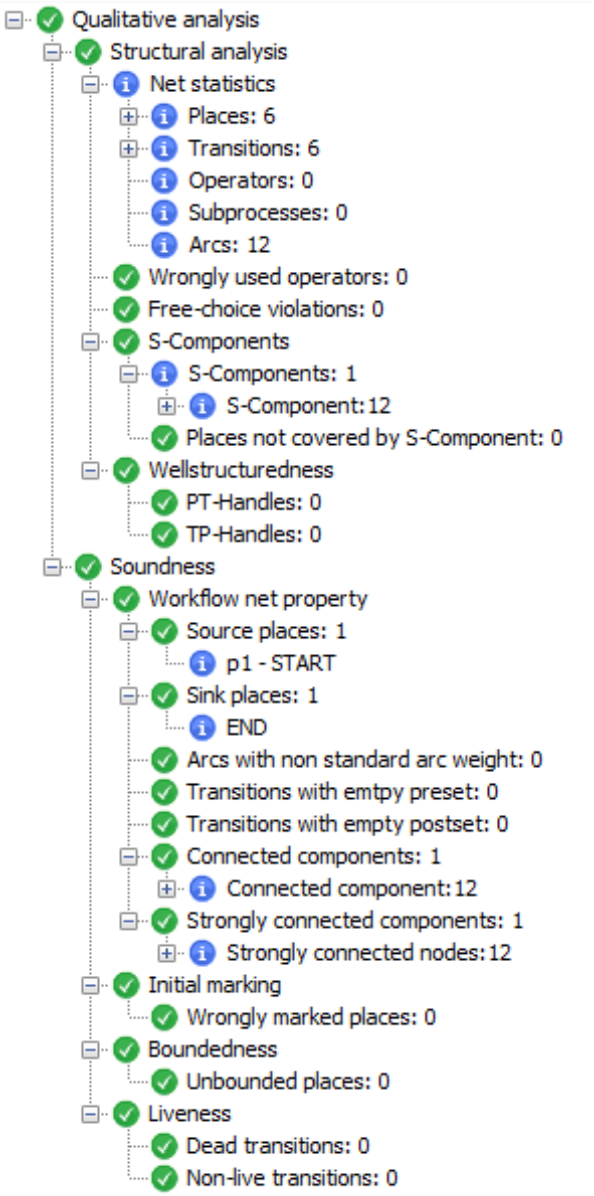


(c) Coverability Graph of the Supervisor (Base Model).

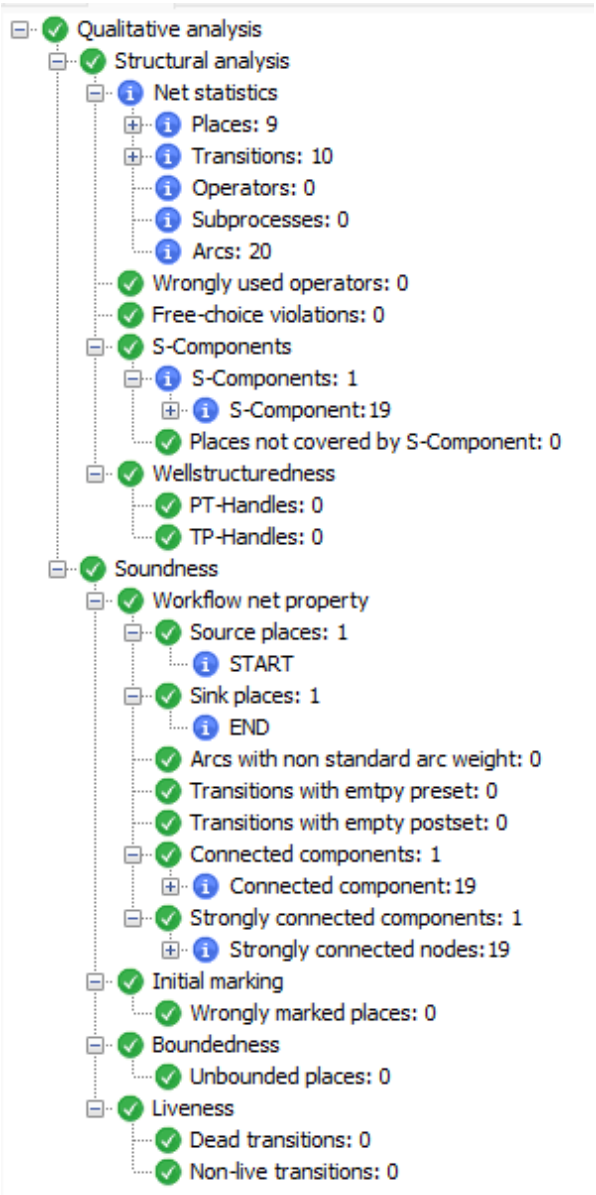


(d) Coverability Graph of the Supervisor (Variant).

Figure 3: Comparison of the Workflow Nets and Coverability Graphs for the Supervisor. The base model (a) represents the standard process, while the variant (b) introduces an extension mechanism to prolong the internship period. Subfigures (c) and (d) show the corresponding coverability graphs.



(a) Qualitative analysis of the Supervisor's Workflow Net (Base Model).



(b) Qualitative analysis of the Supervisor's Workflow Net (Variant).

Figure 4: Comparison of the qualitative analysis for the Supervisor's Workflow Net. The base model (a) maintains standard workflow properties, while the variant (b) integrates an internship extension mechanism, increasing process complexity.

2.2 Intern

The Petri net representing the process of the **Intern** is shown in Figure 5a (a) for the base model and in Figure 5a (b) for the variant. Below, the main structural and behavioral properties of both models are analyzed.

- The base model consists of **11 places**, **11 transitions**, and **22 arcs**. The variant extends the model with **14 places**, **15 transitions**, and **30 arcs** to incorporate the internship extension mechanism.
- Both models are **workflow nets**, ensuring a structured execution with a well-defined start and end.
- The nets satisfy the properties of **liveness** and **boundedness**, ensuring that the execution does not reach deadlock and that every transition remains potentially fireable.
- The nets are classified as **S-systems**, meaning that each transition has exactly one input and one output place, allowing for the identification of an **S-invariant**, which ensures process consistency.
- The nets do not belong to the **T-system** class, as multiple places have more than one transition in their preset or postset.
- Both models maintain the **free-choice property**, meaning that transitions sharing preset places do so in a controlled manner, ensuring a structured decision-making process.
- Both workflow nets are **well-structured**, as they do not contain TP-handles or PT-handles.
- The models are **deadlock-free** and **strongly connected**, ensuring full reachability between all states.
- Both nets are **S-coverable**, meaning that a single **S-component** includes all places in their respective networks. The base model has **S-Component: 22**, while the variant increases this number to **S-Component: 29**, reflecting its higher complexity.
- The models are **bounded**, with no unbounded places, ensuring a finite reachability graph that coincides with the coverability graph as we can see in Figure 5c.

As shown in Figure 6a, the qualitative analysis confirms that both models satisfy all required correctness properties. The base model remains structurally sound and behaviorally correct, while the variant still **maintains** these properties despite its increased complexity.

2.2.1 Intern Coverability Graph

Base Model. Figure 5c depicts the base coverability graph for the *Intern* Net. The following quantitative attributes characterize it:

- **Number of vertices:** `11`, **Number of arcs:** `11`

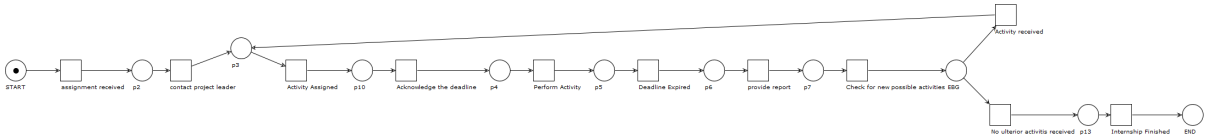
- **Presence of cycles:** Yes. There is a loop centered on receiving an activity, performing it, and reporting back. Although the net contains a cycle, it does not produce an infinite loop because the final marking is always reachable.

The Net is *bounded* and *live* and we can find exactly one terminal node, which corresponds to the intern completing all tasks and reaching the end of the internship. Hence, the base net is confirmed to be also*sound*.

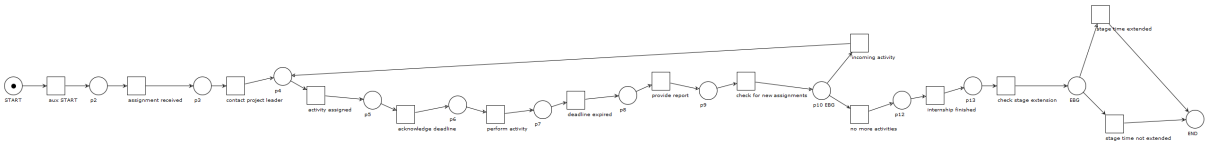
Variant Figure 5d shows the extended coverability graph for the *Intern* Net when the *Supervisor* may decide to prolong the internship. We note:

- **Number of vertices (states):** `14`, **Number of arcs:** `14`
- **Presence of cycles:** The loops from the base model persist (repeated tasks), now augmented by a conditional branch that checks if the internship is extended. These additions do not create unbounded cycles because the net design forces a final choice between termination or continuing with newly assigned tasks.

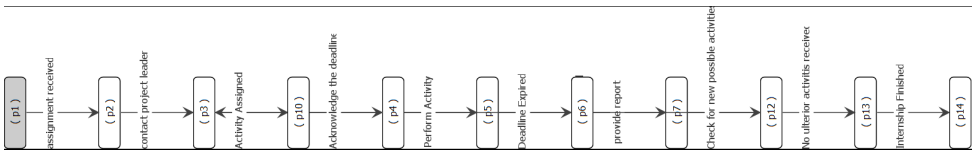
As for the base net, also the variant one is *bounded*, *live* and *sound*.



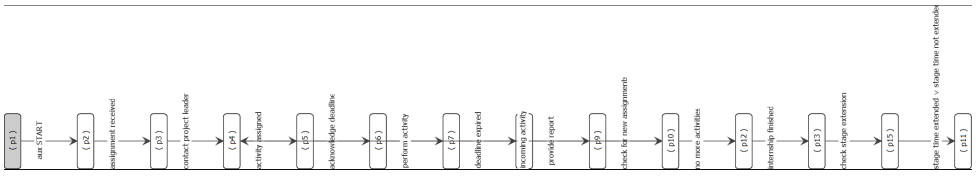
(a) Workflow Net of the Intern (Base Model).



(b) Workflow Net of the Intern (Variant).

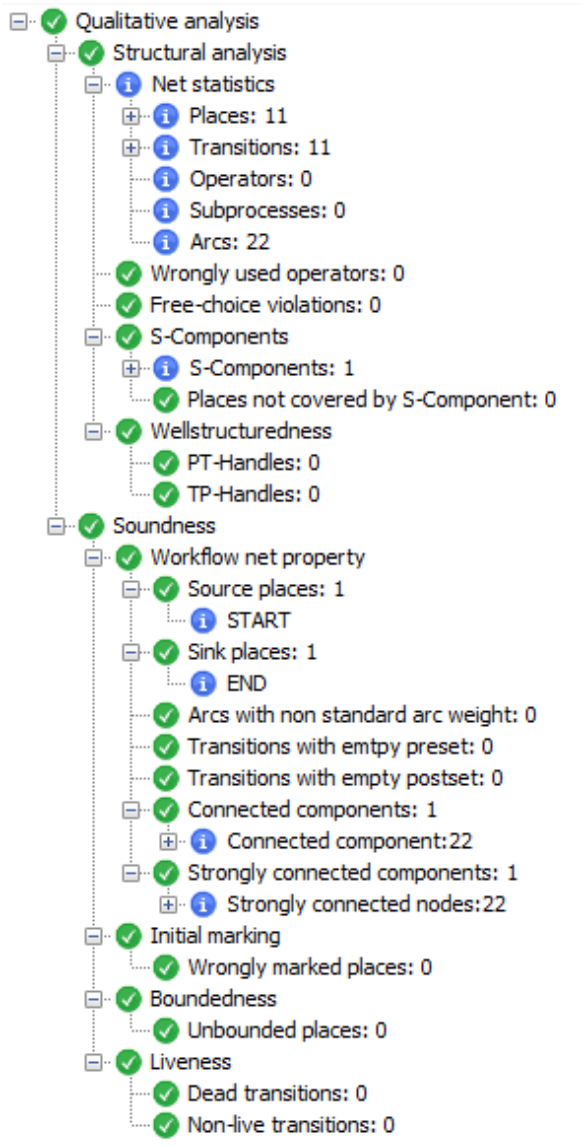


(c) Coverability Graph of the Intern (Base Model).

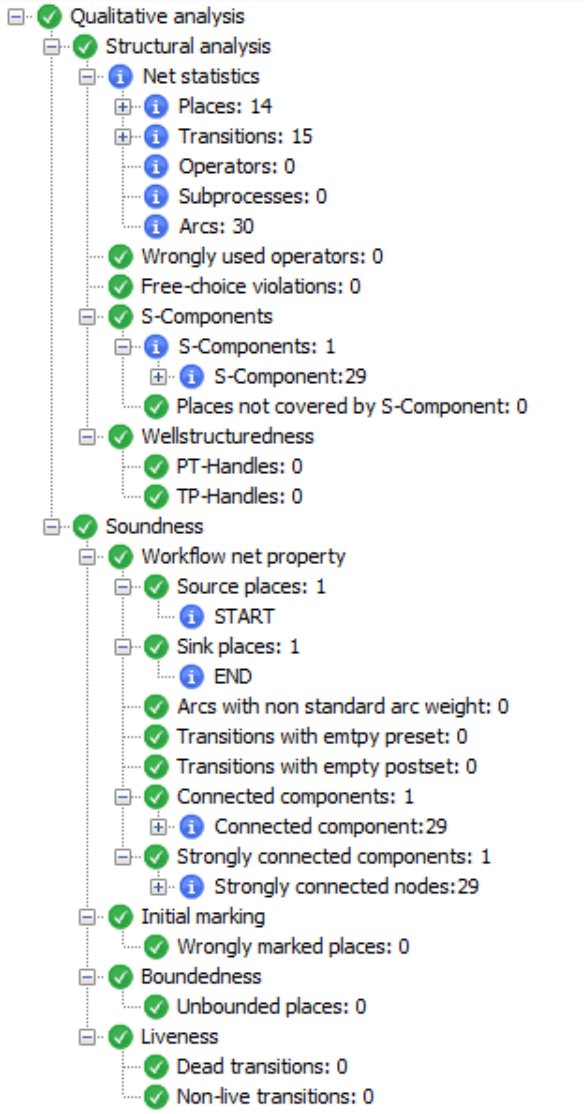


(d) Coverability Graph of the Intern (Variant).

Figure 5: The base model (a) represents the standard internship process, while the variant (b) introduces the possibility of extending the internship period. Sub figures (c) and (d) show the corresponding coverability graphs.



(a) Qualitative analysis of the Intern's Workflow Net (Base Model).



(b) Qualitative analysis of the Intern's Workflow Net (Variant).

Figure 6: Comparison of the qualitative analysis for the Intern's Workflow Net. The base model (a) follows the standard process, while the variant (b) integrates the internship extension mechanism.

2.3 Project Leader

The Petri net representing the process of the **Project Leader** is depicted in Figure 7a (a) for the base model and in Figure 7a (b) for the variant. The following key structural properties characterize these models:

- The base model consists of **20 places**, **22 transitions** and **44 arcs**. In the variant, the complexity increases, leading to **24 places**, **28 transitions**, and **56 arcs**.
- Both models are **Workflow Nets**, ensuring a structured execution with a unique start and end place.
- The nets satisfy **boundedness** and **liveness** properties, meaning that the system does not reach deadlocks and that every transition remains potentially fireable.
- Both models belong to the **S-system** class, as they exhibit a single **S-component** covering all places, ensuring global structural consistency.
- The nets are not **T-systems**, as some places have multiple transitions in their preset or postset.
- Both models satisfy the **free-choice** property, ensuring that every shared preset place corresponds to a structured decision-making mechanism.
- The networks exhibit **well-structuredness**, with no **PT-handles** or **TP-handles**, confirming an absence of structural irregularities.
- Both models are **deadlock-free** and **strongly connected**, ensuring that any reachable marking in the system allows progress.
- The nets are **S-coverable**, with the base model containing **42 strongly connected nodes**, increasing to **52** in the variant, further reinforcing process consistency.
- Given that both models are **bounded**, their reachability graph is finite and aligns with the coverability graph.

The qualitative analysis, as illustrated in Figure 8a, validates that both models uphold all critical correctness criteria, thereby guaranteeing a **sound**, **live**, and **well-defined** workflow execution.

2.3.1 Project Leader Coverability Graph

Base Model. In Figure 7c, the coverability graph of the *Project Leader* exhibits more branching than that of the supervisor or intern, due to multiple cycles handling ongoing tasks, deadlines, and evaluations. Numerically:

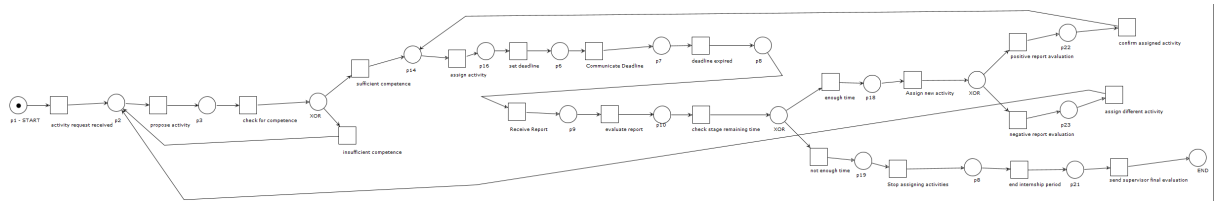
- **Number of vertices:** `20`, **Number of edges:** `22`
- **Presence of cycles:** Yes, there are 3 cycles. The most prominent cycles revolve around the project leader iteratively requesting updates or assigning new tasks. Nevertheless, none of these cycles are infinite because each iteration depends on the net's internal state and external messages, ultimately converging on the final evaluation.

Only one terminal state is found, corresponding to delivering the final report to the Supervisor and exiting the net. No deadlocks or alternative terminal markings are detected, consistent with soundness. The net is then *bounded*, *live* and *sound*.

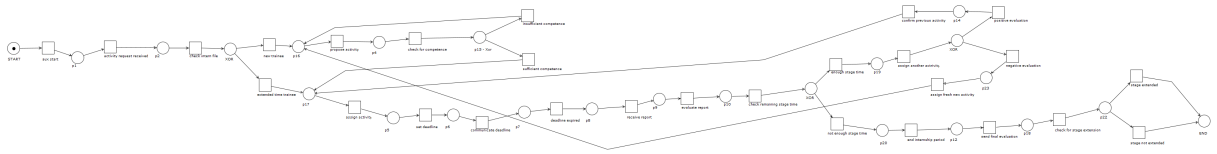
Variant. Finally, in Figure 7d, the extended net's coverability graph accommodates additional transitions for the *internship extension* request and acknowledgment. Key figures include:

- **Number of vertices:** 24, **Number of edges:** 27
- **Presence of cycles:** Yes. 3 cycles arising from repeatedly proposing, assigning, and evaluating activities.

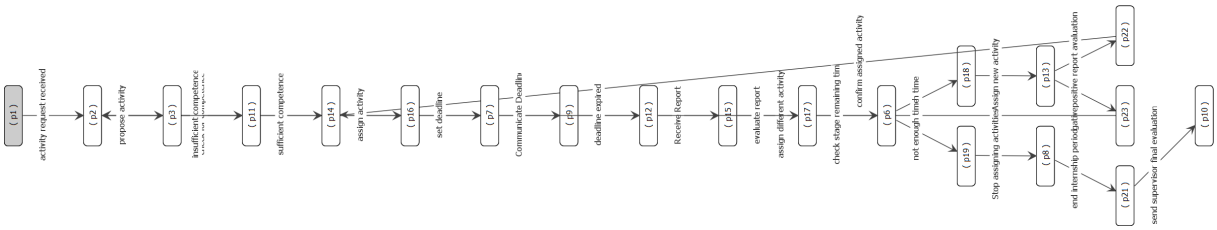
As with the base version, only one global exit condition exists, demonstrating that the net is still *sound*, *bounded*, and *live*, with no unreachable marked places or permanent deadlocks.



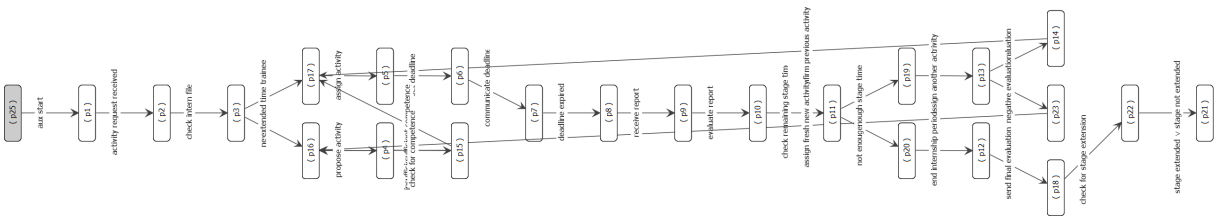
(a) Workflow Net of the Project Leader (Base Model).



(b) Workflow Net of the Project Leader (Variant).

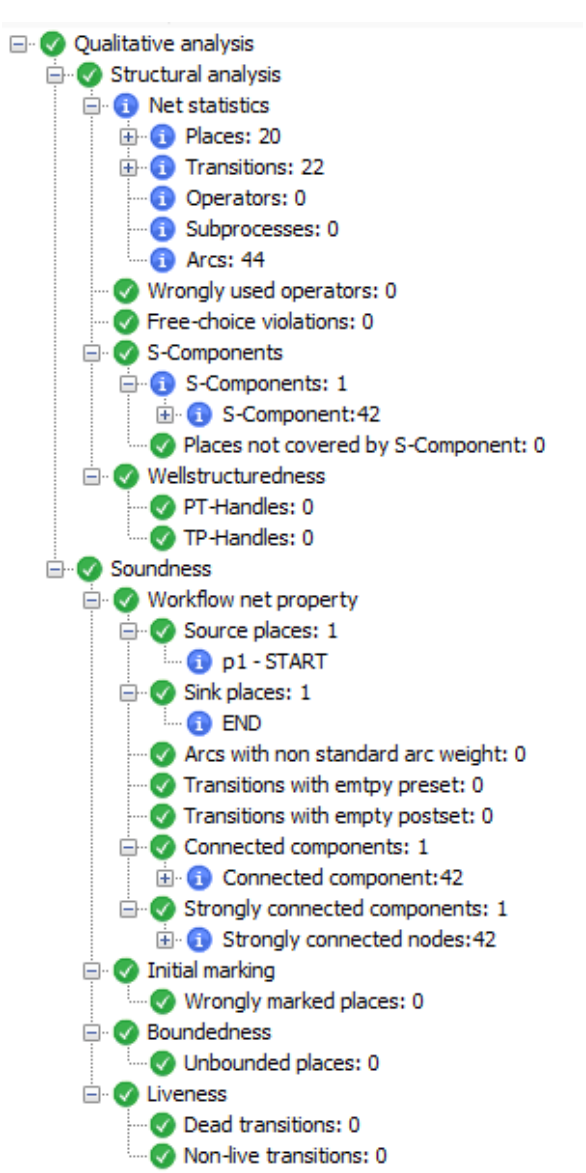


(c) Coverability Graph of the Project Leader (Base Model).

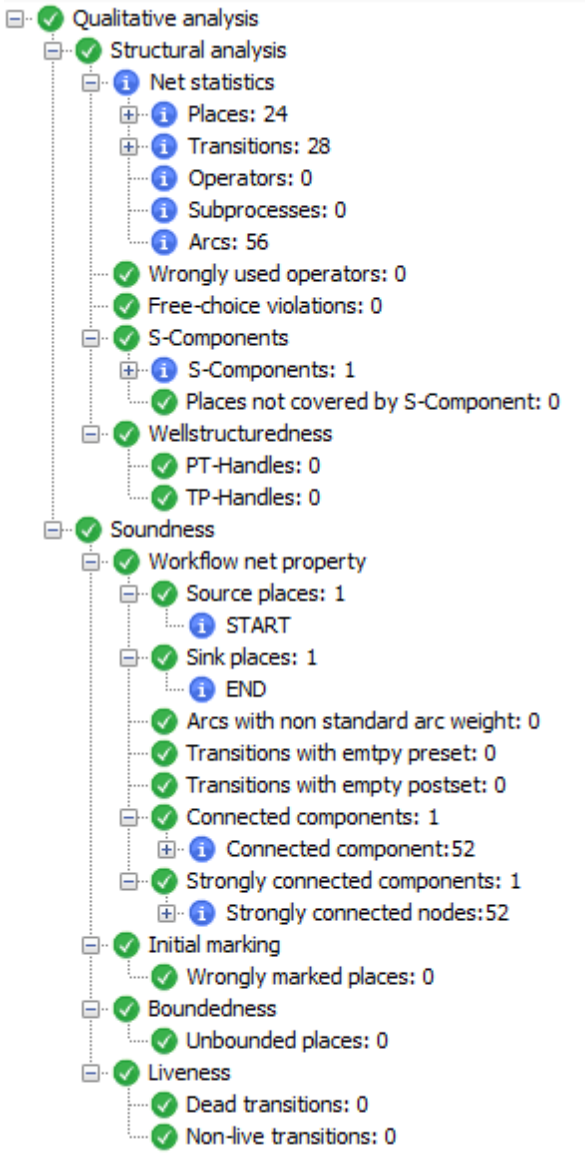


(d) Coverability Graph of the Project Leader (Variant).

Figure 7: Comparison of the Workflow Nets and Coverability Graphs for the Project Leader. The base model (a) represents the standard process, while the variant (b) introduces the possibility of extending the internship period. Subfigures (c) and (d) show the corresponding coverability graphs.



(a) Qualitative analysis of the Project Leader's Workflow Net (Base Model).



(b) Qualitative analysis of the Project Leader's Workflow Net (Variant).

Figure 8: Comparison of the qualitative analysis for the Project Leader's Workflow Net. The base model (a) represents the standard process, while the variant (b) introduces additional mechanisms for internship extension.

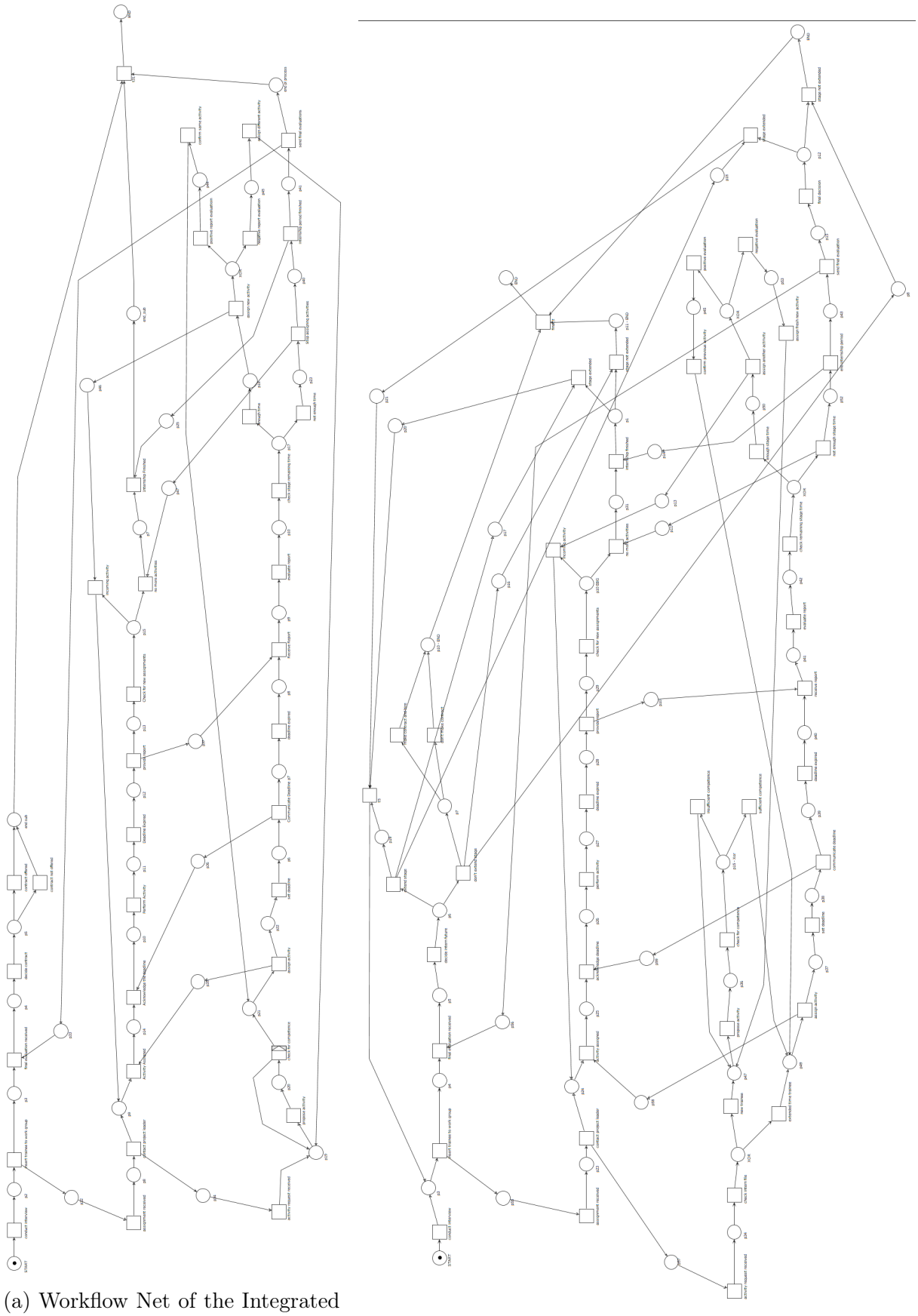
2.4 Workflow System

The Workflow System integrates the individual Workflow Nets of the **Supervisor**, **Intern**, and **Project Leader** into a unified Petri net representation. This model captures the interdependencies and synchronization points between roles, ensuring a well-coordinated execution of the internship process. Figures 9a (a) and (b) illustrate the base model and its variant.

The integration is achieved by introducing additional places and arcs that model inter-process communication. The structural properties of the system can be summarized as follows:

- The **base model** consists of **44 places**, **39 transitions**, and **96 arcs**. In the **variant**, the complexity increases, leading to **57 places**, **50 transitions**, and **128 arcs** due to the introduction of the internship extension mechanism.
- Both models conform to the definition of **Workflow Nets**, ensuring structured execution with a unique start and end place.
- The nets satisfy the properties of **boundedness** and **liveness**, preventing deadlocks and guaranteeing that each transition remains fireable under appropriate conditions.
- Neither model qualifies as an **S-system** or a **T-system**, as multiple places have more than one transition in their preset or postset.
- The workflow system does **not satisfy the free-choice property**, as some transitions share input places without strict separation, creating interdependencies that affect execution. A true *free-choice net* requires that no two transitions share the same input place unless they do so exclusively (i.e., the place does not lead to any other transitions). For this reason, both of our Workflow Systems do not exhibit the free-choice property.
- The networks are **not well-structured**, as evidenced by the presence of **16 PT-handles and 15 TP-handles** in the base model, increasing to **40 PT-handles and 24 TP-handles** in the variant, highlighting the additional complexity introduced by the internship extension mechanism.
- Both models are **S-coverable**, meaning every place belongs to at least one S-component. The base model contains **18 S-components**, while the variant increases this number to **58**, reflecting the added structural elements necessary for managing the extended internship.
- The Workflow System is **strongly connected** when extended with the net N^*, which introduces a reset transition from the final place back to the initial place. This ensures that every node remains reachable from any other node, preserving full connectivity.
- Since both models are **bounded**, their reachability graph is finite and coincides with the coverability graph.

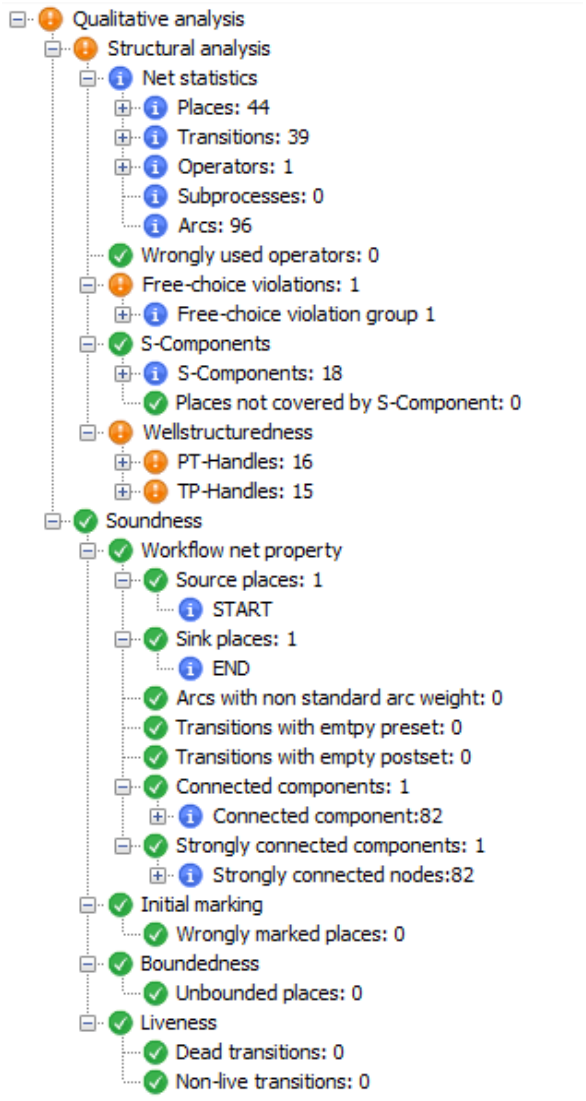
The qualitative analysis, shown in Figure 10a, confirms that both models satisfy key correctness properties, ensuring a **sound** and reliable workflow execution. Despite the added complexity of the variant, the fundamental guarantees—**boundedness, liveness, and strong connectivity**—remain intact.



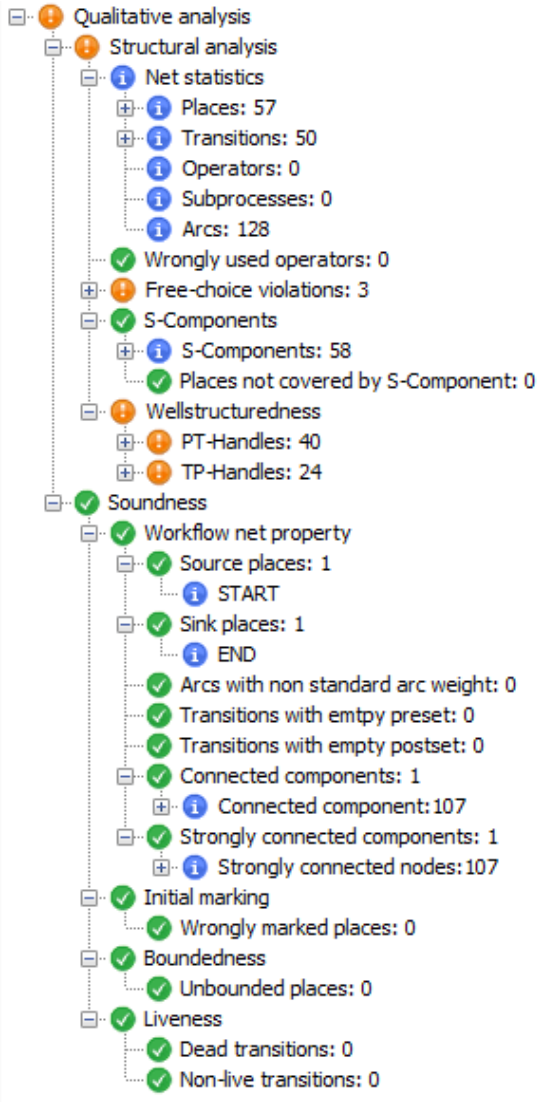
(a) Workflow Net of the Integrated System (Base Model).

(b) Workflow Net of the Integrated System (Variant).

Figure 9: Comparison of the Workflow System. The base model (a) represents the standard process flow, while the variant (b) integrates the internship extension mechanism.



(a) Qualitative analysis of the Workflow System (Base Model).



(b) Qualitative analysis of the Workflow System (Variant).

Figure 10: Comparison of the qualitative analysis for the Workflow System. The base model (a) follows the standard process, while the variant (b) integrates additional synchronization mechanisms for internship extension.