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Politecnico di Milano
Faculty of Automation and Control Engineering


# Harbour crane analysis

Yearwork


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# Contents

# Harbour crane analysis

Figure 2 shows the model of the harbour crane depicted in Figure 1.

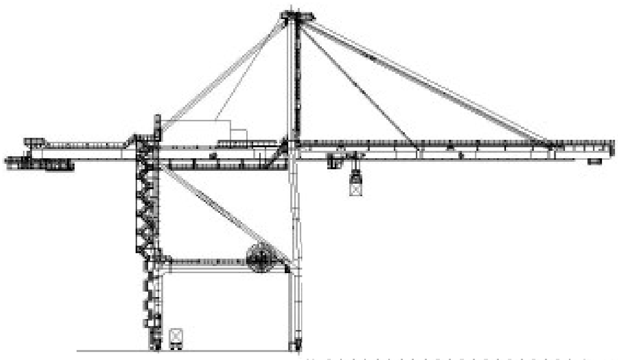


Figure 1: harbour crane

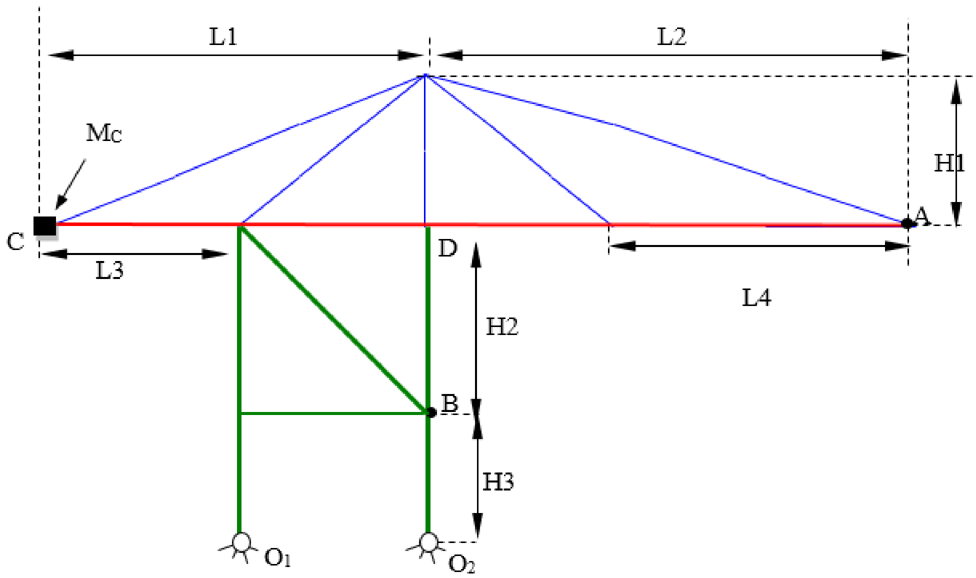



**Figure 2: harbour crane model**

Structural damping is to be accounted for according to the proportional damping assumption, $[C]=\alpha[M]+\beta[K]$, with values $\alpha$ and $\beta$ assigned in the input data table.

<u>The students are asked to:</u>

1) Define a finite element model valid in the frequency range 0÷10 Hz

2) Compute the system's natural frequencies and related modes of vibration in the frequency range 0÷10 Hz

3) Compute the following frequency response functions (FRF) in the frequency range 0 ÷ 10 Hz with step 0.01 Hz:

   a. Input: vertical force at point A; output: vertical displacement of point A;
   b. Input: vertical force at point A; output: horizontal displacement of point B;
   c. Input: vertical force at point A; output: vertical component of the constraint force in the hinge $O_2$;
   d. Input: vertical force at point C; output: vertical component of the constraint force in the hinge $O_2$.

4) Compute the frequency response functions (FRF) in the frequency range 0 ÷ 10 Hz, step 0.01 Hz for a horizontal distributed load (magnitude 1 N/m) applied on the right leg of the crane between points B and D. Outputs:
   a. horizontal displacement of point A;
   b. vertical displacement of point A;
   c. vertical displacement of point C.
5) Assume a winch is placed at A and moves up and down a mass $M_A$ according to a periodic time history $y_{rel}(t)$:

$$y_{rel}(t) = \sum_{i=1,3} A_i \cos\left( i\frac{2\pi}{T}t + \varphi_i \right)$$

   which represents the periodic vertical relative displacement of the load with respect to point A (positive if upwards directed). Compute the corresponding time history of the absolute vertical displacement of point A.
6) Modify the structure in order to reduce at least by 50 % the maximum value of the constraint force computed at point 3, without increasing the total mass of the crane more than 5 % of its original value. Any change of the material and/or of the constraints (by adding or moving them) is not allowed. In case of change of one or more beams sections, both the inertial and the stiffness parameters m, EA, EJ have to be computed taking into account a real geometry of the new sections and the physical properties of steel material.

## Input data:

| | | |
|---|---|---|
| Main horizontal red beam: linear mass (kg/m) | $m_1$ | 312 |
| Main horizontal red beam: axial stiffness (N) | $EA_1$ | 8.2 E9 |
| Main horizontal red beam: bending stiffness ($Nm^2$) | $EJ_1$ | 1.40 E9 |
| Vertical, diagonal and secondary horizontal green beams: linear mass (kg/m) | $m_2$ | 200 |
| Vertical, diagonal and secondary horizontal green beams: axial stiffness (N) | $EA_2$ | 5.4 E9 |
| Vertical, diagonal and secondary horizontal green beams: bending stiffness ($Nm^2$) | $EJ_2$ | 4.5 E8 |
| Blue beams: linear mass (kg/m) | $m_3$ | 90.0 |
| Blue beams: axial stiffness (N) | $EA_3$ | 2.4E9 |
| Blue beams: bending stiffness ($Nm^2$) | $EJ_3$ | 2.0 E8 |
| Point mass in C (kg) | $M_C$ | 2000 |
| Moment of inertia of the point mass in C ($kgm^2$) | $J_C$ | 320 |
| $L_1$ length (m) | $L_1$ | 30 |
| $L_2$ length (m) | $L_2$ | 39 |
| $L_3$ length (m) | $L_3$ | 15 |
| $L_4$ length (m) | $L_4$ | 24 |
| $H_1$ height (m) | $H_1$ | 12 |
| $H_2$ height (m) | $H_2$ | 15 |
| $H_3$ height (m) | $H_3$ | 10 |
| Moving load $M_A$ (kg) | $M_A$ | 800 |
| Point 5, amplitude of the first harmonic (m) | $A_1$ | 0.25 |
| Point 5, amplitude of the second harmonic (m) | $A_2$ | 0.25 |
| Point 5, amplitude of the third harmonic (m) | $A_3$ | 0.15 |
| Point 5, phase of the first harmonic (rad) | $\varphi_1$ | 0 |
| Point 5, phase of the second harmonic (rad) | $\varphi_2$ | $\pi$ |
| Point 5, phase of the third harmonic (rad) | $\varphi_3$ | $\pi$ |
| Point 5: period (s) | T | 1.2 |
| Coefficient for the structural damping evaluation | $\alpha$ | 0.1 |
| Coefficient for the structural damping evaluation | $\beta$ | 2.0e-4 |

# Development

This section aims to the study and resolution of the all assigned questions.

## 2.1 Question 1

Generally, the subdivision method of the system into finite elements (FEM) uses every two neighboring nodal section (o node) to define a beam. But also, this is not the only rule to take into consideration. In the case of dynamics load, in order to get a good approximation of the displacement of the system, each finite element has to work in the QUASI-STATIC field, which means at frequencies well below their first proper resonance.

$$\omega_{max} << \omega_k = \left(\frac{\pi}{L_k}\right)^2 \sqrt{\frac{EJ_k}{m_k}} \tag{2.1}$$

, where $\omega_k$ is the lowest natural frequency of the k-th element and $\omega_{max}$ is the maximum circular frequency, i.e. frequency range of the model.
This implies that the maximum length of k-th beam type $Lk_{max}$ is defined as

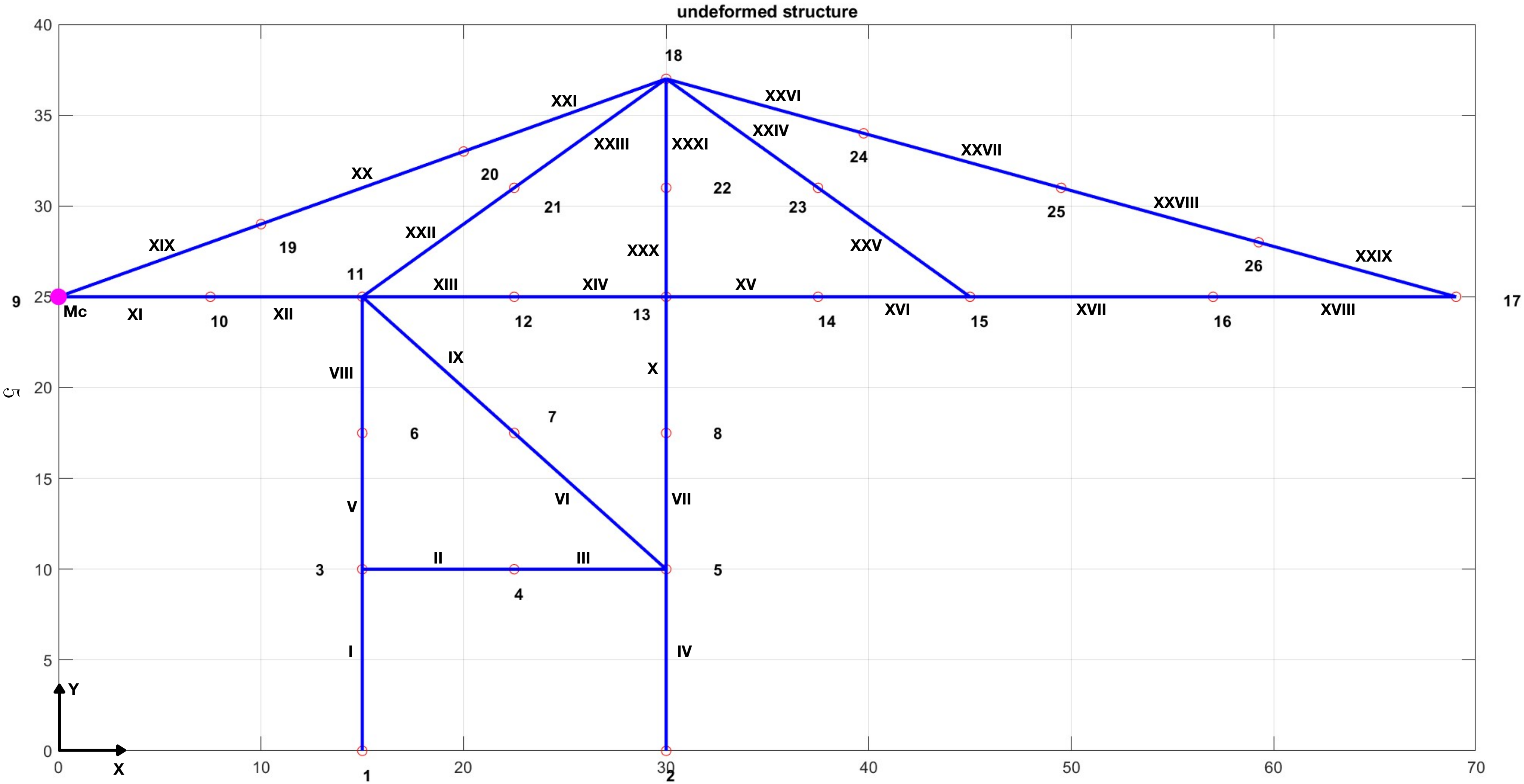
$$Lk_{max} = \frac{\pi \left(\frac{EJ_k}{m_k}\right)^{\frac{1}{4}}}{\sqrt{\omega_k}} \rightarrow L_k < Lk_{max} \tag{2.2}$$

, where $\omega_k = sc * (2\pi\omega_{max})$ is in radiant, and the safe coefficient $sc := 2$. In the specific case provided by the assignment, the structure is build through three different types of beams (RED, GREEN and BLUE). The following script made the computation of $Lk_{max}$ and the chose of $L_k$ dimension faster for each beam.

```
clear all
close all
clc

% Beam's type linear mass and bending stiffness
% Red beams
m1 = 312;
EJ1 = 1.40e9;
% Green beams
m2 = 200;
```

```
EJ2 = 4.5e8;
% Blue beams
m3 = 90;
EJ3 = 2.0e8;

% Lk_max computing
sc = 2; % safe coefficient
omegaMax = 10;
omegaMax_rad = 2*pi*omegaMax;

Lk_max_red = computeLkMax(sc,omegaMax_rad,EJ1,m1)
Lk_max_green = computeLkMax(sc,omegaMax_rad,EJ2,m2)
Lk_max_blue = computeLkMax(sc,omegaMax_rad,EJ3,m3)

% Lk_max computing function
function Lk_max = computeLkMax(sc,omegaMax_rad,EJk,mk)
Lk_max = (pi*(EJk/mk)^(1/4))/sqrt(sc*omegaMax_rad);
end
```

The obtained parameters $Lk\_max\_red$, $Lk\_max\_green$ and $Lk\_max\_blue$ were used for the meshing operation (FEM subdivision)of the harbour crane undeformed structure. Next, the results were used to define the requested .inp file, loaded in the $DMB\_FEM2$ program.

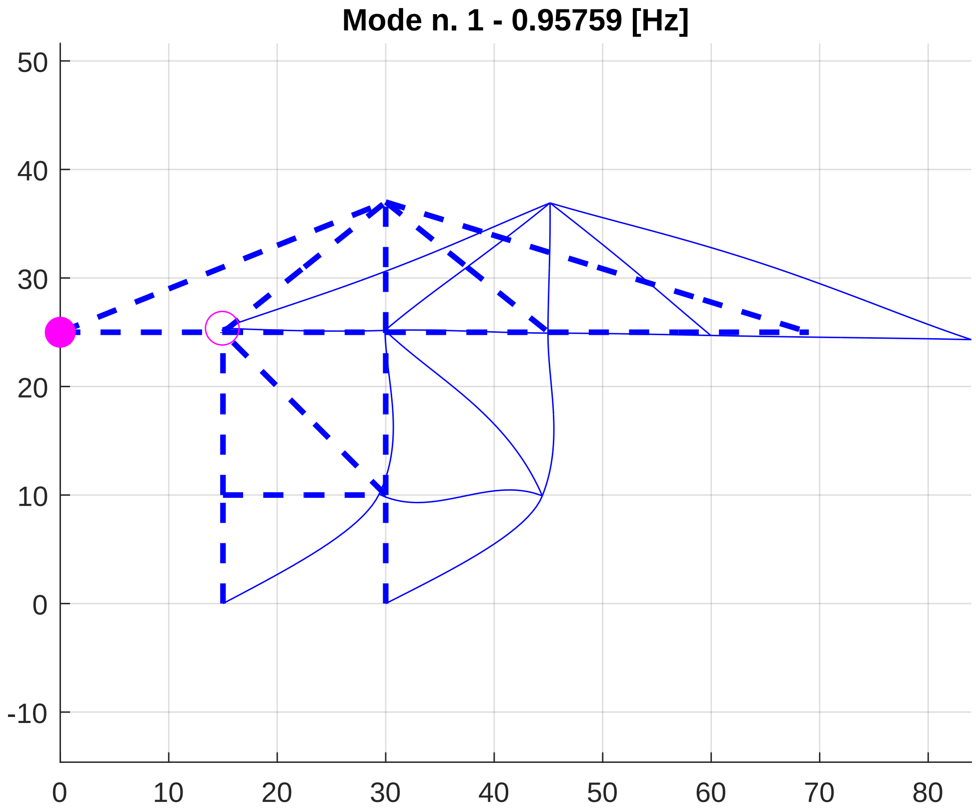

undeformed structure
Mc
X
Y
0
10
20
30
40
50
60
70
0
5
10
15
20
25
30
35
40
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
I
II
III
IV
V
VI
VII
VIII
IX
X
XI
XII
XIII
XIV
XV
XVI
XVII
XVIII
XIX
XX
XXI
XXII
XXIII
XXIV
XXV
XXVI
XXVII
XXVIII
XXIX
XXX
XXXI

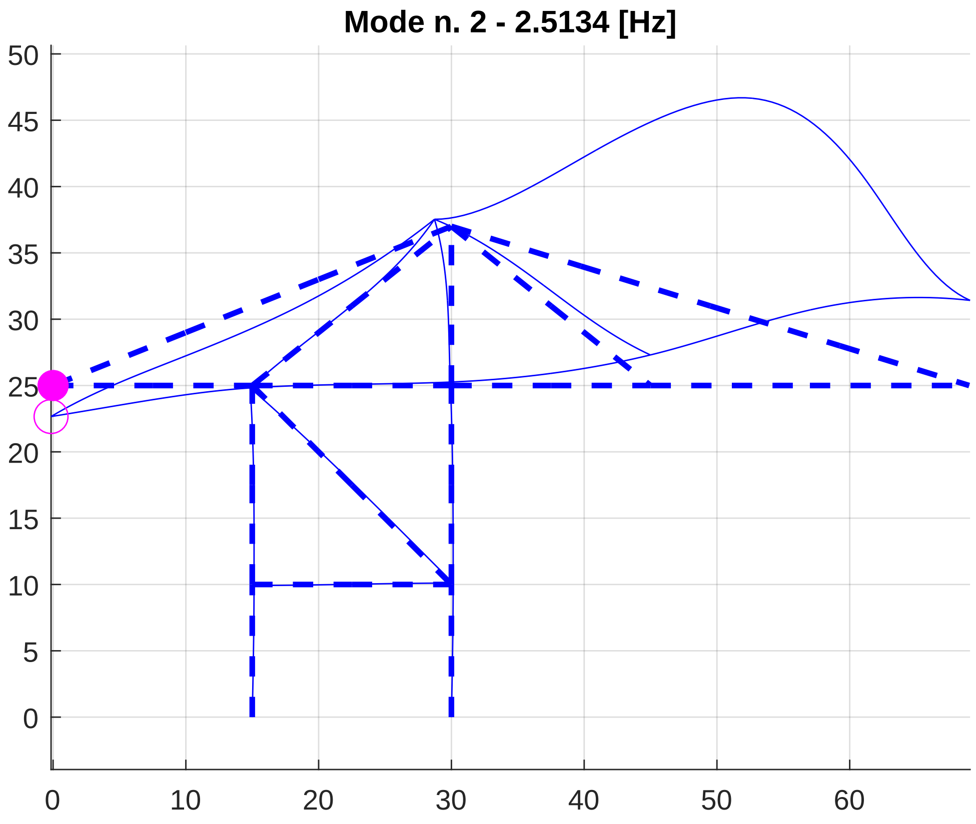
```
! Harbour crane

! NODES LIST:
! ( node nr. \ boundary conditions codes: x,y,theta - x - y )
*NODES
1       1 1 0    15.0   0.0
2       1 1 0    30.0   0.0
3       0 0 0    15.0   10.0
4       0 0 0    22.5   10.0
5       0 0 0    30.0   10.0
6       0 0 0    15.0   17.5
7       0 0 0    22.5   17.5
8       0 0 0    30.0   17.5
9       0 0 0    0.0    25.0
10      0 0 0    7.5    25.0
11      0 0 0    15.0   25.0
12      0 0 0    22.5   25.0
13      0 0 0    30.0   25.0
14      0 0 0    37.5   25.0
15      0 0 0    45.0   25.0
16      0 0 0    57.0   25.0
17      0 0 0    69.0   25.0
18      0 0 0    30.0   37.0
19      0 0 0    10.0   29.0
20      0 0 0    20.0   33.0
21      0 0 0    22.5   31.0
22      0 0 0    30.0   31.0
23      0 0 0    37.5   31.0
24      0 0 0    39.75  34.0
25      0 0 0    49.5   31.0
26      0 0 0    59.25  28.0
*ENDNODES

! BEAMS LIST:
! ( beam nr. \ i-th node nr. - j-th node nr. - mass[kg/m] - EA[N] -
   EJ[Nm^2] )
*BEAMS
! Green beams
1       1    3    200    5.4e9    4.5e8
2       3    4    200    5.4e9    4.5e8
3       4    5    200    5.4e9    4.5e8
4       2    5    200    5.4e9    4.5e8
5       3    6    200    5.4e9    4.5e8
6       5    7    200    5.4e9    4.5e8
7       5    8    200    5.4e9    4.5e8
8       6    11   200    5.4e9    4.5e8
9       7    11   200    5.4e9    4.5e8
10      8    13   200    5.4e9    4.5e8
! Red beams
11      9    10   312    8.2e9    1.40e9
12      10   11   312    8.2e9    1.40e9
13      11   12   312    8.2e9    1.40e9
14      12   13   312    8.2e9    1.40e9
15      13   14   312    8.2e9    1.40e9
16      14   15   312    8.2e9    1.40e9
17      15   16   312    8.2e9    1.40e9
```

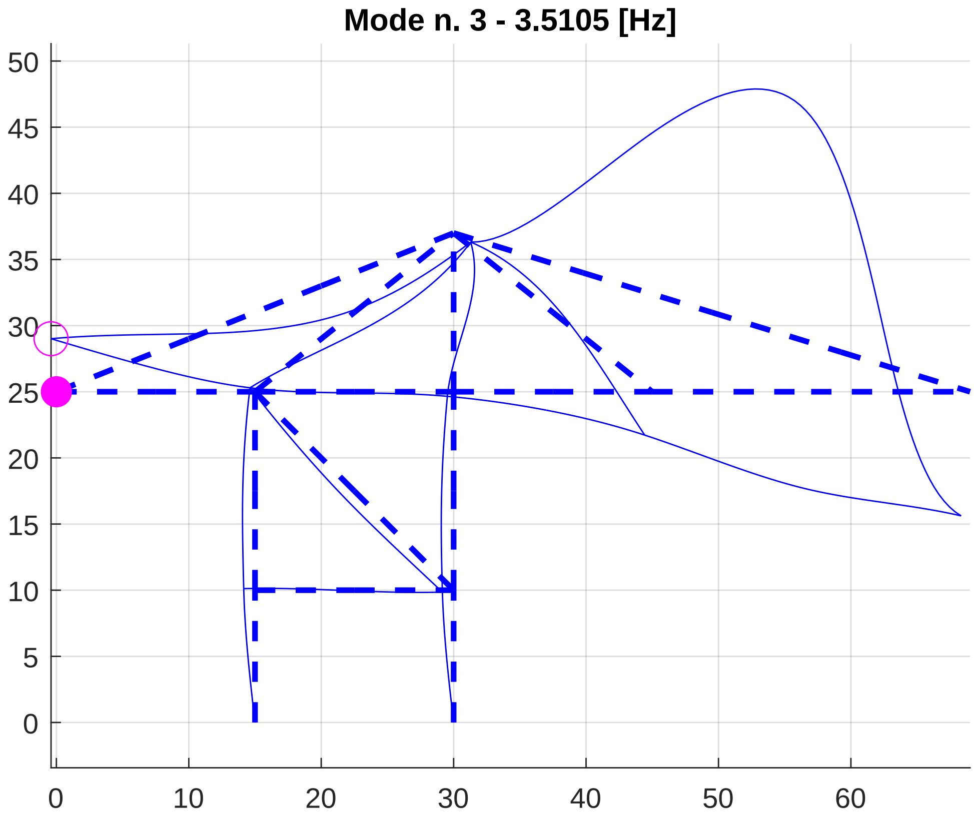
```
18      16   17   312    8.2e9    1.40e9
! Blue beams
19      9    19   90     2.4e9    2.0e8
20      19   20   90     2.4e9    2.0e8
21      20   18   90     2.4e9    2.0e8
22      11   21   90     2.4e9    2.0e8
23      21   18   90     2.4e9    2.0e8
24      18   23   90     2.4e9    2.0e8
25      23   15   90     2.4e9    2.0e8
26      18   24   90     2.4e9    2.0e8
27      24   25   90     2.4e9    2.0e8
28      25   26   90     2.4e9    2.0e8
29      26   17   90     2.4e9    2.0e8
30      13   22   90     2.4e9    2.0e8
31      22   18   90     2.4e9    2.0e8
*ENDBEAMS

! ALPHA AND BETA VALUES (DAMPING MATRIX):
! ( alpha - beta )
*DAMPING
0.1    2.0e-4

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

! RIGID BODY DATA: ATTACHED RIGID MASS AT NODE NR. 9
! ( mass nr. \ node nr. - mass[kg] - J[kgm^2] )
*MASSES
1      9    2000    320
*ENDMASSES
```

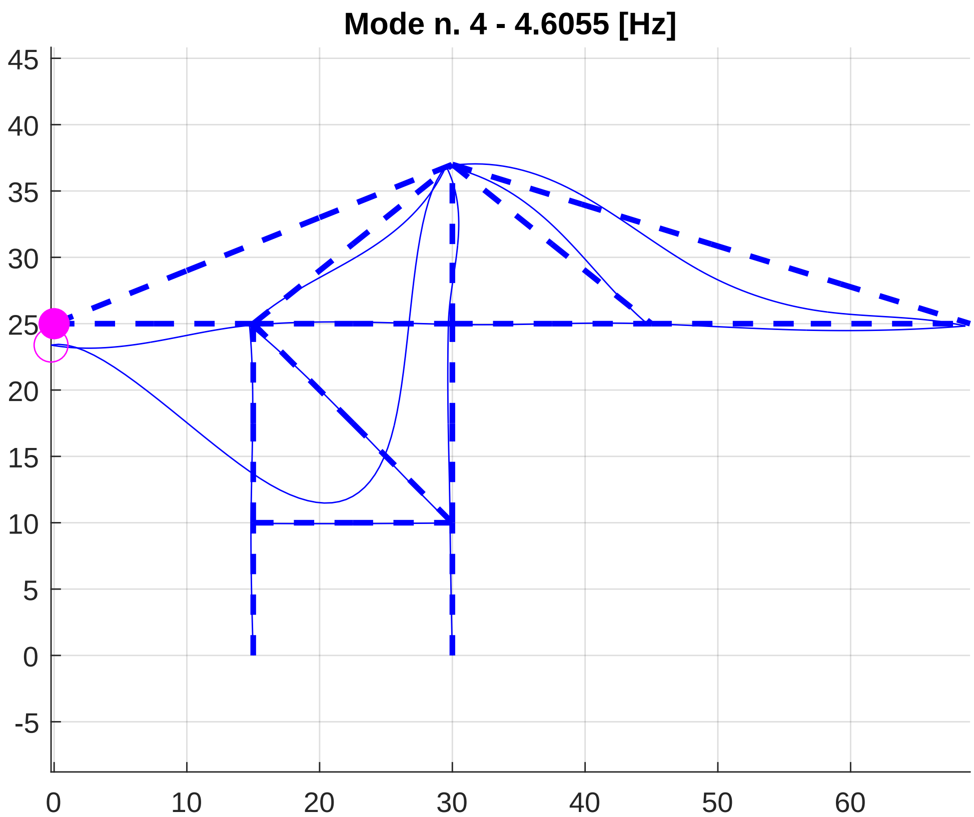
## 2.2 Question 2

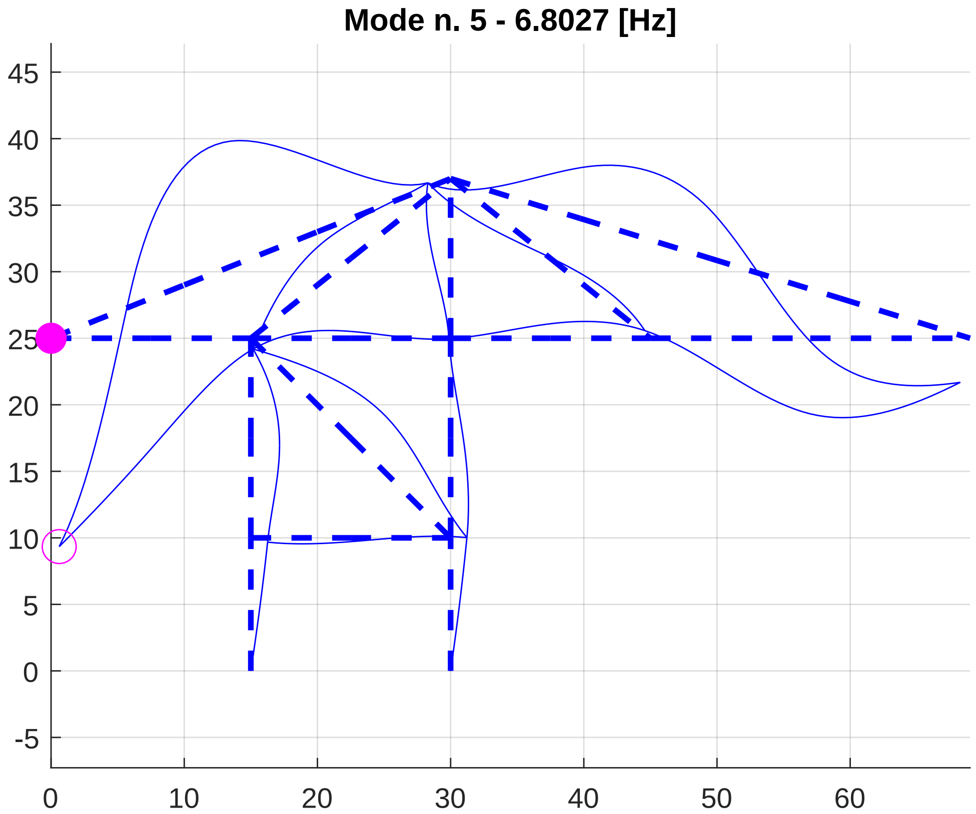
The $DMB\_FEM2$ program has the feature to rapidly compute and plot all frequencies and modes of vibration of the imported system in .inp file format.
Due to the settled frequency limit ($\leq 10\ Hz$), we can find six valid modes for the system in analysis.

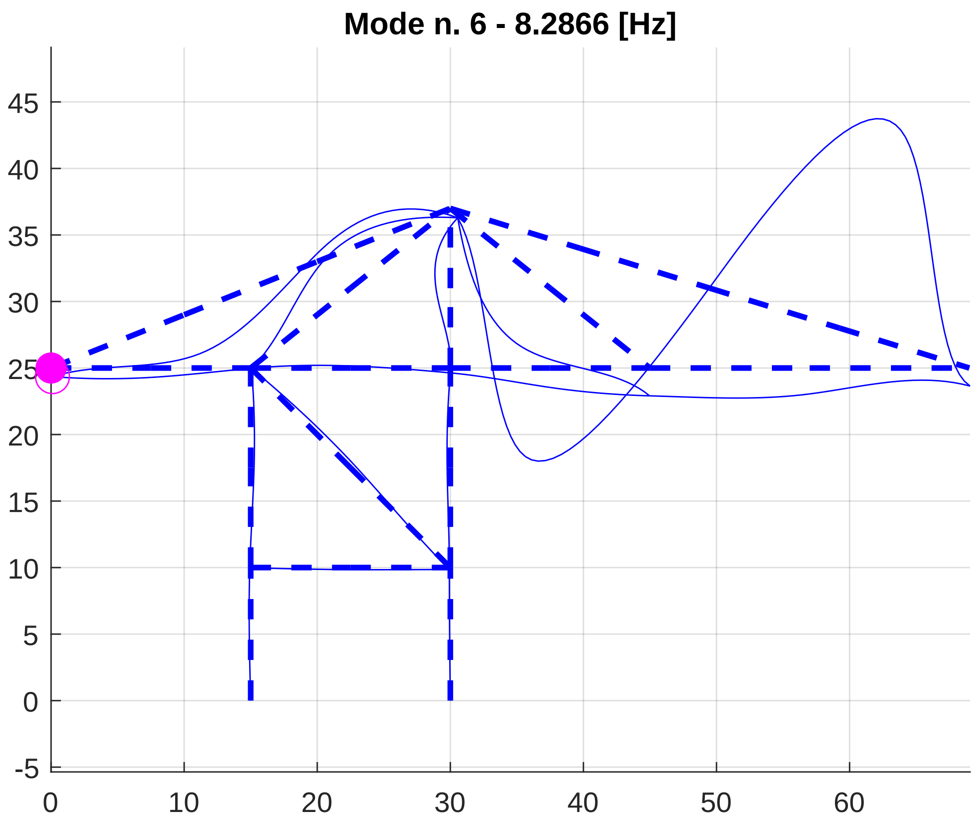

Mode n. 1 - 0.95759 [Hz]
50
40
30
20
10
0
-10
0
10
20
30
40
50
60
70
80


Mode n. 2 - 2.5134 [Hz]
50
45
40
35
30
25
20
15
10
5
0
0
10
20
30
40
50
60


Mode n. 3 - 3.5105 [Hz]
50
45
40
35
30
25
20
15
10
5
0
0
10
20
30
40
50
60


Mode n. 4 - 4.6055 [Hz]
45
40
35
30
25
20
15
10
5
0
-5
0
10
20
30
40
50
60


Mode n. 5 - 6.8027 [Hz]
45
40
35
30
25
20
15
10
5
0
-5
0
10
20
30
40
50
60


Mode n. 6 - 8.2866 [Hz]
45
40
35
30
25
20
15
10
5
0
-5
0
10
20
30
40
50
60

## 2.3 Question 3

Four FRF plots are requested here:

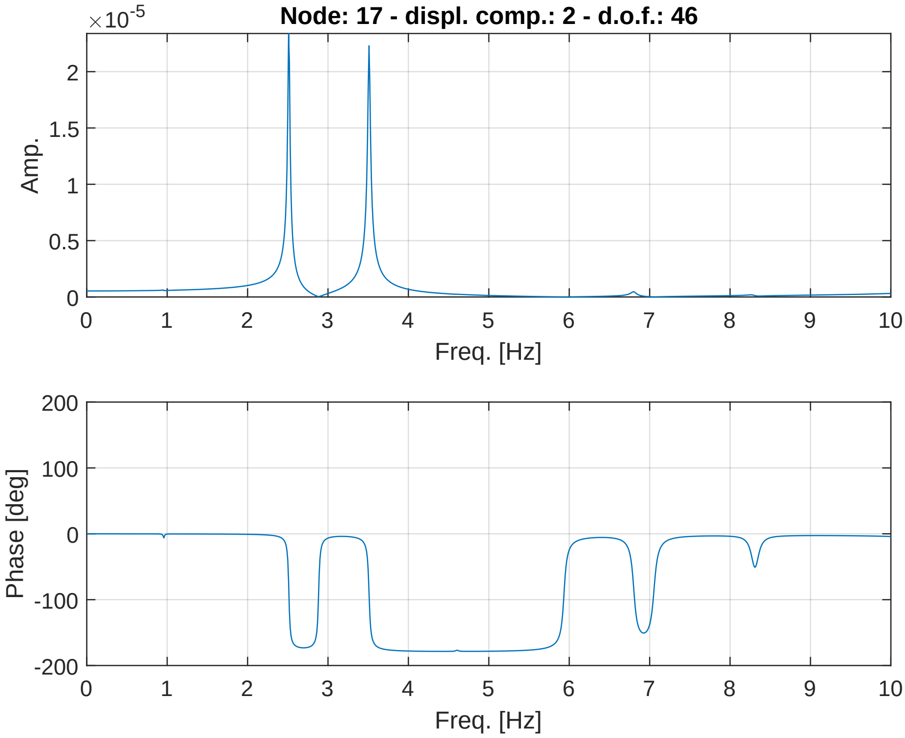
a) input: vertical force at point A; output: vertical displacement of point A;

b) input: vertical force at point A; output: horizontal displacement of point B;

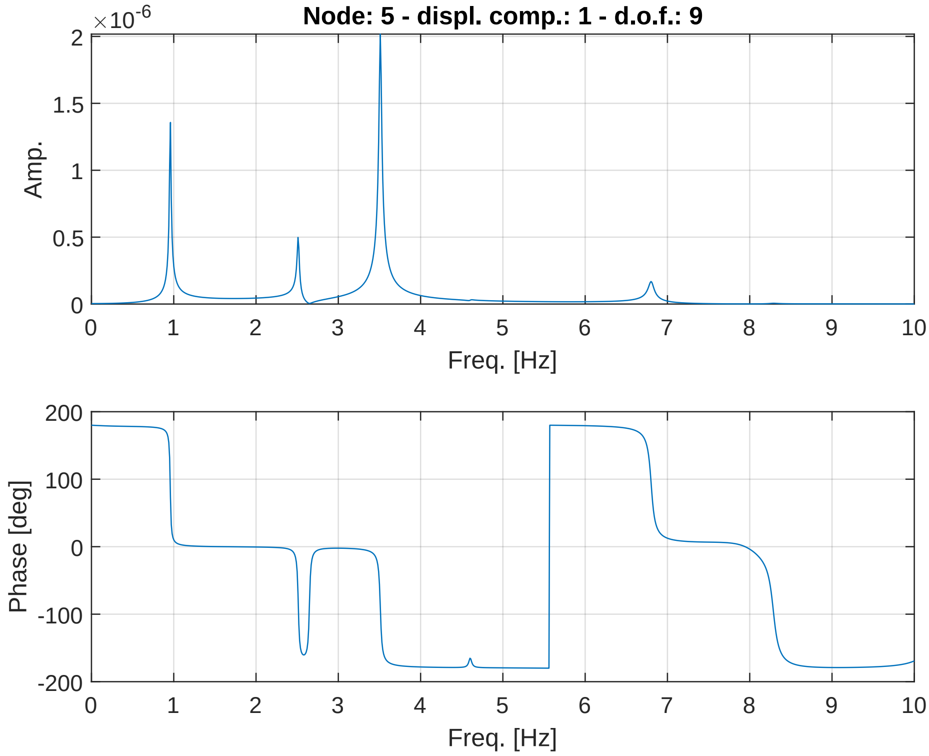
c) input: vertical force at point A; output: vertical component of the constraint force in the hinge O2;

d) input: vertical force at point C; output: vertical component of the constraint force in the hinge O2.

The first two plots a) and b) can be computed use the $DMB\_FEM2$ directly. Moreover, it's necessary to look up on the .inp file to find the corresponding nodal sections with the requested points. In this case, we can see from the undeformed structure that point A was defined as the node 17 of the .inp file. The same reasoning can be find for the corresponding between the requested outputs with the reported nodes plots.




*a) input: vertical force at point A; output: vertical displacement of point A.*




*b) input: vertical force at point A; output: horizontal displacement of point B.*

On the other way, different considerations have to be done for what concerns the computation of the following c) and d) plots. The $DMB\_FEM2$ script does not implement something about the FRF analysis of forces. To reach the goal, all was done "by hand" through the following Matlab file.

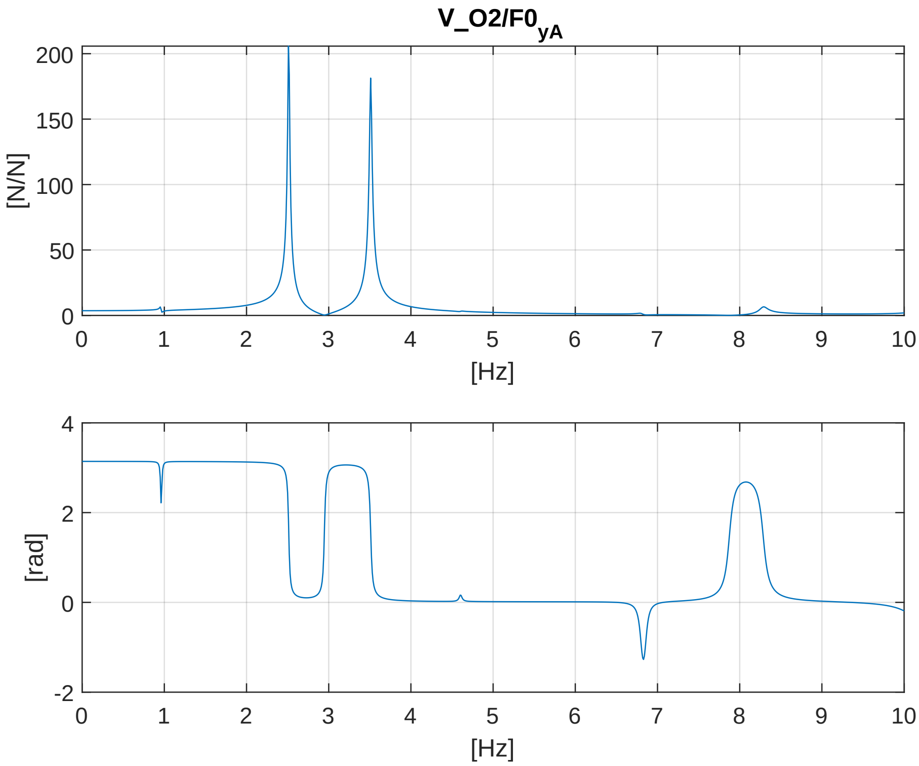
```
clear all
close all
clc

load("Harbour_crane_structure_mkr")

% Structural matrices definition

MFF=M(1:74,1:74);
CFF=R(1:74,1:74);
KFF=K(1:74,1:74);

MFC=M(1:74,75:78);
CFC=R(1:74,75:78);
KFC=K(1:74,75:78);

MCF=M(75:78,1:74);
CCF=R(75:78,1:74);
KCF=K(75:78,1:74);
```

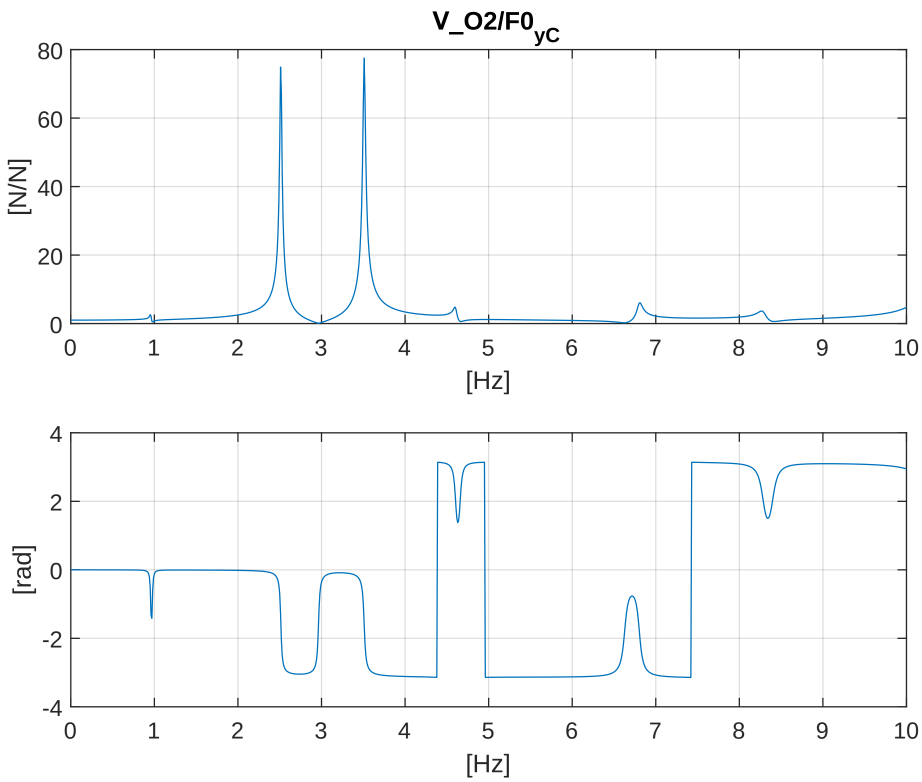
```
MCC=M(75:78,75:78);
CCC=R(75:78,75:78);
KCC=K(75:78,75:78);

%%

i=sqrt(-1);
vett_f=0:0.01:10;
LqF_A=zeros(1,74);
LqF_A(46)=1;
LqF_C=zeros(1,74);
LqF_C(22)=1;
F0=1;
QF_A=LqF_A'*F0;
QF_C=LqF_C'*F0;
for k=1:length(vett_f)
    ome=2*pi*vett_f(k);
    A=-ome^2*MFF+i*ome*CFF+KFF;
    xF_A=A\QF_A;
    xF_C=A\QF_C;
    QCF_A=(-ome^2*MCF+i*ome*CCF+KCF)*xF_A;
    QCF_C=(-ome^2*MCF+i*ome*CCF+KCF)*xF_C;
    mod1(k)=abs(QCF_A(4));
    phase1(k)=angle(QCF_A(4));
    mod2(k)=abs(QCF_C(4));
    phase2(k)=angle(QCF_C(4));
end

figure
subplot
   211;plot(vett_f,mod1);grid;xlabel('[Hz]');ylabel('[N/N]');title('V_O2/F0_y_A')
subplot 212;plot(vett_f,phase1);grid;xlabel('[Hz]');ylabel('[rad]')
figure
subplot
   211;plot(vett_f,mod2);grid;xlabel('[Hz]');ylabel('[N/N]');title('V_O2/F0_y_C')
subplot 212;plot(vett_f,phase2);grid;xlabel('[Hz]');ylabel('[rad]')
```

The vectors $QCF\_A$ and $QCF\_C$ (properly, due to force application in point A and C) explain the output forces of the constraints due to the input forces application. Using the *idb* matrix legend and taking to account the partition matrices, we find that the vertical component of the constraint force in the hinge $O_2$ corresponds to $QCF(4)$.

$$\begin{cases} [M_{FF}]\underline{\ddot{x}}_F + [C_{FF}]\underline{\dot{x}}_F + [K_{FF}]\underline{x}_F = \underline{Q}_F - \left([M_{FC}]\underline{\ddot{x}}_C + [C_{FC}]\underline{\dot{x}}_C + [K_{FC}]\underline{x}_C\right) = \underline{Q}_F + \underline{Q}_{FC} \\ [M_{CF}]\underline{\ddot{x}}_F + [M_{CC}]\underline{\ddot{x}}_C + [C_{CF}]\underline{\dot{x}}_F + [C_{CC}]\underline{\dot{x}}_C + [K_{CF}]\underline{x}_F + [K_{CC}]\underline{x}_C = \underline{Q}_C \end{cases}$$

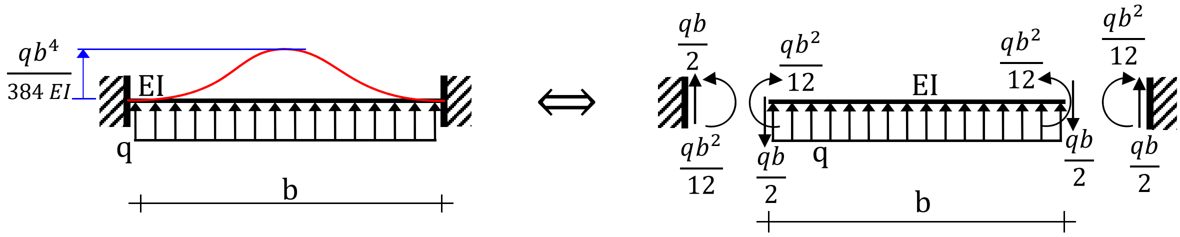
Also, what the script works because all the constraints accelerations vectors of the second equation of matrix above are equals to 0 (by definition), and so $Q_C = Q_{CF}$.


V_O2/F0$_{yA}$
[N/N]
[Hz]
[rad]
[Hz]

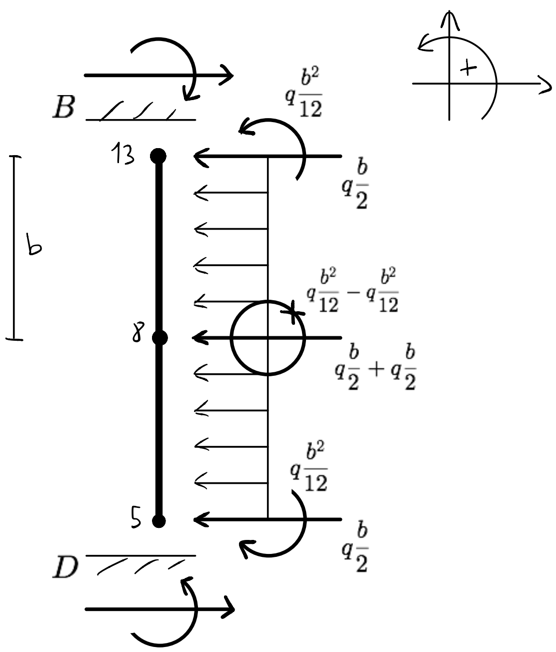

V_O2/F0$_{yC}$
[N/N]
[Hz]
[rad]
[Hz]

## 2.4 Question 4

As we did in the previous question, the computation of three FRF plots is requested here too. But in this case, the input force that we have to take into consideration is not a single harmonic force in a specific point, but an horizontal distributed load applied on the right leg of the crane between points B and D.
The first thing in which we have to think about is how mathematical understand the input force as a distributed infinitesimal force along all the specified beam. The shape functions come to help us. They are used to express the motion of all sections in the beam element as function of the nodal coordinates.
As long as the segment $BD$ can be considered a pinned-pinned beam, we can find on books the following graphical explanation.




The figure on the left side shows the maximum displacement (bending) of the beam and its shape that is subjected to the perpendicular forces and the torsional moment explained in the right figure. Applying what we seen to the case in examination, we obtain the following representation.

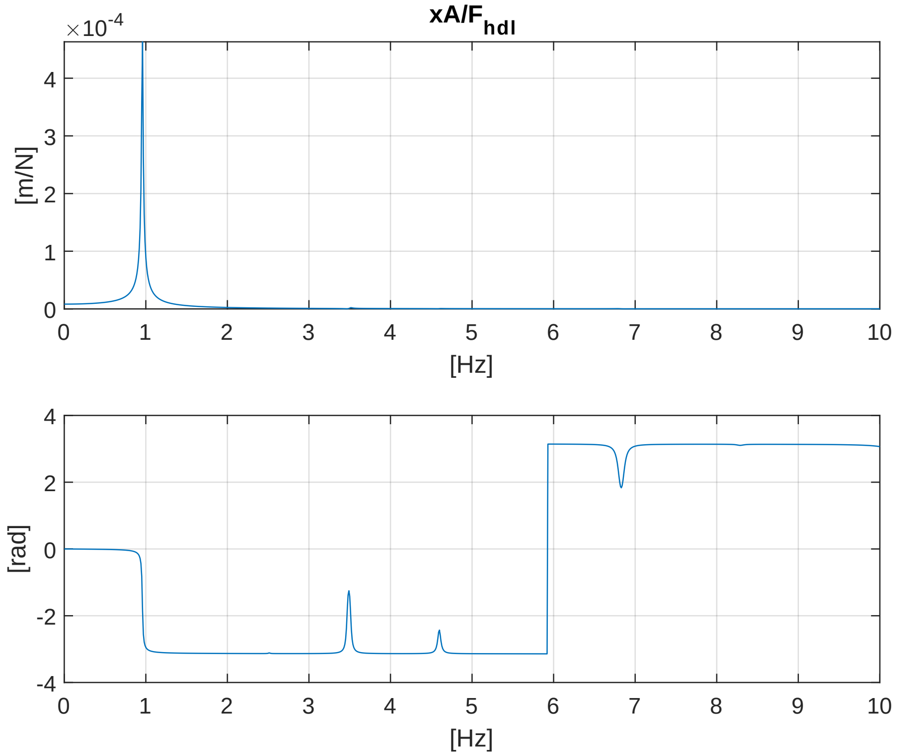


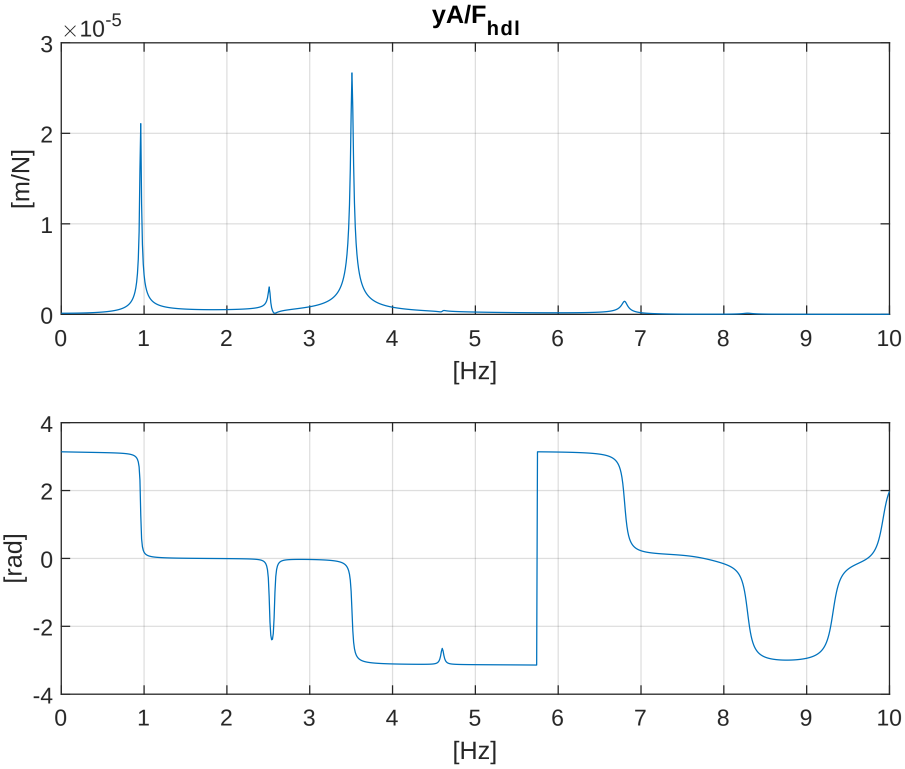

$b$ and $q$ are the length of the finite element and the distributed load force respectively.

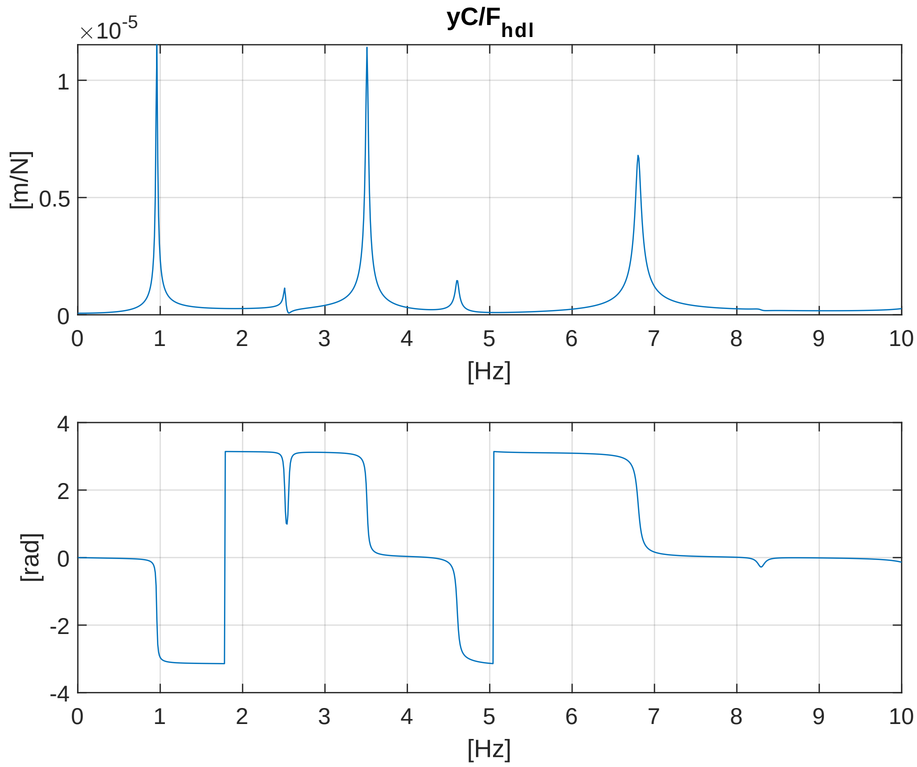
Notice that the node 8 is shared between the two FE in which the $BD$ beam was divided. This implies a null torsional moment for this nodal section.
Next, using the imported idb matrix is easy to associate the $BD$'s nodes with each of their d.o.f involved, and so do the requested plots computation.

```
clear all
close all
clc

load("Harbour_crane_structure_mkr")

% Structural matrices definition

MFF=M(1:74,1:74);
CFF=R(1:74,1:74);
KFF=K(1:74,1:74);

MFC=M(1:74,75:78);
CFC=R(1:74,75:78);
KFC=K(1:74,75:78);

MCF=M(75:78,1:74);
CCF=R(75:78,1:74);
KCF=K(75:78,1:74);

MCC=M(75:78,75:78);
CCC=R(75:78,75:78);
KCC=K(75:78,75:78);

%%

i=sqrt(-1);
vett_f=0:0.01:10;
QF=zeros(74,1);
F0=1; % magnitude of horizontal distributed load
FE_len=7.5; % lenght of each FE
% Distributed load force partitioning
QF(9,1)=F0*FE_len/2; % Node 5 (x)
QF(11,1)=F0*FE_len^2/12; % Node 5 (theta)
QF(18,1)=F0*FE_len; % Node 8 (x)
QF(33,1)=F0*FE_len/2; % Node 13 (x)
QF(35,1)=-F0*FE_len^2/12; % Node 13 (theta)
for k=1:length(vett_f)
    ome=2*pi*vett_f(k);
    A=-ome^2*MFF+i*ome*CFF+KFF;
    xF=A\QF;
    xA=xF(45,1);
    yA=xF(46,1);
    xC=xF(21,1);
    yC=xF(22,1);
    mod1(k)=abs(xA);
    phase1(k)=angle(xA);
    mod2(k)=abs(yA);
    phase2(k)=angle(yA);
```

```
    mod3(k)=abs(yC);
    phase3(k)=angle(yC);
end

figure
subplot
   211;plot(vett_f,mod1);grid;xlabel('[Hz]');ylabel('[m/N]');title('xA/F_h_d_l')
subplot 212;plot(vett_f,phase1);grid;xlabel('[Hz]');ylabel('[rad]')
figure
subplot 211;plot(vett_f,mod2);grid;xlabel('[Hz]');ylabel ('[m/N]');title
    ('yA/F_h_d_l')
subplot 212;plot(vett_f,phase2);grid;xlabel('[Hz]');ylabel('[rad]')
figure
subplot 211;plot(vett_f,mod3);grid;xlabel('[Hz]');ylabel ('[m/N]');title
    ('yC/F_h_d_l')
subplot 212;plot(vett_f,phase3);grid;xlabel('[Hz]');ylabel('[rad]')
```

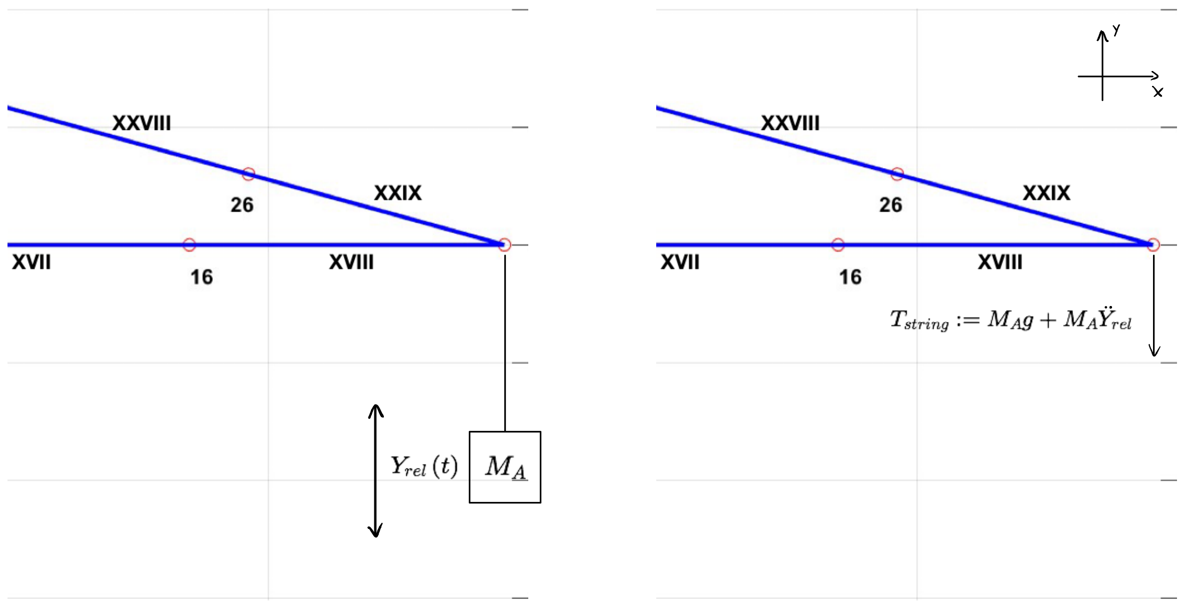



yA/F$_{hdl}$
×10$^{-5}$
[m/N]
[Hz]
[rad]
[Hz]


yC/F$_{hdl}$
×10$^{-5}$
[m/N]
[Hz]
[rad]
[Hz]

## 2.5 Question 5

The request of this point is to analize the displacement of the point A (node 17) given the presence of an hanging mass $M_A$ that moves up and down according to a periodic time history $Y_{rel}(t)$. The following drawing and Matlab script file were used to reach the goal.




```
clear all
close all
clc

load("Harbour_crane_structure_5_mkr") % Static weight force added in node
    17 (A) [Fg = Ma*g]

% Structural matrices definition

MFF=M(1:74,1:74);
CFF=R(1:74,1:74);
KFF=K(1:74,1:74);

MFC=M(1:74,75:78);
CFC=R(1:74,75:78);
KFC=K(1:74,75:78);

MCF=M(75:78,1:74);
CCF=R(75:78,1:74);
KCF=K(75:78,1:74);

MCC=M(75:78,75:78);
CCC=R(75:78,75:78);
KCC=K(75:78,75:78);
```

```
%%

i=sqrt(-1);
T=1.2;
dt=0.001;
vett_t=0:dt:T;
Ma=800;
g=9.81;
A_y_rel=[0.25 0.25 0.15];
phi_y_rel=[0 pi pi];
vett_yA=zeros(1,length(vett_t));
vett_y_rel=zeros(1,length(vett_t));
QF=zeros(74,1);
for k=1:3
    ome=k*2*pi/T;
    A=-ome^2*MFF+i*ome*CFF+KFF;
    y_rel=A_y_rel(k)*exp(i*phi_y_rel(k));
    y_rel_dd=-ome^2*y_rel;
    Tstring=Ma*y_rel_dd;
    QF(46,1)=-Tstring; % Force on point A. Negative due to the reference
        system
    xF=A\QF;
    yA=xF(46,1);
    vett_y_rel=vett_y_rel+(abs(y_rel)*cos(ome*vett_t+angle(y_rel)));
    vett_yA=vett_yA+(abs(yA)*cos(ome*vett_t+angle(yA)));
end

figure
plot(vett_t,vett_y_rel);grid;xlabel('[s]');title ('y_r_e_l(t)')
figure
plot(vett_t,vett_yA);grid;xlabel('[s]');title ('y_A(t)')

%%

fft_y_rel=fft(vett_y_rel);
fft_y_a=fft(vett_yA);
N=length(vett_y_rel); % N=length(vett_yA)
% df=1/T;
% fmax=(N/2-1)*df;
fmax=10; % fs=20 (Shannon theorem)
df=fmax/(N/2-1);
vett_freq=0:df:fmax;

mod_y_rel(1)=1/N*abs(fft_y_rel(1));
mod_y_rel(2:N/2)=2/N*abs(fft_y_rel(2:N/2));
phase_y_rel(1:N/2)=angle(fft_y_rel(1:N/2));

mod_y_a(1)=1/N*abs(fft_y_a(1));
mod_y_a(2:N/2)=2/N*abs(fft_y_a(2:N/2));
phase_y_a(1:N/2)=angle(fft_y_a(1:N/2));

figure
subplot
   211;stem(vett_freq,mod_y_rel);xlabel('[Hz]');ylabel('[m]');title('y_r_e_l')
subplot 212;stem(vett_freq,phase_y_rel);grid;xlabel('[Hz]');ylabel('[rad]')
figure
```

```
subplot
   211;stem(vett_freq,mod_y_a);grid;xlabel('[Hz]');ylabel('[m/N]');title('yA/-Tstring');

subplot 212;stem(vett_freq,phase_y_a);grid;xlabel('[Hz]');ylabel('[rad]')
x_zoom=0.1;
figure
subplot
   211;stem(vett_freq,mod_y_rel);xlabel('[Hz]');ylabel('[m]');title("y_r_e_l
   (zoomed to "+x_zoom+"Hz)");xlim([0 x_zoom])
subplot
   212;stem(vett_freq,phase_y_rel);grid;xlabel('[Hz]');ylabel('[rad]');xlim([0
   x_zoom])
figure
subplot
   211;stem(vett_freq,mod_y_a);grid;xlabel('[Hz]');ylabel('[m/N]');title("yA/-Tstring
   (zoomed to "+x_zoom+"Hz)");xlim([0 x_zoom])
subplot
   212;stem(vett_freq,phase_y_a);grid;xlabel('[Hz]');ylabel('[rad]');xlim([0
   x_zoom])
```

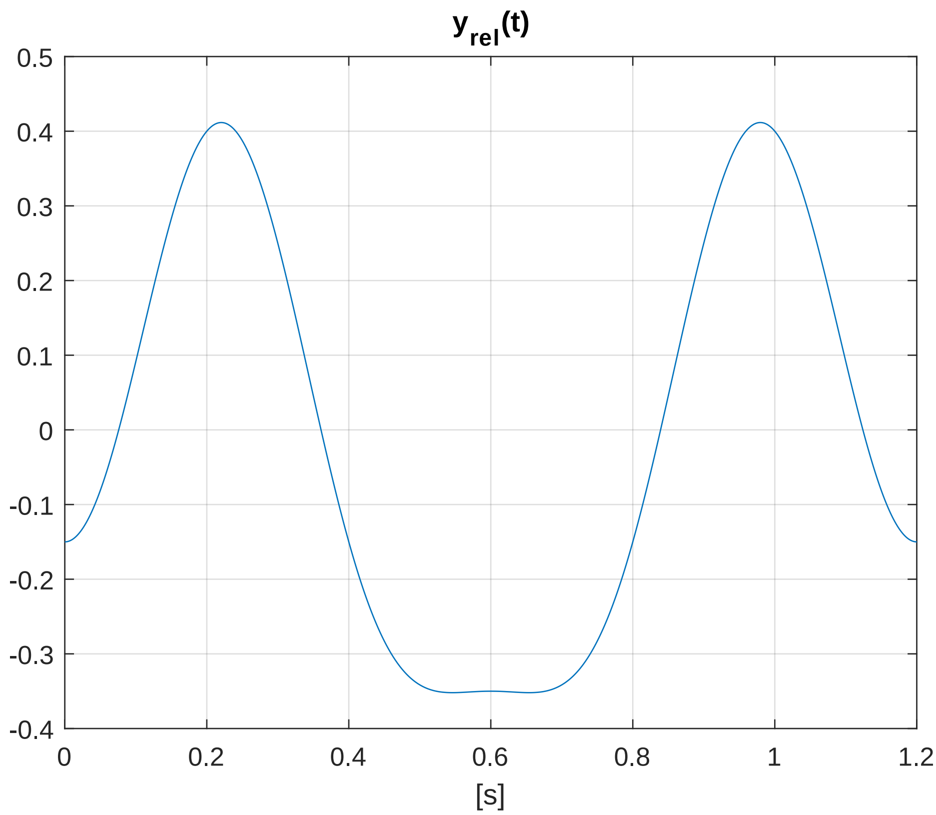
The input force $T_{String}$ in the point A is defined through a static and a dynamic component. The first one is proper of the attached mass and it was introduced into the structure under the specific keyword $MASSES$, in a new .inp file shown below.

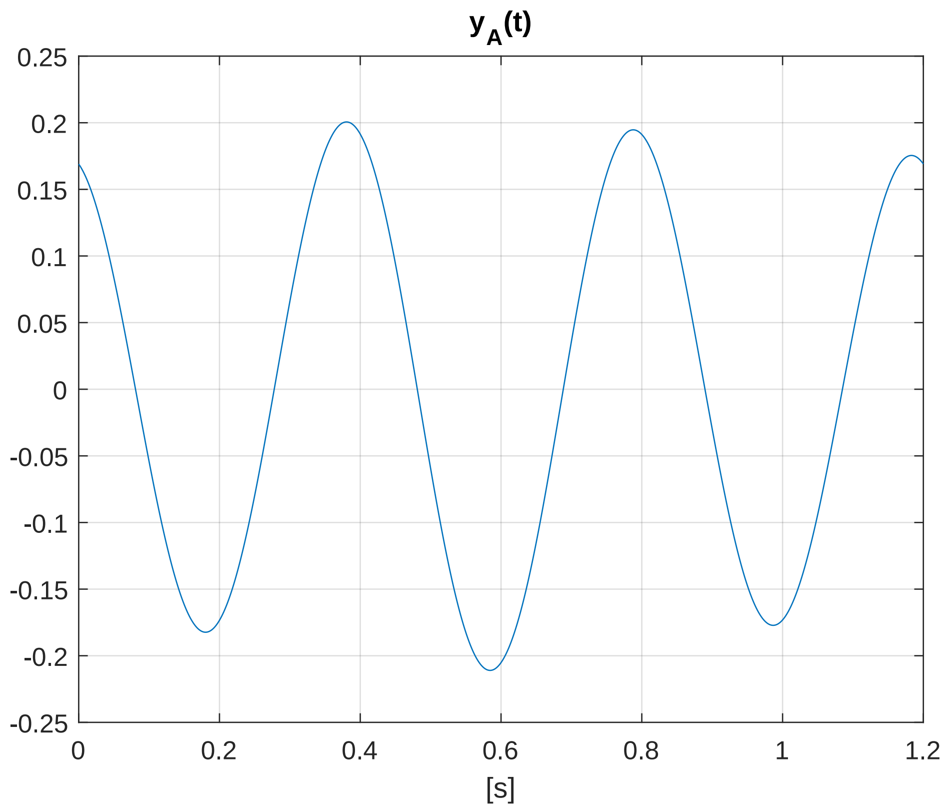
```
! Harbour crane

! NODES LIST:
! ( node nr. \ boundary conditions codes: x,y,theta - x - y )
*NODES
1       1 1 0    15.0    0.0
2       1 1 0    30.0    0.0
3       0 0 0    15.0    10.0
4       0 0 0    22.5    10.0
5       0 0 0    30.0    10.0
6       0 0 0    15.0    17.5
7       0 0 0    22.5    17.5
8       0 0 0    30.0    17.5
9       0 0 0    0.0     25.0
10      0 0 0    7.5     25.0
11      0 0 0    15.0    25.0
12      0 0 0    22.5    25.0
13      0 0 0    30.0    25.0
14      0 0 0    37.5    25.0
15      0 0 0    45.0    25.0
16      0 0 0    57.0    25.0
17      0 0 0    69.0    25.0
18      0 0 0    30.0    37.0
19      0 0 0    10.0    29.0
20      0 0 0    20.0    33.0
21      0 0 0    22.5    31.0
22      0 0 0    30.0    31.0
23      0 0 0    37.5    31.0
24      0 0 0    39.75   34.0
25      0 0 0    49.5    31.0
```

```
26     0 0 0    59.25  28.0
*ENDNODES

! BEAMS LIST:
! ( beam nr. \ i-th node nr. - j-th node nr. - mass[kg/m] - EA[N] -
   EJ[Nm^2] )
*BEAMS
1      1    3    200    5.4e9    4.5e8
2      3    4    200    5.4e9    4.5e8
3      4    5    200    5.4e9    4.5e8
4      2    5    200    5.4e9    4.5e8
5      3    6    200    5.4e9    4.5e8
6      5    7    200    5.4e9    4.5e8
7      5    8    200    5.4e9    4.5e8
8      6    11   200    5.4e9    4.5e8
9      7    11   200    5.4e9    4.5e8
10     8    13   200    5.4e9    4.5e8
11     9    10   312    8.2e9    1.40e9
12     10   11   312    8.2e9    1.40e9
13     11   12   312    8.2e9    1.40e9
14     12   13   312    8.2e9    1.40e9
15     13   14   312    8.2e9    1.40e9
16     14   15   312    8.2e9    1.40e9
17     15   16   312    8.2e9    1.40e9
18     16   17   312    8.2e9    1.40e9
19     9    19   90     2.4e9    2.0e8
20     19   20   90     2.4e9    2.0e8
21     20   18   90     2.4e9    2.0e8
22     11   21   90     2.4e9    2.0e8
23     21   18   90     2.4e9    2.0e8
24     18   23   90     2.4e9    2.0e8
25     23   15   90     2.4e9    2.0e8
26     18   24   90     2.4e9    2.0e8
27     24   25   90     2.4e9    2.0e8
28     25   26   90     2.4e9    2.0e8
29     26   17   90     2.4e9    2.0e8
30     13   22   90     2.4e9    2.0e8
31     22   18   90     2.4e9    2.0e8
*ENDBEAMS

! ALPHA AND BETA VALUES (DAMPING MATRIX):
! ( alpha - beta )
*DAMPING
0.1    2.0e-4

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

! RIGID BODY DATA (ATTACHED RIGID MASS AT NODE NR.  )
! ( mass nr. \ node nr. - mass[kg] - J[kgm^2] )
*MASSES
1      9    2000    320
2      17   800     0
*ENDMASSES
```

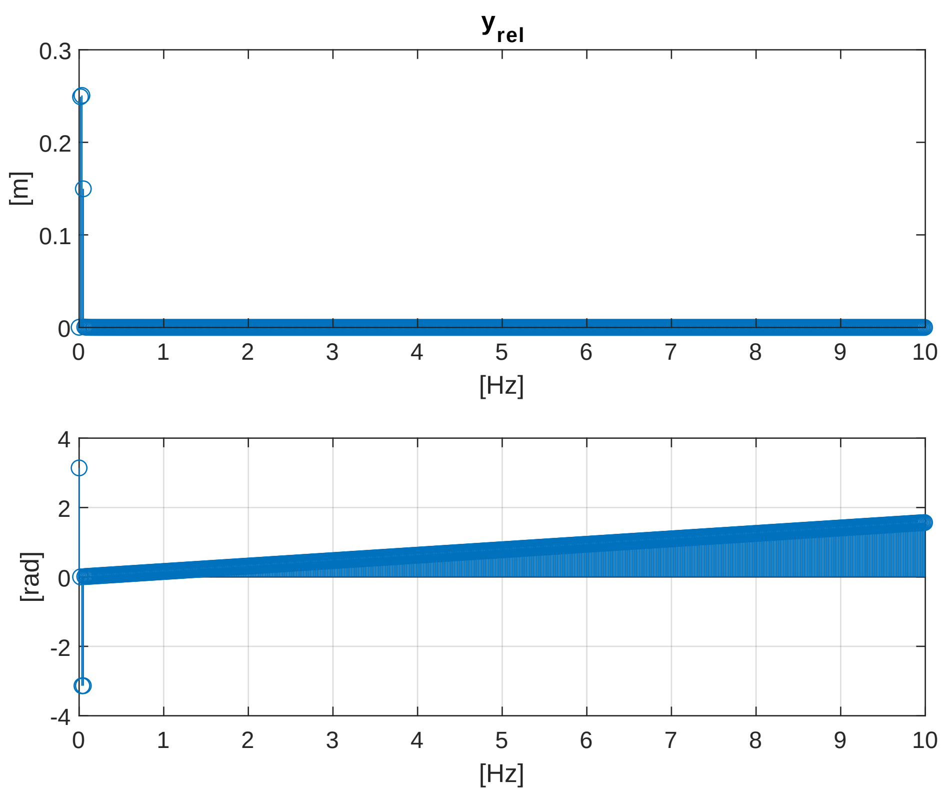
On the other side, the dynamic component was easily computed multiplying the acceleration $\ddot{Y}_{rel}$ with the corresponding mass $M_A$. Notice that the obtained force $T_{string}$ must

be negative considered according to the chosen reference system.
Here the requested plots.

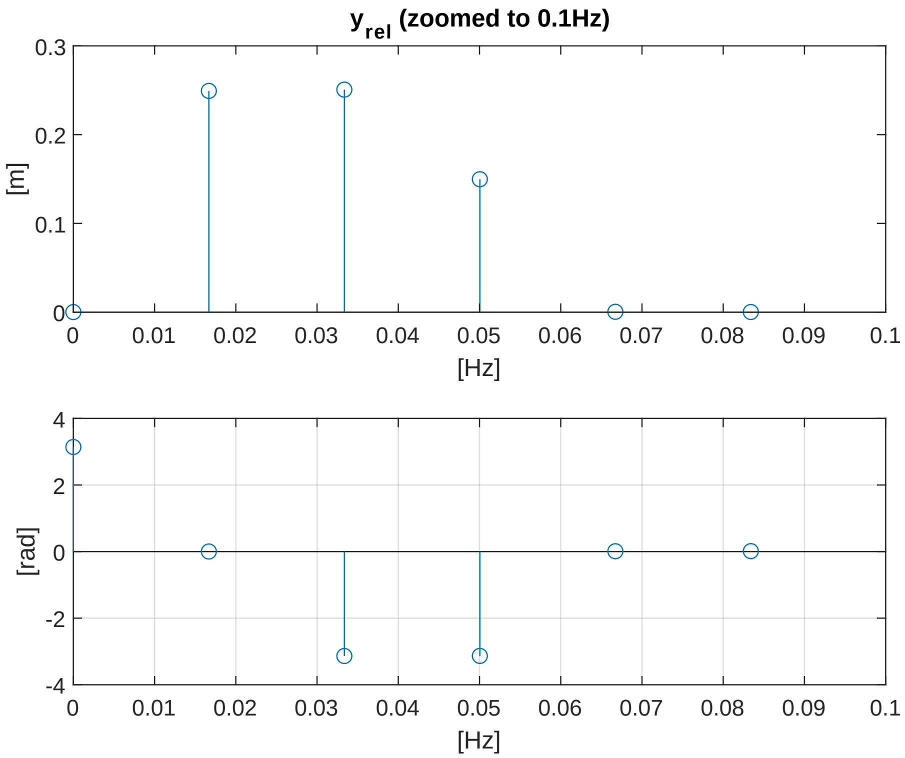


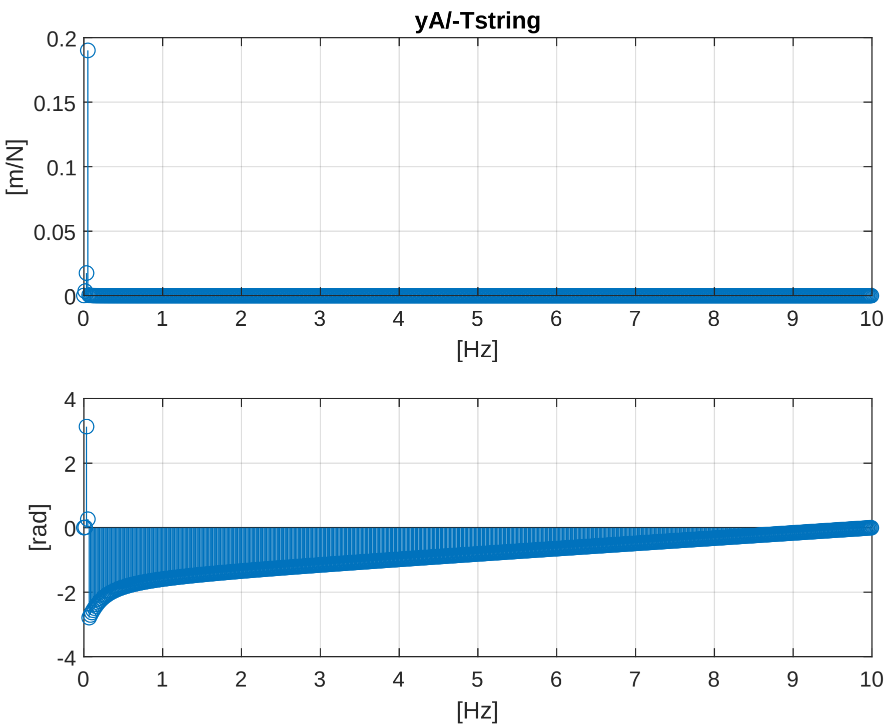


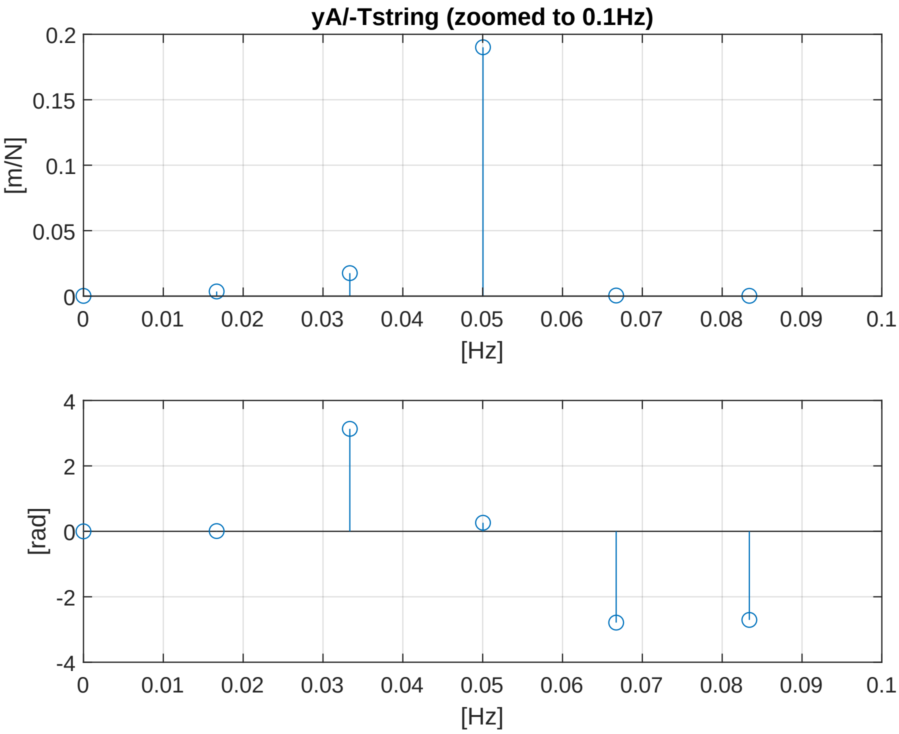
$y_A$(t)
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
0
0.2
0.4
0.6
0.8
1
1.2
[s]

$y_{rel}$
0.3
0.2
0.1
0
[m]
0
1
2
3
4
5
6
7
8
9
10
[Hz]
4
2
0
-2
-4
[rad]
0
1
2
3
4
5
6
7
8
9
10
[Hz]


$y_{rel}$ (zoomed to 0.1Hz)
[m]
0.3
0.2
0.1
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
[Hz]
[rad]
4
2
0
-2
-4
[Hz]


yA/-Tstring
[m/N]
0.2
0.15
0.1
0.05
0
1
2
3
4
5
6
7
8
9
10
[Hz]
[rad]
4
2
0
-2
-4
[Hz]

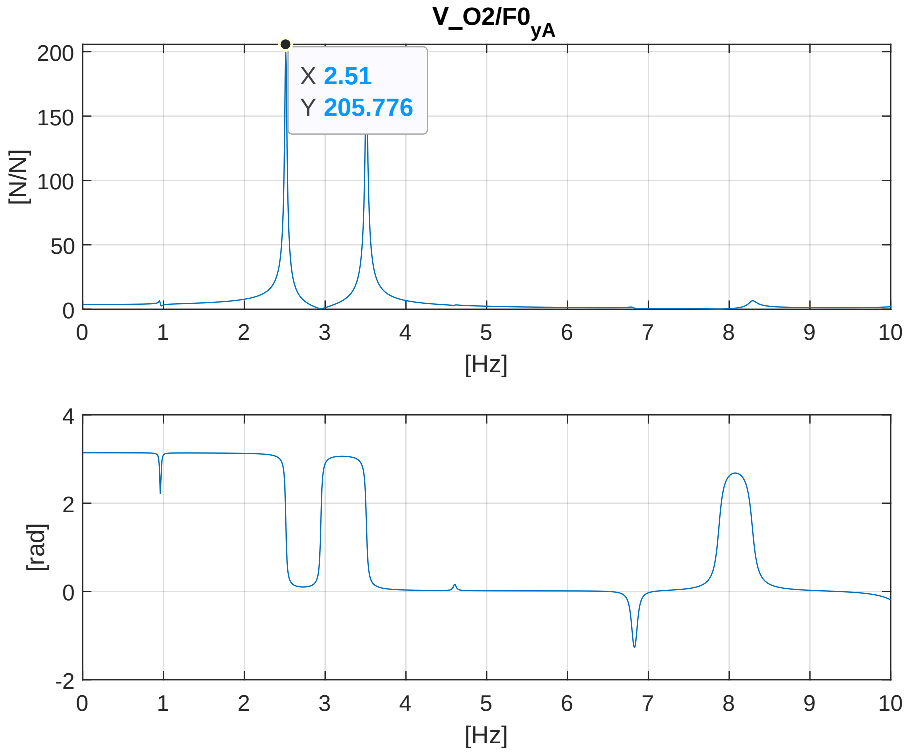



## 2.6 Question 6

The last question of the year-work ask to modify the harbour crane structure proposed in order to reduce at least by 50% the maximum value of the constraint force in the hinge $O_2$ computed in the question 3.
Also, the following modification boundaries are defined:

- the final total mass of the crane must be bigger no more than the 5% of the original value;
- any change of the material and/or of the constraints (by adding or moving them) is not allowed;
- in case of change of one or more beams sections, all computations must be done taking into account a real geometry and the physical properties of steel material.

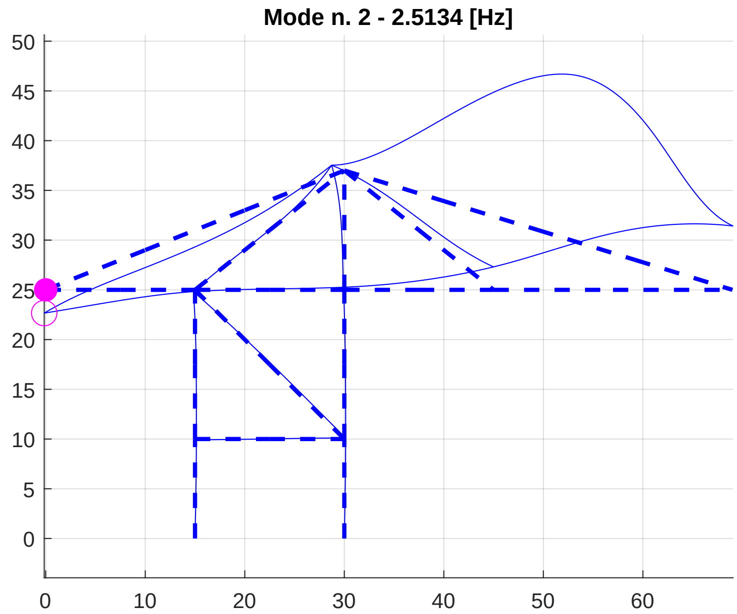
Considerations I made for the solution that I'm going to propose analyzes the case ($c$) of the question 3, where the input force is placed in the point A. The FFT analysis leads to this plot:

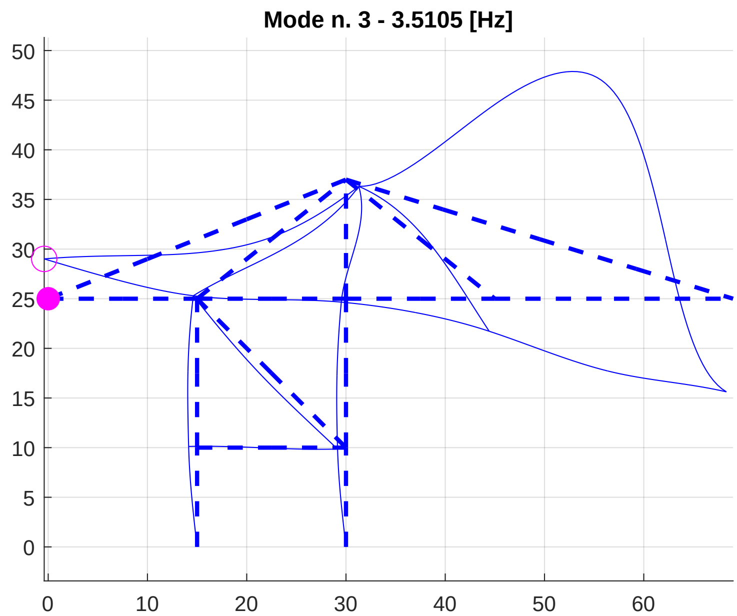



So in other words, the goal asked here is to get a final peak (or peaks) of the constraint force $V\_O2$ that must be $\leq 102.888N$.
The solution described on the next pages probably is suitable for a faster approach to the problem. Clearly it is not the best one in relation to the exercise boundaries, because it provides the use of a **damper system** of which the **weight is totally neglected** by the $DMB\_FEM2$ program.

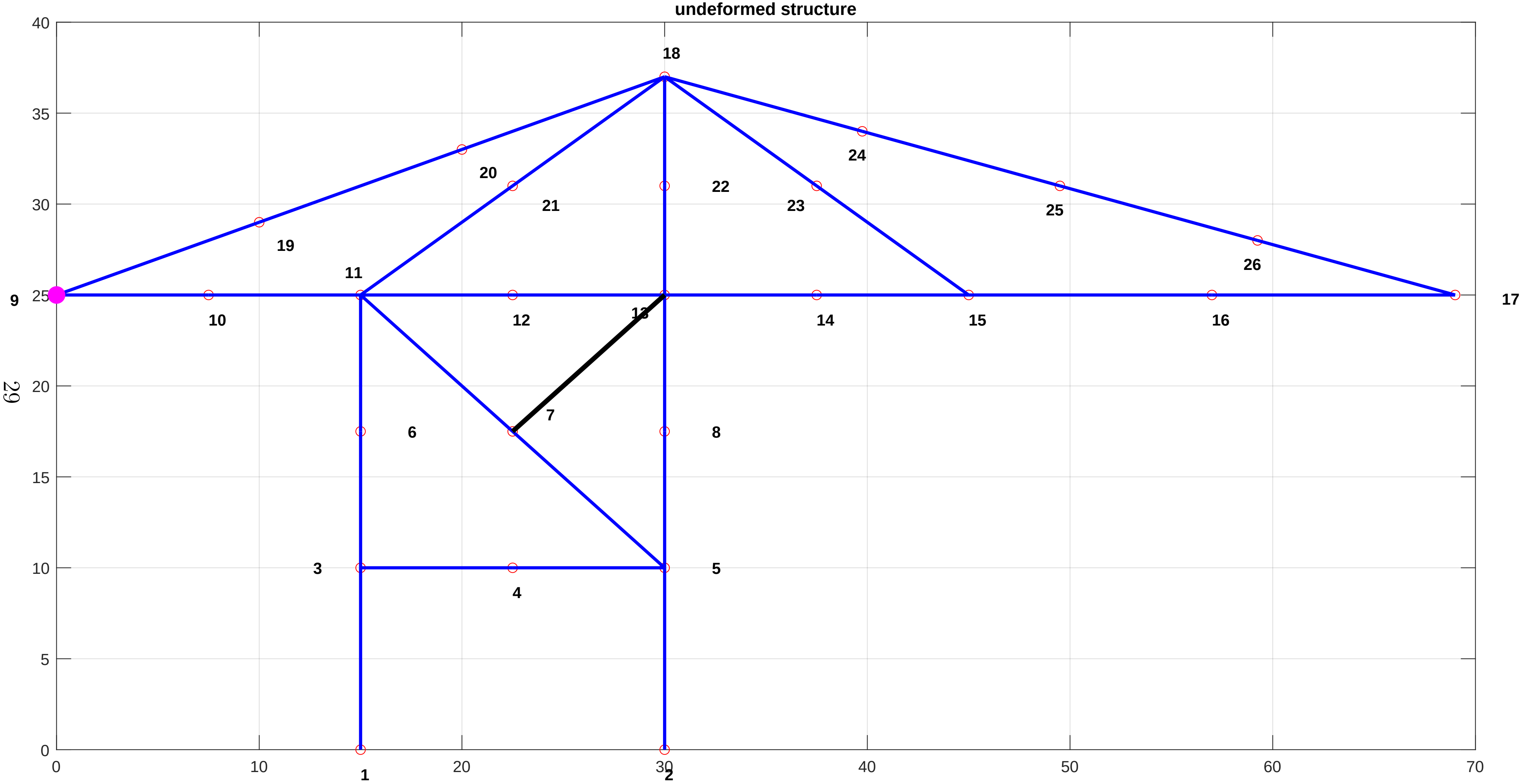
### 2.6.1 Explanations about the use of a damper system

Watching the plot $V_O2/F0_{yA}$ of the previous figure, we can notice that higher value of the constraint force matches with the resonance peak at the **second proper frequency** of the system; the other relevant one is around the **third** mode of the harbour crane.




It's clear that the peaks on the hinge $O_2$ are caused by an "huge" vibration of the second vertical half of the structure. How to mitigate the propagation of that vibrations up to the base of the structure (where the hinge of the case is)? A working solution concerns to place a damper system that links the node 7 and the node 13, as is shown down (the black line).


undeformed structure
18
20
21
22
23
24
25
26
19
11
9
10
12
13
14
15
16
17
6
7
8
3
4
5
1
2

### 2.6.2 Implementation of a proper damper system

According to the 2nd Newton's law,

$$F_d = c\dot{\Delta L} \tag{2.3}$$

is the elastic force corresponds to the type of resistance to motion and energy dissipation that is encountered when a piston is moved through a cylinder filled with a viscous fluid, for example oil.
So given a damper build with a maximum supported force $F_d$ and a maximum displacement in time $\dot{\Delta L}$, the damping coefficient $c$ can be easily calculate rearranging the previous equation:

$$c = \frac{F_d}{\dot{\Delta L}}. \tag{2.4}$$

It's necessary now to define a criteria for a better search of a suitable damper system. Here I just found a sort of "lower bound" for the maximum supported force capacity by the candidate dampers.
Consider as example a static mass load $M_A = 800\ Kg$ that hangs on the point A, as in the question 5. If the 205.776 value of the peak is due to a conventional $F0_{yA} = 1$ force, we can make a "symbolic" proportion as

$$F0_{yA} : 205.776 = Fst_{M_A} : x;\ x \approx 1614930\ N \approx 1615\ kN \tag{2.5}$$

to say that the damper that we will choose should work properly in the order $10^3\ kN$. According to this, the device that we will use is under the name of $TSTD\ 2000/\pm 50$ from *Tensa - Dampers & stus* catalog.

**TYPICAL DIMENSIONS FOR BUILDINGS**

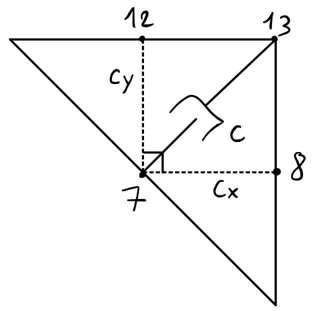
| DEVICE | F (ULS) | d (ULS) | D | L | $L_T$ | $L_V$ | $F_P$ | $L_{aT}$ | $L_{bT}$ | $i_{aT}$ | $i_{bT}$ | N° | $L_{aV}$ | $L_{bV}$ | $i_{aV}$ | $i_{bV}$ | N° | H |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | [kN] | [±mm] | [mm] | [mm] | [mm] | [mm] | [mm] | [mm] | [mm] | [mm] | [mm] | | [mm] | [mm] | [mm] | [mm] | | [mm] |
| TSTD 500/±25 | 500 | 25 | 185 | 760 | 195 | 330 | 60 | 250 | 300 | 190 | 240 | 4 M24 | 250 | 400 | 190 | 160 | 6 M30 | 200 |
| TSTD 500/±50 | 500 | 50 | 185 | 850 | 195 | 330 | 60 | 250 | 300 | 190 | 240 | 4 M24 | 250 | 400 | 190 | 160 | 6 M30 | 200 |
| TSTD 750/±25 | 750 | 25 | 215 | 850 | 220 | 330 | 70 | 300 | 350 | 220 | 275 | 4 M30 | 300 | 410 | 220 | 160 | 6 M36 | 200 |
| TSTD 750/±50 | 750 | 50 | 215 | 940 | 220 | 330 | 70 | 300 | 350 | 220 | 275 | 4 M30 | 300 | 410 | 220 | 160 | 6 M36 | 200 |
| TSTD 1000/±25 | 1000 | 25 | 250 | 960 | 245 | 330 | 80 | 325 | 350 | 230 | 260 | 4 M36 | 325 | 420 | 230 | 155 | 6 M42 | 200 |
| TSTD 1000/±50 | 1000 | 50 | 250 | 1040 | 245 | 330 | 80 | 325 | 350 | 230 | 260 | 4 M36 | 325 | 420 | 230 | 155 | 6 M42 | 200 |
| TSTD 1250/±25 | 1250 | 25 | 290 | 1080 | 265 | 430 | 90 | 350 | 400 | 260 | 310 | 4 M36 | 350 | 530 | 260 | 205 | 6 M48 | 250 |
| TSTD 1250/±50 | 1250 | 50 | 290 | 1160 | 265 | 430 | 90 | 350 | 400 | 260 | 310 | 4 M36 | 350 | 530 | 260 | 205 | 6 M48 | 250 |
| TSTD 1500/±25 | 1500 | 25 | 300 | 1180 | 280 | 425 | 90 | 400 | 450 | 290 | 345 | 4 M42 | 400 | 530 | 290 | 205 | 6 M48 | 250 |
| TSTD 1500/±50 | 1500 | 50 | 300 | 1260 | 280 | 425 | 90 | 400 | 450 | 290 | 345 | 4 M42 | 400 | 530 | 290 | 205 | 6 M48 | 250 |
| TSTD 2000/±25 | 2000 | 25 | 350 | 1365 | 325 | 505 | 110 | 450 | 500 | 330 | 380 | 4 M48 | 450 | 630 | 330 | 170 | 8 M48 | 275 |
| TSTD 2000/±50 | 2000 | 50 | 350 | 1445 | 325 | 505 | 110 | 450 | 500 | 330 | 380 | 4 M48 | 450 | 630 | 330 | 170 | 8 M48 | 275 |

$$F_d = 2000 \times 10^3\ N;\ \dot{\Delta L} = 50 \times 10^{-3}\ \frac{m}{s} \tag{2.6}$$

Using (2.4)

$$c = \frac{2000 \times 10^3}{50 \times 10^{-3}} = 4 \times 10^7\ \frac{Ns}{m} \tag{2.7}$$

Once defined the damping coefficient from the chosen damper system, we need now to put this last one into our harbour crane structure. As before, we use a modified .inp file having a new line under the $SPRINGS$ keyword. This one contains the linked nodes by the damper and the damping coefficient $c$ in the form of its $x$ and $y$ coordinates.

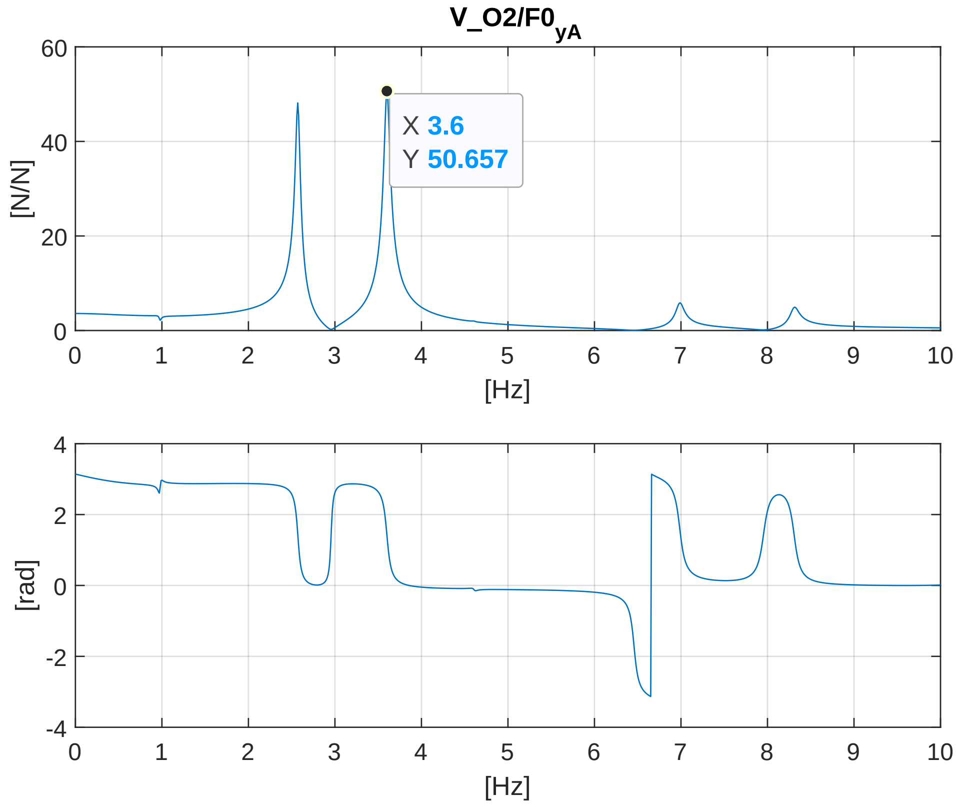



$$Cx = C\cos(45) = C\sin(45) = Cy = 4\times10^7\cdot\frac{\sqrt{2}}{2} = 28284271,25\frac{Ns}{m}$$

```
! Harbour crane

! NODES LIST:
! ( node nr. \ boundary conditions codes: x,y,theta - x - y )
*NODES
1       1 1 0     15.0    0.0
2       1 1 0     30.0    0.0
3       0 0 0     15.0    10.0
4       0 0 0     22.5    10.0
5       0 0 0     30.0    10.0
6       0 0 0     15.0    17.5
7       0 0 0     22.5    17.5
8       0 0 0     30.0    17.5
9       0 0 0     0.0     25.0
10      0 0 0     7.5     25.0
11      0 0 0     15.0    25.0
12      0 0 0     22.5    25.0
13      0 0 0     30.0    25.0
14      0 0 0     37.5    25.0
15      0 0 0     45.0    25.0
16      0 0 0     57.0    25.0
17      0 0 0     69.0    25.0
18      0 0 0     30.0    37.0
19      0 0 0     10.0    29.0
20      0 0 0     20.0    33.0
21      0 0 0     22.5    31.0
22      0 0 0     30.0    31.0
23      0 0 0     37.5    31.0
24      0 0 0     39.75   34.0
25      0 0 0     49.5    31.0
26      0 0 0     59.25   28.0
*ENDNODES

! BEAMS LIST:
! ( beam nr. \ i-th node nr. - j-th node nr. - mass[kg/m] - EA[N] -
   EJ[Nm^2] )
*BEAMS
! Green beams
1       1    3    200    5.4e9    4.5e8
```

```
2      3    4    200    5.4e9    4.5e8
3      4    5    200    5.4e9    4.5e8
4      2    5    200    5.4e9    4.5e8
5      3    6    200    5.4e9    4.5e8
6      5    7    200    5.4e9    4.5e8
7      5    8    200    5.4e9    4.5e8
8      6    11   200    5.4e9    4.5e8
9      7    11   200    5.4e9    4.5e8
10     8    13   200    5.4e9    4.5e8
! Red beams
11     9    10   312    8.2e9    1.40e9
12     10   11   312    8.2e9    1.40e9
13     11   12   312    8.2e9    1.40e9
14     12   13   312    8.2e9    1.40e9
15     13   14   312    8.2e9    1.40e9
16     14   15   312    8.2e9    1.40e9
17     15   16   312    8.2e9    1.40e9
18     16   17   312    8.2e9    1.40e9
! Blue beams
19     9    19   90     2.4e9    2.0e8
20     19   20   90     2.4e9    2.0e8
21     20   18   90     2.4e9    2.0e8
22     11   21   90     2.4e9    2.0e8
23     21   18   90     2.4e9    2.0e8
24     18   23   90     2.4e9    2.0e8
25     23   15   90     2.4e9    2.0e8
26     18   24   90     2.4e9    2.0e8
27     24   25   90     2.4e9    2.0e8
28     25   26   90     2.4e9    2.0e8
29     26   17   90     2.4e9    2.0e8
30     13   22   90     2.4e9    2.0e8
31     22   18   90     2.4e9    2.0e8
*ENDBEAMS

! ALPHA AND BETA VALUES (DAMPING MATRIX):
! ( alpha - beta )
*DAMPING
0.1    2.0e-4

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

! RIGID BODY DATA: ATTACHED RIGID MASS AT NODE NR. 9
! ( mass nr. \ node nr. - mass[kg] - J[kgm^2] )
*MASSES
1      9    2000    320
*ENDMASSES

! SPRING AND DAMPER SYSTEMS DATA: ATTACHED DAMPER BETWEEN NODES NR. 7 AND
   13
! ( spring/damper nr. \ 1st node nr. - 2nd node number - k_x[N/m] -
   k_x[N/m] - Tk(theta)[Nm/rad] - c_x[Ns/m] - c_y[Ns/m] - Tc[Nms/rad] )
*SPRINGS
1     7   13     0   0   0     28284271.25   28284271.25   0
*ENDSPRINGS
```

### 2.6.3 Results




The effect of the damper in the system is evident. The maximum resonance peak due to the second mode of vibration was mitigate of more than 75% of its original value. Also, its amplitude was "distributed" among the other proper frequencies because the shape of vibration of the system is changed.