Supplementary Document for- "Real-World Multi-Objective Constrained Optimisation Competition"

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1. Mathematical Description of All RWCMOPs

1.1. Mechanical Design Problems

1.1.1. Pressure Vessel Design (RCM01) [1]

This constrained optimization problem contains discrete, integer, and continuous variables. This problem's main objective is to obtain the shape of a helical compression spring having the least volume.
Minimize:

$$f_1 = 1.7781 z_2 x_3^2 + 0.6224 z_1 x_3 x_4 + 3.1661 z_1^2 x_4 + 19.84 z_1^2 x_3 \tag{1}$$

$$f_2 = -\pi x_3^2 x_4 - \frac{4}{3}\pi x_3^3 \tag{2}$$

subject to :

$$g_1(\bar{x}) = 0.00954 x_3 \leq z_2,$$

$$g_2(\bar{x}) = 0.0193 x_3 \leq z_1,$$

where :

$$z_1 = 0.0625 x_1,$$
$$z_2 = 0.0625 x_2.$$

with bounds :

$$10 \leq x_4, x_3 \leq 200$$
$$1 \leq x_2, x_1 \leq 99 \text{ (integer variables)}.$$

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1.1.2. Vibrating Platform Design (RCM02) [2]

Originally vibrating platform problem is formulated as a single-objective constrained problem with the maximization of the fundamental frequency. This problem is modified to include the cost of operations as the second objective.
Minimize:

$$f_1 = -\frac{\pi}{2L^2}\sqrt{\frac{EI}{\mu}} \tag{3}$$

$$f_2 = 2bL\,(c_1 d_1 + c_2(d_2 - d_1) + c_3(d_3 - d_2)) \tag{4}$$

subject to:

$$g_1 = \mu L - 2800 \leq 0,$$

$$g_2 = d_1 - d_2 \leq 0,$$
$$g_3 = d_2 - d_1 - 0.15 \leq 0,$$
$$g_4 = d_2 - d_3 \leq 0,$$
$$g_5 = d_3 - d_2 - 0.01 \leq 0$$

where,

$$EI = \frac{2b}{3}\left(E_1 d_1^3 + E_2(d_2^3 - d_1^3) + E_3(d_3^3 - d_2^3)\right),$$

$$\mu = 2b\,(\rho_1 d_1 + \rho_2(d_2 - d_1) + \rho_3(d_3 - d_2))$$
$$\rho_1 = 100, \quad \rho_2 = 2770, \rho_3 = 7780,$$
$$E_1 = 1.6, \quad E_2 = 70, E_3 = 200,$$
$$c_1 = 500, \quad c_2 = 1500, c_3 = 800$$

with bounds:

$$0.05 \leq d_1 \leq 0.5$$
$$0.2 \leq d_2 \leq 0.5$$
$$0.2 \leq d_3 \leq 0.6$$
$$0.35 \leq b \leq 0.5$$
$$3 \leq L \leq 6$$

1.1.3. Two Bar Truss Design (RCM03) [3]

This problem involves the design of a two-bar truss. Originally, this problem is developed as a single-objective problem. The problem has been transformed into a bi-objective problem.
Minimize:

$$f_1(x) = x_1\sqrt{16 + x_3^2} + x_2\sqrt{1 + x_3^2}, \tag{5}$$

$$f_2(x) = \frac{20\sqrt{16 + x_3^2}}{x_3 x_1} \tag{6}$$

subject to:

$$g_1(x) = f_1(x) - 0.1 \leq 0,$$

$$g_2(x) = f_2(x) - 10^5 \leq 0,$$

$$g_3(x) = \frac{80\sqrt{1+x_3^2}}{x_3 x_2} - 10^5 \leq 0$$

with bounds:

$$10^{-5} \leq x_1 \leq 100,$$
$$10^{-5} \leq x_2 \leq 100,$$
$$1 \leq x_3 \leq 3$$

1.1.4. Welded Beam Design (RCM04) [4]

This problem is already well-studied as a single-objective optimization problem, where four design variables need to be optimized for which the beam's cost is minimum. However, the objective functions regarding minimum cost and maximum rigidity are conflicting with each other. Therefore, this problem is redefined as a bi-objective problem. Minimize:

$$f_1(x) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14 + x_2), \tag{7}$$

$$f_2(x) = \frac{4PL^3}{Ex_4x_3^3} \tag{8}$$

subject to:

$$g_1(x) = \tau(x) - \tau_{max} \leq 0,$$
$$g_2(x) = \sigma(x) - \sigma_{max} \leq 0,$$
$$g_3(x) = x_1 - x_4 \leq 0,$$
$$g_4(x) = P - P_c(x) \leq 0,$$

where,

$$\tau(x) = \sqrt{(\tau')^2 + \frac{2\tau'\tau'' x_2}{2R} + (\tau'')^2},$$

$$\tau' = \frac{P}{\sqrt{2}x_1x_2},$$

$$M = P\left(L + \frac{x_2}{2}\right),$$

$$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2}$$

$$J = 2\left(\sqrt{2}x_1x_2\left(\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right)\right),$$

$$\sigma(x) = \frac{6PL}{x_4x_3^2}$$

$$P_c(x) = \frac{4.013E\sqrt{\frac{x_3^2x_6^4}{36}}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right),$$

$$P = 6000,$$

$$L = 14,$$

$E = 30 \times 10^6$,

$\tau_{max} = 13600$.

$\sigma_{max} = 30,000$.

with bounds:

$0.125 \leq x_1 \leq 5,$
$0.1 \leq x_2 \leq 10,$
$0.1 \leq x_3 \leq 10,$
$0.125 \leq x_4 \leq 5.$

1.1.5. Disc Brake Design (RCM05) [5]

This design problem aims to reduce the brake weight and minimize the stopping time. The variables are the radius of internal and external disks, the force of engagement, and the number of friction surfaces. The design constraints involve the maximum break length, friction, temperature, and torque limits.
Minimize:

$$f_1(x) = 4.9 \times 10^{-5} \left(x_2^2 - x_1^2\right)(x_4 - 1), \tag{9}$$

$$f_2(x) = 9.82 \times 10^6 \left(\frac{x_2^2 - x_1^2}{x_3 x_4 \left(x_2^3 - x_1^3\right)}\right), \tag{10}$$

subject to:

$$g_1(x) = 20 - (x_2 - x_1) \leq 0,$$

$$g_2(x) = \frac{x_3}{3.14\left(x_2^2 - x_1^2\right)} - 0.4 \leq 0,$$

$$g_3(x) = \frac{2.22 \times 10^{-3} x_3 \left(x_2^3 - x_1^3\right)}{\left(x_2^2 - x_1^2\right)^2} - 1 \leq 0,$$

$$g_4(x) = 900 - 2.66 \times 10^{-2} \frac{x_3 x_4 \left(x_2^3 - x_1^3\right)}{(x_2^2 - x_1^2)} \leq 0$$

with bounds:

$55 \leq x_1 \leq 80$
$75 \leq x_2 \leq 110$
$1000 \leq x_3 \leq 3000$
$11 \leq x_4 \leq 20.$

1.1.6. Speed Reducer Design (RCM06) [6]

This problem is a bi-objective constrained optimization problem. Here, one of the constraints of the original problem is assumed an extra objective of the problem.
Minimize:

$$\begin{aligned} f_1(x) =& 0.7854 x_1 x_2^2 \left(\frac{10 x_3^2}{3} + 14.933 x_3 - 43.0934\right) \\ &- 1.508 x_1 (x_6^2 + x_7^2) + 7.477 (x_6^3 + x_7^3) \\ &+ 0.7854 (x_4 x_6^2 + x_5 x_7^2) \end{aligned} \tag{11}$$

$$f_2(x) = \frac{\sqrt{\left(\frac{745x_4}{x_2x_3}\right)^2 + 1.69 \times 10^7}}{0.1x_6^3} \tag{12}$$

subject to:

$$\begin{aligned}
g_1(x) &= \frac{1}{x_1x_2^2x_3} - \frac{1}{27} \leq 0,\\
g_2(x) &= \frac{1}{x_1x_2^2x_3^2} - \frac{1}{397.5} \leq 0,\\
g_3(x) &= \frac{x_4^3}{x_2x_3x_6^4} - \frac{1}{1.93} \leq 0,\\
g_4(x) &= \frac{x_5^3}{x_2x_3x_7^4} - \frac{1}{1.93} \leq 0,\\
g_5(x) &= x_2x_3 - 40 \leq 0,\\
g_6(x) &= \frac{x_1}{x_2} - 12 \leq 0,\\
g_7(x) &= -\frac{x_1}{x_2} + 5 \leq 0,\\
g_8(x) &= 1.9 - x_4 + 1.5x_6 \leq 0,\\
g_9(x) &= 1.9 - x_5 + 1.1x_7 \leq 0,\\
g_{10}(x) &= f_2(x) - 1300 \leq 0,\\
g_{11}(x) &= \frac{\sqrt{\left(\frac{745x_5}{x_2x_3}\right)^2 + 1.575 \times 10^8}}{0.1x_7^3} - 110 \leq 0.
\end{aligned}$$

with bounds:

$$\begin{aligned}
&2.6 \leq x_1 \leq 3.6\\
&0.7 \leq x_2 \leq 0.8\\
&x_3 \in \{17, ..., 28\}(integer)\\
&7.3 \leq x_4 \leq 8.3\\
&7.3 \leq x_5 \leq 8.3\\
&2.9 \leq x_6 \leq 3.9\\
&5 \leq x_7 \leq 5.5
\end{aligned}$$

1.1.7. Gear Train Design (RCM07) [7]

This design problem involves minimization of gears' ratio and size (inner and outer radius of gears). The ratio of gear trains can be defined as the ratio of input and output shafts' angular velocities.
Minimize:

$$f_1(x) = \left|6.931 - \frac{x_3x_4}{x_1x_2}\right|, \tag{13}$$

$$f_2(x) = max\,\{x_1, x_2.x_3, x_4\} \tag{14}$$

subject to:

$$g_1(x) = \frac{f_1(x)}{6.931} - 0.5 \leq 0$$

with bounds:

$$x_1.x_2, x_3, x_4 \in \{12, ..., 60\}(interger)$$

1.1.8. Car Side Impact Design (RCM08) [8]

This design problem is a three-objective constrained optimization problem, where all objectives are minimization type. This problem contains seven variables and 10 inequality constraints.
Minimize:

$$f_1(x) = 1.98 + 4.9x_1 6.67x_2 + 6.98x_3 + 4.01x_4 + 1.78x_5 + 10^{-5}x_6 + 2.73x_7, \tag{15}$$

$$f_2(x) = 4.72 - 0.5x_4 - 0.19x_2x_3, \tag{16}$$

$$f_3(x) = 0.5\,(V_{MBP}(x) + V_{FD}(x)) \tag{17}$$

subject to:

$$g_1(x) = -1 + 1.16 - 0.3717x_2x_4 - 0.0092928x_3 \leq 0,$$

$$g_2(x) = -0.32 + 0.261 - 0.0159x_1x_2 - 0.06486x_1 - 0.019x_2x_7 + 0,0144x_3x_5 + 0.0154464x_6 \leq 0,$$

$$g_3(x) = -0.32 + 0.74 - 0.61x_2 - 0.031296x_3 - 0.031872x_7 + 0.227x_2^2 \leq 0,$$

$$\begin{aligned} g_4(x) = &- 0.32 + 0.214 + 0.00817x_5 - 0.045195x_1 - 0.0135168x_1 + 0.03099x_2x_6 - 0.018x_2x_7 \\ &+ 0.007176x_3 + 0.023232x_3 - 0.00364x_5x_6 - 0.018x_2^2 \leq 0, \end{aligned}$$

$$g_5(x) = -32 + 33.86 + 2.95x_3 - 5.057x_1x_2 - 3.795x_2 - 3.4431x_7 + 1.45728 \leq 0,$$

$$g_6(x) = -32 + 28.98 + 3.818x_3 - 4.2x_1x_2 + 1.27296x_6 - 2.68065x_7 \leq 0,$$

$$g_7(x) = -32 + 46.36 - 9.9x_2 - 4.4505x_1 \leq 0,$$

$$g_8(x) = f_1(x) - 4 \leq 0,$$

$$g_9(x) = V_{MBP} - 9.9 \leq 0,$$

$$g_{10}(x) = V_{FD}(x) - 15.7 \leq 0$$

where,

$$V_{MBP}(x) = 10.58 - 0.674x_1x_2 - 0.67275x_2,$$

$$V_{FD}(x) = 16.45 - 0.489x_3x_7 - 0.843x_5x_6$$

with bounds:

$$0.5 \leq x_1 \leq 1.5$$
$$0.45 \leq x_2 \leq 1.35$$
$$0.5 \leq x_3 \leq 1.5$$
$$0.5 \leq x_4 \leq 1.5$$
$$0.875 \leq x_5 \leq 2.625$$
$$0.4 \leq x_6 \leq 1.2$$
$$0.4 \leq x_7 \leq 1.2$$

1.1.9. Four Bar Plane Truss (RCM09) [9]

Four Bar Plane Truss is a bi-objective bound-constrained optimization problem. This problem has four variables. Minimize:

$$f_1(x) = L\left(2x_1 + \sqrt{(2)}x_2 + \sqrt{(2)}x_3 + x_4\right), \tag{18}$$

$$f_2(x) = \frac{FL}{E}\left(\frac{2}{x_1} + \frac{2\sqrt{(2)}}{x_2} - \frac{2\sqrt{(2)}}{x_3} + \frac{2}{x_4}\right), \tag{19}$$

with bounds:

$$\begin{aligned}
&\frac{F}{\sigma} \leq x_1 \leq 3\frac{F}{\sigma}\\
&\sqrt{2}\frac{F}{\sigma} \leq x_2 \leq 3\frac{F}{\sigma}\\
&\sqrt{2}\frac{F}{\sigma} \leq x_3 \leq 3\frac{F}{\sigma}\\
&\frac{F}{\sigma} \leq x_4 \leq 3\frac{F}{\sigma}
\end{aligned}$$

where,

$$F = 10kN, \;\; E = 2 \times 10^5 kN/cm^2, \;\; L = 200cm, \;\; \sigma = 10kN/cm^2.$$

1.1.10. Two Bar Plane Truss (RCM10)

This problem is a bi-objective constrained optimization problem, where all objectives are minimization type. This problem involves two variables and two inequality constraints.
Minimize:

$$f_1(x) = 2\rho h x_2 \sqrt{1 + x_1^2}, \tag{20}$$

$$f_2(x) = \frac{\rho h(1 + x_1^2)^{1.5}(1 + x_1^4)^{0.5}}{2\sqrt{2}Ex_1^2 x_2}, \tag{21}$$

subject to:

$$\begin{aligned}
g_1 &= \frac{P(1 + x_1)(1 + x_1^2)^{0.5}}{2\sqrt{2}x_1 x_2} - \sigma_0 \leq 0,\\
g_2 &= \frac{P(-x_1 + 1)(1 + x_1^2)^{0.5}}{2\sqrt{2}x_1 x_2} - \sigma_0 \leq 0
\end{aligned}$$

with bounds:

$$\begin{aligned}
&0.1 \leq x_1 \leq 2,\\
&0.5 \leq x_2 \leq 2.5
\end{aligned}$$

where,

$$\rho = 0.283lb/in^3, \;\; h = 100in, \;\; P = 104lb, \;\; E = 3 \times 10^7 lb/in^2,$$

$$\sigma_0 = 2 \times 10^4 lb/in^2, \;\; A_{min} = 1in^2.$$

1.1.11. Water Resources Management (RCM11)

This mechanical design problem involves minimization of the five objectives of the problem by satisfying design constraints. This problem has three variables and seven inequality constraints.
Minimize:

$$f_1 = 106780.37\left(x_2 + x_3\right) + 61704.67, \tag{22}$$

$$f_2 = 3000x_1, \tag{23}$$

$$f_3 = 2.62314586 \times 10^3 x_2, \tag{24}$$

$$f_4 = 572250e^{-3.975x_2+9.9x_3+2.74}, \tag{25}$$

$$f_5 = 25\left(\frac{1.39}{x_1x_2} + 4940x_3 - 80\right). \tag{26}$$

subject to:

$$g_1 = -1 + \left(\frac{0.00139}{x_1x_2} + 4.94x_3 - 0.08\right),$$

$$g_2 = -1 + \left(\frac{0.000306}{x_1x_2} + 1.082x_3 - 0.0986\right),$$

$$g_3 = -50000 + \left(\frac{12.307}{x_1x_2} + 49408.24x_3 + 4051.02\right),$$

$$g_4 = -16000 + \left(\frac{12.098}{x_1x_2} + 8046.33x_3 - 696.71\right),$$

$$g_5 = -10000 + \left(\frac{2.138}{x_1x_2} + 7883.39x_3 - 705.04\right),$$

$$g_6 = -2000 + \left(0.417x_1x_2 + 1721.26x_3 - 136.54\right),$$

$$g_7 = -550 + \left(\frac{0.164}{x_1x_2} + 631.13x_3 - 54.48\right).$$

with bounds:

$$0.01 \le x_1 \le 0.45$$
$$0.01 \le x_2 \le 0.1$$
$$0.01 \le x_3 \le 0.1.$$

1.1.12. Simply Supported I-beam Design (RCM12) [10]

I-beam design problem includes minimization of the two objective functions of the problem through settling design constraints. This problem involves four variables and one inequality constraints.
Minimize:

$$f_1 = 2x_2x_4 + x_3\left(x_1 - 2x_4\right), \tag{27}$$

$$f_2 = \frac{PL^3}{4E\left(x_3\left(x_1 - 2x_4\right)^3 + 2x_2x_4\left(4x_4^2 + 3x_1\left(x_1 - 2x_4\right)\right)\right)}. \tag{28}$$

where,

$$P = 600, \quad L = 200, \quad E = 20000,$$

subject to:

$$g_1 = -16 + \frac{180000x_1}{x_3\left(x_1 - 2x_4\right)^3 + 2x_2x_4\left(4x_4^2 + 3x_1\left(x_1 - 2x_4\right)\right)} + \frac{15000x_2}{\left(\left(x_1 - 2x_4\right)x_3^3 + 2x_4x_2^3\right)},$$

with bounds:

$$10 \le x_1 \le 80,$$
$$10 \le x_2 \le 50,$$
$$0.9 \le x_3 \le 5,$$
$$0.9 \le x_4 \le 5.$$

1.1.13. Gear Box Design (RCM13)

Minimize:

$$\begin{aligned}
f_1 &= 0.7854x_2^2x_1\left(\frac{14.9334}{x_3} - 43.0934 + 3.3333x_3^2\right) + 0.7854(x_5x_7^2 + x_4x_6^2) - 1.508x_1(x_7^2 + x_6^2) + 7.477(x_7^3 + x_6^3)\\
f_2 &= 10x_6^{-3}\sqrt{16.91 \times 10^6 + (745x_4x_2^{-1}x_3^{-1})^2}\\
f_3 &= 10x_7^{-3}\sqrt{157.5 \times 10^6 + (745x_5x_2^{-1}x_3^{-1})^2}
\end{aligned} \tag{29}$$

subject to:

$$g_1(\bar{x}) = \frac{1}{x_1x_2^2x_3} - \frac{1}{27} \le 0,$$

$$g_2(\bar{x}) = \frac{1}{x_1x_2^2x_3^2} - \frac{1}{397.5} \le 0,$$

$$g_3(\bar{x}) = \frac{1}{x_2x_6^4x_3x_4^{-3}} - \frac{1}{1.93} \le 0,$$

$$g_4(\bar{x}) = \frac{1}{x_2x_7^4x_3x_5^{-3}} - \frac{1}{1.93} \le 0,$$

$$g_5(\bar{x}) = 10x_6^{-3}\sqrt{16.91 \times 10^6 + (745x_4x_2^{-1}x_3^{-1})^2} - 1100 \le 0,$$

$$g_6(\bar{x}) = 10x_7^{-3}\sqrt{157.5 \times 10^6 + (745x_5x_2^{-1}x_3^{-1})^2} - 850 \le 0,$$

$$g_7(\bar{x}) = x_2x_3 - 40 \le 0,$$

$$g_8(\bar{x}) = -x_1x_2^{-1} + 5 \le 0,$$

$$g_9(\bar{x}) = x_1x_2^{-1} - 12 \le 0,$$

$$g_{10}(\bar{x}) = 1.5x_6 - x_4 + 1.9 \le 0,$$

$$g_{11}(\bar{x}) = 1.1x_7 - x_5 + 1.9 \le 0,$$

with bounds:

$$0.7 \le x_2 \le 0.8, x_3 \in \{17, 28\}, 2.6 \le x_1 \le 3.6,$$

$$5 \le x_7 \le 5.5, 7.3 \le x_5, x_4 \le 8.3, 2.9 \le x_6 \le 3.9.$$

1.1.14. Multiple Disk Clutch Brake Design (RCM14) [11]

Minimize:

$$f_1 = \pi(x_2^2 - x_1^2)x_3(x_5 + 1)\rho, \tag{30}$$

$$f_2 = T. \tag{31}$$

subject to:

$$g_1(\bar{x}) = -p_{max} + p_{rz} \leq 0,$$

$$g_2(\bar{x}) = p_{rz}V_{sr} - V_{sr,max}p_{max} \leq 0,$$

$$g_3(\bar{x}) = \triangle R + x_1 - x_2 \leq 0,$$

$$g_4(\bar{x}) = -L_{max} + (x_5 + 1)(x_3 + \delta) \leq 0,$$

$$g_5(\bar{x}) = sM_s - M_h \leq 0,$$

$$g_6(\bar{x}) = T \geq 0,$$

$$g_7(\bar{x}) = -V_{sr,max} + V_{sr} \leq 0,$$

$$g_7(\bar{x}) = T - T_{max} \leq 0,$$

where,

$$M_h = \frac{2}{3}\mu x_4 x_5 \frac{x_2^3 - x_1^3}{x_2^2 - x_1^2} \text{ N.mm},$$

$$\omega = \frac{\pi n}{30} \text{ rad/s},$$

$$A = \pi(x_2^2 - x_1^2) \text{ mm}^2,$$

$$p_{rz} = \frac{x_4}{A} \text{ N/mm}^2,$$

$$V_{sr} = \frac{\pi R_{sr} n}{30} \text{ mm/s},$$

$$R_{sr} = \frac{2}{3}\frac{x_2^3 - x_1^3}{x_2^2 x_1^2} \text{ mm},$$

$$T = \frac{I_z\omega}{M_h + M_f},$$

$$\triangle R = 20 \text{ mm}, \ L_{max} = 30 \text{ mm}, \ \mu = 0.6,$$

$$V_{sr,max} = 10 \text{ m/s}, \ \delta = 0.5 \text{ mm}, \ s = 1.5,$$

$$T_{max} = 15 \text{ s}, \ n = 250 \text{ rpm}, \ I_z = 55 \text{ Kg.m}^2,$$

$$M_s = 40 \text{ Nm}, \ M_f = 3 \text{ Nm}, \text{ and } p_{max} = 1.$$

with bounds:

$$60 \leq x_1 \leq 80, \ 90 \leq x_2 \leq 110, \ 1 \leq x_3 \leq 3,$$

$$0 \leq x_4 \leq 1000, \ 2 \leq x_5 \leq 9.$$

1.1.15. Spring Design (RCM15) [1]

Minimize:

$$f_1 = \frac{\pi^2 x_2 x_3^2 (x_1+2)}{4}, \tag{32}$$

$$f_2 = \frac{8000 C_f x_2}{\pi x_3^3}. \tag{33}$$

subject to:

$$g_1(\bar{x}) = \frac{8000 C_f x_2}{\pi x_3^3} - 189000 \leq 0,$$

$$g_2(\bar{x}) = l_f - 14 \leq 0,$$

$$g_3(\bar{x}) = 0.2 - x_3 \leq 0,$$

$$g_4(\bar{x}) = x_2 - 3 \leq 0,$$

$$g_5(\bar{x}) = 3 - \frac{x_2}{x_3} \leq 0,$$

$$g_6(\bar{x}) = \sigma_p - 6 \leq 0,$$

$$g_7(\bar{x}) = \sigma_p + \frac{700}{K} + 1.05(x_1+2)x_3 - l_f \leq 0,$$

$$g_8(\bar{x}) = 1.25 - \frac{700}{K} \leq 0,$$

where,

$$C_f = \frac{4\frac{x_2}{x_3} - 1}{4\frac{x_2}{x_3} - 4} + \frac{0.615 x_3}{x_2},\ K = \frac{11.5 \times 10^6 x_3^4}{8 x_1 x_2^3},\ \sigma_p = \frac{300}{K},\ l_f = \frac{1000}{K} + 1.05(x_1+2)x_3.$$

with bounds:

$1 \leq x_1$ (integer) ≤ 70,

x_3 (discreate) $\in \{0.009, 0.0095, 0.0104, 0.0118, 0.0128, 0.0132, 0.014, 0.015, 0.0162, 0.0173, 0.018, 0.020,$
$0.023, 0.025, 0.028, 0.032, 0.035, 0.041, 0.047, 0.054, 0.063, 0.072, 0.080, 0.092, 0.0105,$
$0.120, 0.135, 0.148, 0.162, 0.177, 0.192, 0.207, 0.225, 0.244, 0.263, 0.283, 0.307, 0.0331,$
$0.362, 0.394, 0.4375, 0.500\}$

$0.6 \leq x_2$ (continuous) ≤ 3.

1.1.16. Cantilever Beam Design (RCM16) [12]

Minimize:

$$f_1 = 0.25 \rho \pi x_2 x_1^2, \tag{34}$$

$$f_2 = \frac{64 P x_2^3}{3 E \pi x_1^4} \tag{35}$$

where,

$$P = 1,\ E = 307 \times 10^8, \rho = 7800.$$

subject to:

$$g_1 = -Sy + \frac{32Px_2}{\pi x_1^3},$$

$$g_2 = -\delta_{max} + \frac{64Px_2^3}{3E\pi x_1^4}$$

where,

$$Sy = 3 \times 10^5,\ \delta_{max} = 0.05.$$

with bounds:

$$0.01 \leq x_1 \leq 0.05,\ 0.20 \leq x_2 \leq 1.$$

1.1.17. Bulk Carrier Design (RCM17) [13]

Minimize:

$$f_1 = \frac{(C_c + C_r + C_v)}{ac}, \tag{36}$$

$$f_2 = ls, \tag{37}$$

$$f_3 = -ac \tag{38}$$

where,

$$\begin{aligned}
a &= 4977.06C_B^2 - 8105.61C_B + 4456.51,\\
b &= -10847.2C_B^2 + 12817C_B - 6960.32,\\
F_n &= \frac{0.5144}{(9.8065L)^{0.5}},\\
P &= \frac{(1.025LBTC_B)^{0.67}V_k^3}{a + bF_n},\\
W_s &= 0.034L^{1.7}B^{0.6}D^{0.4}C_B^{0.5},\\
W_o &= L^{0.8}B^{0.6}D^{0.3}C_B^{0.1},\\
W_m &= 0.17P^{0.9},\\
ls &= W_s + W_o + W_m,\\
D_{wt} &= 1.025LBTC_B - ls,\\
F_c &= 4.56 \times 10^{-5}P + 0.2,\\
D_{cwt} &= D_{wt} - F_c\left(\frac{5000V_k}{24} + 5\right) - 2D_{wt}^{0.5},\\
R_{trp} &= \frac{350}{\frac{5000.*V_k}{24} + 2\left(\frac{D_{cwt}}{8000} + 0.5\right)},\\
ac &= D_{cwt}R_{trp},\\
S_d &= \frac{5000V_k}{24},\\
C_c &= 0.26\left(2000W_s^{0.85} + 3500W_o + 2400P^{0.8}\right),\\
C_r &= 40000D_{wt}^{0.3},\\
C_v &= \left(105F_cS_d + 6.3D_{wt}^{0.8}\right)R_{trp}
\end{aligned} \tag{39}$$

subject to:

$$g_1 = -\frac{L}{B} + 6,$$

$$g_1 = -15 + \frac{L}{D},$$

$$g_3 = -19 + \frac{L}{T},$$

$$g_4 = -0.45 D_{wt}^{0.31} + T,$$

$$g_5 = -0.7D - 0.7 + T,$$

$$g_6 = -0.32 + F_n,$$

$$g_7 = -0.53T - \frac{(0.085. * C_B - 0.002). * B.^2}{(TC_B)} + (1 + 0.52D) + 0.07B,$$

$$g_8 = -D_{wt} + 3000,$$

$$g_9 = -500000 + D_{wt}.$$

with bounds:

$$150 \le L \le 274.32$$
$$20 \le B \le 32.31$$
$$13 \le D \le 25$$
$$10 \le T \le 11.71$$
$$14 \le V_k \le 18$$
$$0.63 \le C_B \le 0.75$$

1.1.18. Front Rail Design (RCM18) [14]

Minimize:

$$f_1 = \frac{Ea}{E}, \tag{40}$$

$$f_2 = \frac{F}{Fa}. \tag{41}$$

where,

$$Ea = 14496.5, \quad Fa = 234.9, E = -70973.4 + 958.656w + 614.173hh - 3.827whh + 57.023wt + 63.274hht - 3.582w^2 - 1.4842hh$$

subject to:

$$g_1 = -(hh - 136)(146 - hh),$$

$$g_2 = -(w - 58)(66 - w),$$

$$g_3 = -(t - 1.4)(2.2 - t).$$

with bounds:

$$136 \le hh \le 146,$$
$$56 \le w \le 68,$$
$$1.4 \le t \le 2.2$$

1.1.19. Multi-product Batch Plant (RCM19) [15]

Minimize:

$$f_1 = \sum_{j=1}^{M} \alpha_j N_j V_j^{\beta_j}, \tag{42}$$

$$f_2 = 65\left(\frac{Q_1}{B_1} + \frac{Q_2}{B_2}\right) + 0.08Q_1 + 0.1Q_2, \tag{43}$$

$$f_3 = Q_1 \frac{T_{L1}}{B_1} + Q_2 \frac{T_{L2}}{B_2}. \tag{44}$$

subject to:

$$g_1(\bar{x}) = S_{ij}B_i - V_j \leq 0,$$

$$g_2(\bar{x}) = -H + \sum_{i=1}^{N} \frac{Q_i T_{Li}}{B_i} \leq 0,$$

$$g_3(\bar{x}) = t_{ij} - N_j T_{Li} \leq 0,$$

with bounds:

$$1 \leq N_i \leq N_j^u,$$
$$V_j^l \leq V_j \leq V_j^u,$$
$$T_{Li}^l \leq T_{Li} \leq T_{Li}^u,$$
$$B_j^l \leq B_j \leq B_j^u.$$

where, $N = 2$, $M = 3$, $\alpha_j = 250$, $H = 6000$, $\beta_j = 0.6$, $N_j^u = 3$, $V_j^l = 250$, and $V_j^u = 2500$. The value of other parameters are calculated by

$$T_{Li}^l = \max\left(\frac{t_{ij}}{N_j^u}\right), \tag{45}$$

$$T_{Li}^u = \max\left(t_{ij}\right), \tag{46}$$

$$B_j^l = \frac{Q_i^* T_{Li}}{H}, \tag{47}$$

$$B_j^u = \min\left(Q_i, \min_j\left(\frac{V_j^u}{S_{ij}}\right)\right) \tag{48}$$

Parameters S_{ij} and t_{ij} are given in Table 1.

Table 1: Values of S_{ij} and t_{ij}.

S_{ij}				t_{ij}		
2	3	4		8	20	8
4	6	3		16	4	4

1.1.20. Hydro-static Thrust Bearing Design (RCM20) [16]

Minimize:

$$f_1 = \left(\frac{QP_0}{0.7} + E_f\right)\frac{1}{12}, \tag{49}$$

$$f_2 = \frac{0.0307}{386.4P_0}\frac{Q}{2\pi Rh}. \tag{50}$$

subject to:

$$g_1(\bar{x}) = 1000 - P_0 \leq 0,$$

$$g_2(\bar{x}) = W - 101000 \leq 0,$$

$$g_3(\bar{x}) = 5000 - \frac{W}{\pi(R^2 - R_0^2)} \leq 0,$$

$$g_4(\bar{x}) = 50 - P_0 \leq 0,$$

$$g_5(\bar{x}) = 0.001 - \frac{0.0307}{386.4P_0}\left(\frac{Q}{2\pi Rh}\right) \leq 0,$$

$$g_6(\bar{x}) = R - R_0 \leq 0,$$

$$g_7(\bar{x}) = h - 0.001 \leq 0,$$

where,

$$W = \frac{\pi P_0}{2}\frac{R^2 - R_0^2}{ln\left(\frac{R}{R_0}\right)},\ P_0 = \frac{6\mu Q}{\pi h^3} ln\left(\frac{R}{R_0}\right),$$

$$E_f = 9336Q \times 0.0307 \times 0.5\triangle T,\ \triangle T = 2(10^P - 559.7),$$

$$P = \frac{log_{10}log_{10}\left(8.122 \times 10^6\mu + 0.8\right) + 3.55}{10.04},$$

$$h = \left(\frac{2\pi \times 750}{60}\right)^2 \frac{2\pi\mu}{E_f}\left(\frac{R^4}{4} - \frac{R_0^4}{4}\right)$$

with bounds:

$$1 \leq R \leq 16,\ 1 \leq R_0 \leq 16,$$

$$1 \times 10^{-6} \leq \mu \leq 16 \times 10^{-6},\ 1 \leq Q \leq 16.$$

1.1.21. Crash Energy Management for High-speed Train (RCM21) [17]

Minimize:

$$\begin{aligned}
f_1 =&1.3667145844797 - 0.00904459793976106x_1 - 0.0016193573938033x_2 - 0.00758531275221425x_3\\
&- 0.00440727360327102x_4 - 0.00572216860791644x_5 - 0.00936039926190721x_6 + 2.62510221107328\\
&\times 10^{-6}(x_1^2) + 4.92982681358861 \times 10^{-7}(x_2^2) + 2.25524989067108 \times 10^{-6}\\
&(x_3^2) + 1.84605439400301 \times 10^{-6}(x_4^2) + 2.17175358243416 \times 10^{-6}(x_5^2)\\
&+ 3.90158043948054 \times 10^{-6}(x_6^2) + 4.55276994245781 \times 10^{-7}x_1x_2 - 6.37013576290982\\
&\times 10^{-7}x_1x_3 + 8.26736480446359 \times 10^{-7}x_1x_4 + 5.66352809442276 \times 10^{-8}x_1x_5\\
&- 3.20213897443278 \times 10^{-7}x_1x_6 + 1.18015467772812 \times 10^{-8}x_2x_3 + 9.25820391546515\\
&\times 10^{-8}x_2x_4 - 1.05705364119837 \times 10^{-7}x_2x_5 - 4.74797783014687 \times 10^{-7}x_2x_6\\
&- 5.02319867013788 \times 10^{-7}x_3x_4 + 9.54284258085225 \times 10^{-7}x_3x_5 + 1.80533309229454\\
&\times 10^{-7}x_3x_6 - 1.07938022118477 \times 10^{-6}x_4x_5 - 1.81370642220182 \times 10^{-7}x_4x_6\\
&- 2.24238851688047 \times 10^{-7}x_5x_6,
\end{aligned} \tag{51}$$

$$\begin{aligned}
f_2 =&- 1.19896668942683 + 3.04107017009774x_1 + 1.23535701600191x_2 + 2.13882039381528x_3\\
&+ 2.33495178382303x_4 + 2.68632494801975x_5 + 3.43918953617606x_6 - 7.89144544980703\\
&\times 10^{-4}(x_1^2) - 2.06085185698215 \times 10^{-4}(x_2^2) - 7.15269900037858\\
&\times 10^{-4}(x_3^2) - 7.8449237573837 \times 10^{-4}(x_4^2) - 9.31396896237177\\
&\times 10^{-4}(x_5^2) - 1.40826531972195 \times 10^{-3}(x_6^2) - 1.60434988248392\\
&\times 10^{-4}x_1x_2 + 2.0824655419411 \times 10^{-4}x_1x_3 - 3.0530659653553 \times 10^{-4}\\
&x_1x_4 - 8.10145973591615 \times 10^{-5}x_1x_5 + 6.94728759651311 \times 10^{-5}x_1x_6\\
&+ 1.18015467772812 \times 10^{-8}x_2x_3 + 9.25820391546515 \times 10^{-8}x_2x_4\\
&- 1.05705364119837 \times 10^{-7}x_2x_5 + 1.69935290196781 \times 10^{-4}x_2x_6\\
&+ 2.32421829190088 \times 10^{-5}x_3x_4 - 2.0808624041163476 \times 10^{-4}x_3x_5\\
&+ 1.75576341867273 \times 10^{-5}x_3x_6 + 2.68422081654044 \times 10^{-4}x_4x_5\\
&+ 4.39852066801981 \times 10^{-5}x_4x_6 + 2.96785446021357 \times 10^{-5}x_5x_6,
\end{aligned} \tag{52}$$

subject to:

$$\begin{aligned}
g_1 &= f_1 - 5,\\
g_2 &= -f_1,\\
g_3 &= f_2 - 28,\\
g_4 &= -f_2
\end{aligned}$$

with bounds:

$$\begin{aligned}
1.3 &\le x_1 \le 1.7\\
2.5 &\le x_2 \le 3.5\\
1.3 &\le x_3 \le 1.7\\
1.3 &\le x_4 \le 1.7\\
1.3 &\le x_5 \le 1.7\\
1.3 &\le x_6 \le 1.7
\end{aligned}$$

1.2. Chemical Engineering Problems

The practice in chemical engineering includes a variety of non-linear multi-objective constrained optimization problems. Process equipment configuration relationships and mass equations, and heat balance establish non-linearity in the problems. A variety of extremely complex and non-linear chemical process problems were suggested due to various non-linear models. In this work, the following problems are taken into account.

1.2.1. Haverly's Pooling Problem (RCM22) [18]

Minimize:

$$f_1 = -9x_1 - 15x_2 + 6x_3 + 16x_4, \tag{53}$$

$$f_2 = 10(x_5 + x_6). \tag{54}$$

subject to:

$$h_1(\bar{x}) = x_7 + x_8 - x_4 - x_3 = 0,$$

$$h_2(\bar{x}) = x_1 - x_5 - x_7 = 0,$$
$$h_3(\bar{x}) = x_2 - x_6 - x_8 = 0,$$
$$h_4(\bar{x}) = x_9x_7 + x_9x_8 - 3x_3 - x_4 = 0,$$
$$g_1(\bar{x}) = x_9x_7 + 2x_5 - 2.5x_1 \leq 0,$$
$$g_2(\bar{x}) = x_9x_8 + 2x_6 - 1.5x_2 \leq 0,$$

with bounds:

$$0 \leq x_1, x_3, x_4, x_5, x_6, x_8 \leq 100, 0 \leq x_2, x_7, x_9 \leq 200.$$

1.2.2. Reactor Network Design (RCM23) [19]

Minimize:

$$f_1 = -x_4, \tag{55}$$

$$f_2 = x_5^{0.5} + x_6^{0.5}. \tag{56}$$

subject to:

$$h_1(\bar{x}) = k_1x_5x_2 + x_1 - 1 = 0,$$

$$h_2(\bar{x}) = k_3x_5x_3 + x_3 + x_1 - 1 = 0,$$
$$h_3(\bar{x}) = k_2x_6x_2 - x_1 + x_2 = 0,$$
$$h_4(\bar{x}) = k_4x_6x_4 + x_2 - x_1 + x_4 - x_3 = 0,$$
$$g_1(\bar{x}) = x_5^{0.5} + x_6^{0.5} \leq 4$$

with bounds:

$$0 \leq x_4, x_3, x_2, x_1 \leq 1,$$
$$0.00001 \leq x_6, x_5 \leq 16.$$

where, $k_3 = 0.0391908$, $k_4 = 0.9k_3$, $k_1 = 0.09755988$, and $k_2 = 0.99k_1$.

1.2.3. Heat Exchanger Nwteork Design (RCM24) [20]

Minimize:

$$f_1 = 35x_1^{0.6} + 35x_2^{0.6}, \tag{57}$$

$$f_2 = 200x_1x_4 - x3, \tag{58}$$

$$f_3 = 200x_1x_6 - x_5. \tag{59}$$

subject to:

$$h_1(\bar{x}) = 200x_1x_4 - x_3 = 0,$$

$$h_2(\bar{x}) = 200x_2x_6 - x_5 = 0,$$

$$h_3(\bar{x}) = x_3 - 10000(x_7 - 100) = 0,$$

$$h_4(\bar{x}) = x_5 - 10000(300 - x_7) = 0,$$

$$h_5(\bar{x}) = x_3 - 10000(600 - x_8) = 0,$$

$$h_6(\bar{x}) = x_5 - 10000(900 - x_9) = 0,$$

$$h_7(\bar{x}) = x_4\ln(x_8 - 100) - x_4\ln(600 - x_7) - x_8 + x_7 + 500 = 0,$$

$$h_8(\bar{x}) = x_6\ln(x_9 - x_7) - x_6\ln(600) - x_9 + x_7 + 600 = 0$$

with bounds:

$$0 \leq x_1 \leq 10, 0 \leq x_2 \leq 200, 0 \leq x_3 \leq 100, 0 \leq x_4 \leq 200,$$

$$1000 \leq x_5 \leq 2000000, 0 \leq x_6 \leq 600, 100 \leq x_7 \leq 600, 100 \leq x_8 \leq 600,$$

$$100 \leq x_9 \leq 900.$$

1.3. Process, Design and Synthesis Problems

In chemical engineering, process design and synthesis problems are defined as a mixed-integer nonlinear multi-objective constrained optimization problem.

1.3.1. Process Synthesis Problem (RCM25) [21]

Minimize:

$$f_1 = x_2 + 2x_1, \tag{60}$$

$$f_2 = -x_1^2 - x_2 \tag{61}$$

subject to:

$$g_1(\bar{x}) = -x_1^2 - x_2 + 1.25 \leq 0,$$

$$g_2(\bar{x}) = x_1 + x_2 \leq 1.6.$$

with bounds:

$$0 \leq x_1 \leq 1.6$$

$$x_2 \in \{0, 1\}$$

1.3.2. Process Synthesis and Design Problem (RCM26) [22]

Minimize:

$$f_1 = -x_3 + x_2 + 2x_1, \tag{62}$$

$$f_2 = -x_1^2 - x_2 + x_1 x_3. \tag{63}$$

subject to:

$$h_1(\bar{x}) = -2\exp(-x_2) + x_1 = 0,$$

$$g_1(\bar{x}) = x_2 - x_1 + x_3 \leq 0.$$

with bounds:

$$0.5 \leq x_1, x_2 \leq 1.4,$$
$$x_3 \in \{0, 1\}.$$

1.3.3. Process Flow Sheeting Problem (RCM27) [23]

Minimize:

$$f_1 = -0.7x_3 + 0.8 + 5(0.5 - x_1)^2, \tag{64}$$

$$f_2 = x_1 - x_3. \tag{65}$$

subject to:

$$g_1(\bar{x}) = -exp(x_1 - 0.2) - x_2 \leq 0,$$

$$g_2(\bar{x}) = x_2 + 1.1x_3 \leq -1.0,$$

$$g_3(\bar{x}) = x_1 - x_3 \leq 0.2.$$

with bounds:

$$-2.22554 \leq x_2 \leq -1,\ 0.2 \leq x_1 \leq 1,$$
$$x_3 \in \{0, 1\}.$$

1.3.4. Two Reactor Problem (RCM28) [21]

Minimize:

$$f_1 = 7.5x_7 + 5.5x_8 + 7x_5 + 6x_6 + 5(x_1 + x_2), \tag{66}$$

$$f_2 = x_1 + x_2. \tag{67}$$

subject to:

$$h_1(\bar{x}) = x_7 + x_8 - 1 = 0,$$

$$h_2(\bar{x}) = x_3 - 0.9(1 - \exp(0.5x_5))x_1 = 0,$$

$$h_3(\bar{x}) = x_4 - 0.8(1 - \exp(0.4x_6))x_2 = 0,$$

$$h_4(\bar{x}) = x_3 + x_4 - 10 = 0,$$

$$h_5(\bar{x}) = x_3 x_7 + x_4 x_8 - 10 = 0,$$

$$g_1(\bar{x}) = x_5 - 10x_7 \leq 0,$$
$$g_2(\bar{x}) = x_6 - 10x_8 \leq 0,$$
$$g_3(\bar{x}) = x_1 - 20x_7 \leq 0,$$
$$g_4(\bar{x}) = x_2 - 20x_8 \leq 0$$

with bounds:

$$0 \leq x_6, x_5, x_4, x_3, x_2, x_1 \leq 100$$
$$x_8, x_7 \in \{0, 1\}.$$

1.3.5. Process Synthesis Problem (RCM29) [21]

Minimize:

$$f_1 = (1 - x_4)^2 + (1 - x_5)^2 + (1 - x_6)^2 - \ln(1 + x_7) + (1 - x_1)^2 + (2 - x_2)^2 + (3 - x_3)^2, \quad (68)$$

$$f_2 = (1 - x_1)^2 + (2 - x_2)^2 + (3 - x_3)^2. \quad (69)$$

subject to:

$$g_1(\bar{x}) = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 - 5 \leq 0,$$
$$g_2(\bar{x}) = x_6^3 + x_1^2 + x_2^2 + x_3^2 = 5.5 \leq 0,$$
$$g_3(\bar{x}) = x_1 + x_4 - 1.2 \leq 0,$$
$$g_4(\bar{x}) = x_2 + x_5 - 1.8 \leq 0,$$
$$g_5(\bar{x}) = x_3 + x_6 - 2.5 \leq 0,$$
$$g_6(\bar{x}) = x_1 + x_7 - 1.2 \leq 0,$$
$$g_7(\bar{x}) = x_5^2 + x_2^2 - 1.64 \leq 0,$$
$$g_8(\bar{x}) = x_6^2 + x_3^2 - 4.25 \leq 0,$$
$$g_9(\bar{x}) = x_5^2 + x_3^2 - 4.64 \leq 0,$$

with bounds:

$$0 \leq x_2, x_3, x_1 \leq 100,$$
$$x_7, x_6, x_5, x_4 \in \{0, 1\}.$$

1.4. Power Electronics Problems

In power electronic engineering, the synchronous optimum pulse-width module is a rising tool for controlling medium voltage drives. It decreases the switching frequency dramatically without increasing the distortion. It thus reduces the lack of switching that increases the inverter's efficiency. This problem can be defined as a bi-objective constrained optimization problem. The following problems are considered in this work.

1.4.1. Synchronous Optimal Pulse-width Modulation of 3-level Inverters (RCM30) [24]

Minimize:

$$f_1 = \frac{\sqrt{\sum_k (k^{-4})(\sum_{i=1}^{N} s(i)cos(kx_i))^2}}{\sqrt{\sum_k k^{-4}}} \tag{70}$$

$$f_2 = \left(m - \sum_{i=1}^{N} s(i)cos(x_i)\right)^2 \tag{71}$$

where, $k = 5, 7, 11, 13.....97$, $N = \lfloor \frac{f_{s,max}}{f.m} \rfloor$, and $s(i) = (-1)^{i+1}$
subject to:

$$x_{i+1} - x_i - 10^{-5} > 0,\ i = 1, 2, ...N - 1,$$

$$0 < x_i < \frac{\pi}{2},\ , i = 1, 2, ...N.$$

1.4.2. Synchronous Optimal Pulse-width Modulation of 5-level Inverters (RCM31) [25]

Minimize:

$$f_1 = \frac{\sqrt{\sum_k (k^{-4})(\sum_{i=1}^{N} s(i)cos(kx_i))^2}}{2\sqrt{\sum_k k^{-4}}} \tag{72}$$

$$f_2 = \left(2m - \sum_{i=1}^{N} s(i)cos(x_i)\right)^2 \tag{73}$$

where, $k = 5, 7, 11, 13.....97$, $N = \lfloor \frac{2.f_{s,max}}{f.m} \rfloor$, and $s = [1, -1, 1, 1, -1, 1, -1, 1, -1, -1]$
subject to:

$$x_{i+1} - x_i - 10^{-5} > 0,\ i = 1, 2, ...N - 1,$$

$$0 < x_i < \frac{\pi}{2},\ , i = 1, 2, ...N.$$

1.4.3. Synchronous Optimal Pulse-width Modulation of 7-level Inverters (RCM32) [26]

Minimize:

$$f_1 = \frac{\sqrt{\sum_k (k^{-4})(\sum_{i=1}^{N} s(i)cos(kx_i))^2}}{3\sqrt{\sum_k k^{-4}}} \tag{74}$$

$$f_2 = \left(3m - \sum_{i=1}^{N} s(i)cos(x_i)\right)^2 \tag{75}$$

where, $k = 5, 7, 11, 13.....97$, $N = \lfloor \frac{3.f_{s,max}}{f.m} \rfloor$, and $s = [1, -1, 1, 1, 1, -1, -1, -1, 1, 1, -1]$
subject to:

$$x_{i+1} - x_i - 10^{-5} > 0,\ i = 1, 2, ...N - 1,$$

$$0 < x_i < \frac{\pi}{2},\ , i = 1, 2, ...N.$$

1.4.4. Synchronous Optimal Pulse-width Modulation of 9-level Inverters (RCM33) [27]

Minimize:

$$f_1 = \frac{\sqrt{\sum_k (k^{-4})(\sum_{i=1}^{N} s(i)cos(kx_i))^2}}{4\sqrt{\sum_k k^{-4}}} \tag{76}$$

$$f_2 = \left(4m - \sum_{i=1}^{N} s(i)cos(x_i)\right)^2 \tag{77}$$

where, $k = 5, 7, 11, 13.....97$, $N = \lfloor \frac{4.f_{s,max}}{f.m} \rfloor$, and $s = [1, 1, 1, 1, -1, 1, -1 - 1, -1, 1, -1, -1]$
subject to:

$$x_{i+1} - x_i - 10^{-5} > 0,\ i = 1, 2, ...N - 1,$$

$$0 < x_i < \frac{\pi}{2},\ , i = 1, 2, ...N.$$

1.4.5. Synchronous Optimal Pulse-width Modulation of 11-level Inverters (RCM34) [28]

Minimize:

$$f_2 = \frac{\sqrt{\sum_k (k^{-4})(\sum_{i=1}^{N} s(i)cos(kx_i))^2}}{5\sqrt{\sum_k k^{-4}}} \tag{78}$$

$$f_2 = \left(5m - \sum_{i=1}^{N} s(i)cos(x_i)\right)^2 \tag{79}$$

where, $k = 5, 7, 11, 13.....97$, $N = \lfloor \frac{5.f_{s,max}}{f.m} \rfloor$, and $s = [1, -1, 1, 1, 1, -1, -1, -1, 1, 1, 1, 1]$
subject to:

$$x_{i+1} - x_i - 10^{-5} > 0,\ i = 1, 2, ...N - 1,$$

$$0 < x_i < \frac{\pi}{2},\ , i = 1, 2, ...N.$$

1.4.6. Synchronous Optimal Pulse-width Modulation of 13-level Inverters (RCM35) [28]

Minimize:

$$f_1 = \frac{\sqrt{\sum_k (k^{-4})(\sum_{i=1}^{N} s(i)cos(k\alpha_i))^2}}{6\sqrt{\sum_k k^{-4}}} \tag{80}$$

$$f_2 = \left(6m - \sum_{i=1}^{N} s(i)cos(\alpha_i)\right)^2 \tag{81}$$

where, $k = 5, 7, 11, 13.....97$, $N = \lfloor \frac{6.f_{s,max}}{f.m} \rfloor$, and $s = [1, 1, 1, -1, 1, -1, 1, -1, 1, 1, 1, 1]$
subject to:

$$\alpha_{i+1} - \alpha_i - 10^{-5} > 0,\ i = 1, 2, ...N - 1,$$
$$0 < \alpha_i < \frac{\pi}{2},\ ,i = 1, 2, ...N.$$

1.5. Power System Optimization Problems

1.5.1. Optimal Sizing of Single Phase Distribution Generation with Reactive Power support for phase balancing at Main Transformer/Grid and Minimizing Active Power Loss (RCM36) [29]

Minimize:

$$\begin{aligned} f_1 =& \left(I^a_{r,1} + I^b_{r,1} + I^c_{r,1}\right)^2 + \left(I^a_{m,1} + I^b_{m,1} + I^c_{m,1}\right)^2 \\ &+ \left(I^a_{r,1} - 0.5\left(I^b_{r,1} + I^c_{r,1}\right) - 0.5\sqrt{3}\left(I^b_{m,1} - I^c_{m,1}\right)\right)^2 \\ &+ \left(I^a_{m,1} - 0.5\left(I^b_{m,1} + I^c_{m,1}\right) + 0.5\sqrt{3}\left(I^b_{r,1} - I^c_{r,1}\right)\right)^2, \end{aligned} \tag{82}$$

$$f_2 = \sum_{i=1}^{N} \sum_{j\in\{a,b,c\}} P^j_i$$

where,

$$I^s_{r,1} = \sum_{k\in\{a,b,c\}} \sum_{i=1}^{N} \left(G^{sk}_{1,i} V^k_{r,i} - B^{sk}_{1i} V^k_{m,i}\right)$$

$$I^s_{m,1} = \sum_{k\in\{a,b,c\}} \sum_{i=1}^{N} \left(B^{sk}_{1,i} V^k_{r,i} + G^{sk}_{1i} V^k_{m,i}\right)$$

subject to:

$$\sum_{s\in\{a,b,c\}} \sum_{i=1}^{N} \left(G^{js}_{k,i} V^s_{r,i} - B^{js}_{ki} V^s_{m,i}\right) - \frac{P^j_k V^j_{r,k} + Q^j_k V^j_{m,k}}{(V^j_{r,k})^2 + (V^j_{m,k})^2} = 0,\ k = 1, ...N \text{ and } j = \{a, b, c\},$$
$$\sum_{s\in\{a,b,c\}} \sum_{i=1}^{N} (B^{js}_{ki} V^s_{r,i} + G^{js}_{ki} V^s_{m,i}) - \frac{P^j_k V^j_{m,k} - Q^j_k V^j_{r,k}}{(V^j_{r,k})^2 + (V^j_{m,k})^2} = 0,\ k = 1, ...N \text{ and } j = \{a, b, c\},$$
$$P^j_k - P^j_{dg,k} + P^j_{l,k} = 0,\ k = 1, ...N \text{ and } j = \{a, b, c\},$$
$$Q^j_k - Q^j_{dg,k} + Q^j_{l,k} = 0,\ k = 1, ...N \text{ and } j = \{a, b, c\},$$

$V_{min} \leq V^j_{r,k}, V^j_{m,k} ...k = 1, 2, ...N$ and $j = \{a, b, c\} \leq V_{max}$

$P_{min} \leq P^j_k ....k = 1, 2, ...N$ and $j = \{a, b, c\} \leq P_{max}$

$Q_{min} \leq Q^j_k ....k = 1, 2, ...N$ and $j = \{a, b, c\} \leq Q_{max}$

$P_{dg,min} \leq P^j_{dg,k} ....k = 1, 2, ...N$ and $j = \{a, b, c\} \leq P_{dg,max}$

$Q_{dg,min} \leq Q^j_{dg,k} ....k = 1, 2, ...N$ and $j = \{a, b, c\} \leq Q_{dg,max}$

where P^j_i and Q^j_i represent the active and reactive injected power, respectively, at i-th bus in j-th phase, $Ybus^{st}_{ij}(= G^{st}_{ij} + 1jB^{st}_{ij})$ is ij-th element of st-th block of admittance matrix, $V^j_i(= V^j_{r,i} + 1jV^j_{m,i})$ is bus voltage at i-th bus in j-th phase, $P^j_{dg,k}$ and $Q^j_{dg,k}$ represent the active and reactive power generation, respectively, at k-th DG in j-th phase and N represents the total number of buses in system.

1.5.2. Optimal Sizing of Single Phase Distribution Generation with Reactive Power support for phase balancing at Main Transformer/Grid and Minimizing Reactive Power Loss (RCM37) [29]

Minimize:

$$\begin{aligned} f_1 =& \left(I^a_{r,1} + I^b_{r,1} + I^c_{r,1}\right)^2 + \left(I^a_{m,1} + I^b_{m,1} + I^c_{m,1}\right)^2 \\ &+ \left(I^a_{r,1} - 0.5\left(I^b_{r,1} + I^c_{r,1}\right) - 0.5\sqrt{3}\left(I^b_{m,1} - I^c_{m,1}\right)\right)^2 \\ &+ \left(I^a_{m,1} - 0.5\left(I^b_{m,1} + I^c_{m,1}\right) + 0.5\sqrt{3}\left(I^b_{r,1} - I^c_{r,1}\right)\right)^2, \end{aligned} \tag{83}$$

$$f_2 = \sum_{i=1}^{N} \sum_{j\in\{a,b,c\}} Q^j_i$$

where,

$$I^s_{r,1} = \sum_{k\in\{a,b,c\}} \sum_{i=1}^{N} \left(G^{sk}_{1,i} V^k_{r,i} - B^{sk}_{1i} V^k_{m,i}\right)$$

$$I^s_{m,1} = \sum_{k\in\{a,b,c\}} \sum_{i=1}^{N} \left(B^{sk}_{1,i} V^k_{r,i} + G^{sk}_{1i} V^k_{m,i}\right)$$

subject to:

$$\sum_{s\in\{a,b,c\}} \sum_{i=1}^{N} \left(G^{js}_{k,i} V^s_{r,i} - B^{js}_{ki} V^s_{m,i}\right) - \frac{P^j_k V^j_{r,k} + Q^j_k V^j_{m,k}}{(V^j_{r,k})^2 + (V^j_{m,k})^2} = 0,\ k = 1, ...N \text{ and } j = \{a, b, c\},$$

$$\sum_{s\in\{a,b,c\}} \sum_{i=1}^{N} (B^{js}_{ki} V^s_{r,i} + G^{js}_{ki} V^s_{m,i}) - \frac{P^j_k V^j_{m,k} - Q^j_k V^j_{r,k}}{(V^j_{r,k})^2 + (V^j_{m,k})^2} = 0,\ k = 1, ...N \text{ and } j = \{a, b, c\},$$

$$P^j_k - P^j_{dg,k} + P^j_{l,k} = 0,\ k = 1, ...N \text{ and } j = \{a, b, c\},$$

$$Q^j_k - Q^j_{dg,k} + Q^j_{l,k} = 0,\ k = 1, ...N \text{ and } j = \{a, b, c\},$$

$V_{min} \leq V^j_{r,k}, V^j_{m,k} ...k = 1, 2, ...N$ and $j = \{a, b, c\} \leq V_{max}$

$P_{min} \leq P^j_k ....k = 1, 2, ...N$ and $j = \{a, b, c\} \leq P_{max}$

$Q_{min} \leq Q^j_k ....k = 1, 2, ...N$ and $j = \{a, b, c\} \leq Q_{max}$

$P_{dg,min} \leq P^{j}_{dg,k}......k = 1, 2, ...N$ and $j = \{a, b, c\} \leq P_{dg,max}$

$Q_{dg,min} \leq Q^{j}_{dg,k}......k = 1, 2, ...N$ and $j = \{a, b, c\} \leq Q_{dg,max}$

where P^j_i and Q^j_i represent the active and reactive injected power, respectively, at i-th bus in j-th phase, $Ybus^{st}_{ij}(= G^{st}_{ij} + 1jB^{st}_{ij})$ is ij-th element of st-th block of admittance matrix, $V^j_i(= V^j_{r,i} + 1jV^j_{m,i})$ is bus voltage at i-th bus in j-th phase, $P^j_{dg,k}$ and $Q^j_{dg,k}$ represent the active and reactive power generation, respectively, at k-th DG in j-th phase and N represents the total number of buses in system.

1.5.3. Optimal Sizing of Single Phase Distribution Generation with Reactive Power support for Minimizing Active and Reactive Power Loss (RCM38) [29]

Minimize:

$$\begin{aligned} f_1 =& \left(I^a_{r,1} + I^b_{r,1} + I^c_{r,1}\right)^2 + \left(I^a_{m,1} + I^b_{m,1} + I^c_{m,1}\right)^2 \\ &+ \left(I^a_{r,1} - 0.5\left(I^b_{r,1} + I^c_{r,1}\right) - 0.5\sqrt{3}\left(I^b_{m,1} - I^c_{m,1}\right)\right)^2 \\ &+ \left(I^a_{m,1} - 0.5\left(I^b_{m,1} + I^c_{m,1}\right) + 0.5\sqrt{3}\left(I^b_{r,1} - I^c_{r,1}\right)\right)^2, \end{aligned} \tag{84}$$

$$f_2 = \sum_{i=1}^{N} \sum_{j\in\{a,b,c\}} P^j_i$$

$$f_3 = \sum_{i=1}^{N} \sum_{j\in\{a,b,c\}} Q^j_i$$

where,

$$I^s_{r,1} = \sum_{k\in\{a,b,c\}} \sum_{i=1}^{N} \left(G^{sk}_{1,i} V^k_{r,i} - B^{sk}_{1i} V^k_{m,i}\right)$$

$$I^s_{m,1} = \sum_{k\in\{a,b,c\}} \sum_{i=1}^{N} \left(B^{sk}_{1,i} V^k_{r,i} + G^{sk}_{1i} V^k_{m,i}\right)$$

subject to:

$$\sum_{s\in\{a,b,c\}} \sum_{i=1}^{N} \left(G^{js}_{k,i} V^s_{r,i} - B^{js}_{ki} V^s_{m,i}\right) - \frac{P^j_k V^j_{r,k} + Q^j_k V^j_{m,k}}{(V^j_{r,k})^2 + (V^j_{m,k})^2} = 0,\ k = 1, ...N \text{ and } j = \{a, b, c\},$$

$$\sum_{s\in\{a,b,c\}} \sum_{i=1}^{N} (B^{js}_{ki} V^s_{r,i} + G^{js}_{ki} V^s_{m,i}) - \frac{P^j_k V^j_{m,k} - Q^j_k V^j_{r,k}}{(V^j_{r,k})^2 + (V^j_{m,k})^2} = 0,\ k = 1, ...N \text{ and } j = \{a, b, c\},$$

$$P^j_k - P^j_{dg,k} + P^j_{l,k} = 0,\ k = 1, ...N \text{ and } j = \{a, b, c\},$$

$$Q^j_k - Q^j_{dg,k} + Q^j_{l,k} = 0,\ k = 1, ...N \text{ and } j = \{a, b, c\},$$

$V_{min} \leq V^j_{r,k}, V^j_{m,k}...k = 1, 2, ...N$ and $j = \{a, b, c\} \leq V_{max}$

$P_{min} \leq P^j_k......k = 1, 2, ...N$ and $j = \{a, b, c\} \leq P_{max}$

$Q_{min} \leq Q^j_k......k = 1, 2, ...N$ and $j = \{a, b, c\} \leq Q_{max}$

$P_{dg,min} \leq P^j_{dg,k}......k = 1, 2, ...N$ and $j = \{a, b, c\} \leq P_{dg,max}$

$Q_{dg,min} \leq Q^j_{dg,k}$......$k = 1, 2, ...N$ and $j = \{a, b, c\} \leq Q_{dg,max}$

where P^j_i and Q^j_i represent the active and reactive injected power, respectively, at i-th bus in j-th phase, $Ybus^{st}_{ij}(= G^{st}_{ij} + 1jB^{st}_{ij})$ is ij-th element of st-th block of admittance matrix, $V^j_i(= V^j_{r,i} + 1jV^j_{m,i})$ is bus voltage at i-th bus in j-th phase, $P^j_{dg,k}$ and $Q^j_{dg,k}$ represent the active and reactive power generation, respectively, at k-th DG in j-th phase and N represents the total number of buses in system.

1.5.4. Optimal Sizing of Single Phase Distribution Generation with Reactive Power support for phase balancing at Main Transformer/Grid and Minimizing Active and Reactive Power Loss (RCM39) [29]

Minimize:

$$f_1 = \sum_{i=1}^{N} \sum_{j \in \{a,b,c\}} P^j_i$$

$$f_2 = \sum_{i=1}^{N} \sum_{j \in \{a,b,c\}} Q^j_i$$

where,

$$I^s_{r,1} = \sum_{k \in \{a,b,c\}} \sum_{i=1}^{N} \left(G^{sk}_{1,i} V^k_{r,i} - B^{sk}_{1i} V^k_{m,i} \right)$$

$$I^s_{m,1} = \sum_{k \in \{a,b,c\}} \sum_{i=1}^{N} \left(B^{sk}_{1,i} V^k_{r,i} + G^{sk}_{1i} V^k_{m,i} \right)$$

subject to:

$$\sum_{s \in \{a,b,c\}} \sum_{i=1}^{N} \left(G^{js}_{k,i} V^s_{r,i} - B^{js}_{ki} V^s_{m,i} \right) - \frac{P^j_k V^j_{r,k} + Q^j_k V^j_{m,k}}{(V^j_{r,k})^2 + (V^j_{m,k})^2} = 0, \; k = 1, ...N \text{ and } j = \{a, b, c\},$$

$$\sum_{s \in \{a,b,c\}} \sum_{i=1}^{N} (B^{js}_{ki} V^s_{r,i} + G^{js}_{ki} V^s_{m,i}) - \frac{P^j_k V^j_{m,k} - Q^j_k V^j_{r,k}}{(V^j_{r,k})^2 + (V^j_{m,k})^2} = 0, \; k = 1, ...N \text{ and } j = \{a, b, c\},$$

$$P^j_k - P^j_{dg,k} + P^j_{l,k} = 0, \; k = 1, ...N \text{ and } j = \{a, b, c\},$$

$$Q^j_k - Q^j_{dg,k} + Q^j_{l,k} = 0, \; k = 1, ...N \text{ and } j = \{a, b, c\},$$

$V_{min} \leq V^j_{r,k}, V^j_{m,k}$...$k = 1, 2, ...N$ and $j = \{a, b, c\} \leq V_{max}$

$P_{min} \leq P^j_k$.......$k = 1, 2, ...N$ and $j = \{a, b, c\} \leq P_{max}$

$Q_{min} \leq Q^j_k$.......$k = 1, 2, ...N$ and $j = \{a, b, c\} \leq Q_{max}$

$P_{dg,min} \leq P^j_{dg,k}$......$k = 1, 2, ...N$ and $j = \{a, b, c\} \leq P_{dg,max}$

$Q_{dg,min} \leq Q^j_{dg,k}$......$k = 1, 2, ...N$ and $j = \{a, b, c\} \leq Q_{dg,max}$

where P^j_i and Q^j_i represent the active and reactive injected power, respectively, at i-th bus in j-th phase, $Ybus^{st}_{ij}(= G^{st}_{ij} + 1jB^{st}_{ij})$ is ij-th element of st-th block of admittance matrix, $V^j_i(= V^j_{r,i} + 1jV^j_{m,i})$ is bus voltage at i-th bus in j-th phase, $P^j_{dg,k}$ and $Q^j_{dg,k}$ represent the active and reactive power generation, respectively, at k-th DG in j-th phase and N represents the total number of buses in system.

1.5.5. Optimal Power Flow for Minimizing Active and Reactive Power Loss (RCM40) [30]

Minimize:

$$f_1 = \sum_{i=1}^{N} P_i \tag{85}$$

$$f_2 = \sum_{i=1}^{N} Q_i \tag{86}$$

subject to:

$$\begin{aligned}
&\sum_{i=1}^{N}(G_{k,i}V_{r,i} - B_{ki}V_{m,i}) - \frac{P_k V_{r,k} + Q_k V_{m,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,\ k = 1, ...N,\\
&\sum_{i=1}^{N}(B_{ki}V_{r,i} + G_{ki}V_{m,i}) - \frac{P_k V_{m,k} - Q_k V_{r,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,\ k = 1, ...N,\\
&P_k - P_{dg,k} + P_{l,k} = 0,\ k = 1, ...N,\\
&Q_k + Q_{l,k} = 0,\ k = 1, ...N,\\
&V_{min} \leq V_{r,k}, V_{m,k} ...k = 1, 2, ...N \leq V_{max}\\
&P_{min} \leq P_k ....k = 1, 2, ...N \leq P_{max}\\
&Q_{min} \leq Q_k ....k = 1, 2, ...N \leq Q_{max}\\
&P_{min,dg} \leq P_{dg,k} ....k = 1, 2, ...N \leq P_{max,dg}
\end{aligned}$$

where P_i and Q_i represent the active and reactive injected power, respectively, at i-th bus, $Ybus_{ij}(= G_{ij} + 1jB_{ij})$ is ij-th element of admittance matrix, $V_i(= V_{r,i} + 1jV_{m,i})$ is bus voltage at i-th bus, $P_{dg,k}$ represents the active power generation of DG at $k-$th bus and N represents the total number of buses in system.

1.5.6. Optimal Power Flow for Minimizing Voltage deviation, Active and Reactive Power Loss (RCM41) [30]

Minimize:

$$f_1 = \sum_{i=1}^{N} P_i \tag{87}$$

$$f_2 = \sum_{i=1}^{N} Q_i \tag{88}$$

$$f_3 = \sum_{i=1}^{N} (1 - |V_i|) \tag{89}$$

subject to:

$$\begin{aligned}
&\sum_{i=1}^{N}(G_{k,i}V_{r,i} - B_{ki}V_{m,i}) - \frac{P_k V_{r,k} + Q_k V_{m,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,\ k = 1, ...N,\\
&\sum_{i=1}^{N}(B_{ki}V_{r,i} + G_{ki}V_{m,i}) - \frac{P_k V_{m,k} - Q_k V_{r,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,\ k = 1, ...N,\\
&P_k - P_{dg,k} + P_{l,k} = 0,\ k = 1, ...N,\\
&Q_k + Q_{l,k} = 0,\ k = 1, ...N,
\end{aligned}$$

$V_{min} \le V_{r,k}, V_{m,k} ...k = 1, 2, ...N \le V_{max}$

$P_{min} \le P_k ....k = 1, 2, ...N \le P_{max}$

$Q_{min} \le Q_k ....k = 1, 2, ...N \le Q_{max}$

$P_{min,dg} \le P_{dg,k} ....k = 1, 2, ...N \le P_{max,dg}$

where P_i and Q_i represent the active and reactive injected power, respectively, at i-th bus, $Ybus_{ij}(= G_{ij} + 1jB_{ij})$ is ij-th element of admittance matrix, $V_i(= V_{r,i} + 1jV_{m,i})$ is bus voltage at i-th bus, $P_{dg,k}$ represents the active power generation of DG at $k-$th bus and N represents the total number of buses in system.

1.5.7. Optimal Power Flow for Minimizing Voltage deviation, and Active Power Loss (RCM42) [30]

Minimize:

$$f_1 = \sum_{i=1}^{N} P_i \tag{90}$$

$$f_2 = \sum_{i=1}^{N} (1 - |V_i|) \tag{91}$$

subject to:

$$\sum_{i=1}^{N} (G_{k,i} V_{r,i} - B_{ki} V_{m,i}) - \frac{P_k V_{r,k} + Q_k V_{m,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,\ k = 1, ...N,$$

$$\sum_{i=1}^{N} (B_{ki} V_{r,i} + G_{ki} V_{m,i}) - \frac{P_k V_{m,k} - Q_k V_{r,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,\ k = 1, ...N,$$

$$P_k - P_{dg,k} + P_{l,k} = 0,\ k = 1, ...N,$$

$$Q_k + Q_{l,k} = 0,\ k = 1, ...N,$$

$V_{min} \le V_{r,k}, V_{m,k} ...k = 1, 2, ...N \le V_{max}$

$P_{min} \le P_k ....k = 1, 2, ...N \le P_{max}$

$Q_{min} \le Q_k ....k = 1, 2, ...N \le Q_{max}$

$P_{min,dg} \le P_{dg,k} ....k = 1, 2, ...N \le P_{max,dg}$

where P_i and Q_i represent the active and reactive injected power, respectively, at i-th bus, $Ybus_{ij}(= G_{ij} + 1jB_{ij})$ is ij-th element of admittance matrix, $V_i(= V_{r,i} + 1jV_{m,i})$ is bus voltage at i-th bus, $P_{dg,k}$ represents the active power generation of DG at $k-$th bus and N represents the total number of buses in system.

1.5.8. Optimal Power Flow for Minimizing Fuel Cost, and Active Power Loss (RCM43) [30]

Minimize:

$$f_1 = \sum_{i=1}^{N} \left(P_{g,i} - P_{l,i}\right) \tag{92}$$

$$f_2 = \sum_{i=1}^{N} \left(a_i + b_i P_{g,i} + c_i P_{g,i}^2\right) \tag{93}$$

where a_i, b_i, and c_i are the cost coefficient of i-th bus generator,
subject to:

$$P_{g,i} - P_{l,i} - V_i \sum_{j=1}^{N} V_j \left(G_{ij}cos(\delta_i - \delta_j) + B_{ij}sin(\delta_i - \delta_j)\right) = 0, \; i = 1, 2, ...N,$$

$$Q_{g,i} - Q_{l,i} - V_i \sum_{j=1}^{N} V_j \left(G_{ij}sin(\delta_i - \delta_j) - B_{ij}cos(\delta_i - \delta_j)\right) = 0, \; i = 1, 2, ...N,$$

$$V_{min} \leq V_k...k = 1, 2, ..N \leq V_{max},$$

$$\delta_{min} \leq \delta_k...k = 1, 2, ..N \leq \delta_{max},$$

$$P_{min} \leq P_{g,k}.......k = 1, 2, ...N \leq P_{max}$$

$$Q_{min} \leq Q_{g,k}.......k = 1, 2, ...N \leq Q_{max}$$

where $P_i(= P_{g,i} - P_{l,i})$ and $Q_i(= Q_{g,i} - Q_{l,i})$ represent the active and reactive injected power, respectively, at i-th bus, $Ybus_{ij}(= G_{ij} + 1jB_{ij})$ is ij-th element of admittance matrix, $\mathrm{V}_i(= V_i\angle\delta_i)$ is bus voltage at i-th bus and N represents the total number of buses in system.

1.5.9. Optimal Power Flow for Minimizing Fuel Cost, Active and Reactive Power Loss (RCM44) [30]

Minimize:

$$f_1 = \sum_{i=1}^{N} \left(P_{g,i} - P_{l,i}\right) \tag{94}$$

$$f_2 = \sum_{i=1}^{N} \left(Q_{g,i} - Q_{l,i}\right) \tag{95}$$

$$f_3 = \sum_{i=1}^{N} \left(a_i + b_iP_{g,i} + c_iP_{g,i}^2\right) \tag{96}$$

where a_i, b_i, and c_i are the cost coefficient of i-th bus generator,
subject to:

$$P_{g,i} - P_{l,i} - V_i \sum_{j=1}^{N} V_j \left(G_{ij}cos(\delta_i - \delta_j) + B_{ij}sin(\delta_i - \delta_j)\right) = 0, \; i = 1, 2, ...N,$$

$$Q_{g,i} - Q_{l,i} - V_i \sum_{j=1}^{N} V_j \left(G_{ij}sin(\delta_i - \delta_j) - B_{ij}cos(\delta_i - \delta_j)\right) = 0, \; i = 1, 2, ...N,$$

$$V_{min} \leq V_k...k = 1, 2, ..N \leq V_{max},$$

$$\delta_{min} \leq \delta_k...k = 1, 2, ..N \leq \delta_{max},$$

$$P_{min} \leq P_{g,k}.......k = 1, 2, ...N \leq P_{max}$$

$$Q_{min} \leq Q_{g,k}.......k = 1, 2, ...N \leq Q_{max}$$

where $P_i(= P_{g,i} - P_{l,i})$ and $Q_i(= Q_{g,i} - Q_{l,i})$ represent the active and reactive injected power, respectively, at i-th bus, $Ybus_{ij}(= G_{ij} + 1jB_{ij})$ is ij-th element of admittance matrix, $\mathrm{V}_i(= V_i\angle\delta_i)$ is bus voltage at i-th bus and N represents the total number of buses in system.

1.5.10. Optimal Power Flow for Minimizing Fuel Cost, Voltage deviation, and Active Power Loss (RCM45) [30]

Minimize:

$$f_1 = \sum_{i=1}^{N} \left(P_{g,i} - P_{l,i}\right) \tag{97}$$

$$f_2 = \sum_{i=1}^{N} (1 - |V_i|) \tag{98}$$

$$f_3 = \sum_{i=1}^{N} \left(a_i + b_i P_{g,i} + c_i P_{g,i}^2\right) \tag{99}$$

where a_i, b_i, and c_i are the cost coefficient of i-th bus generator,
subject to:

$$P_{g,i} - P_{l,i} - V_i \sum_{j=1}^{N} V_j \left(G_{ij} cos(\delta_i - \delta_j) + B_{ij} sin(\delta_i - \delta_j)\right) = 0,\ i = 1, 2, ...N,$$

$$Q_{g,i} - Q_{l,i} - V_i \sum_{j=1}^{N} V_j \left(G_{ij} sin(\delta_i - \delta_j) - B_{ij} cos(\delta_i - \delta_j)\right) = 0,\ i = 1, 2, ...N,$$

$$V_{min} \leq V_k ...k = 1, 2, ..N \leq V_{max},$$

$$\delta_{min} \leq \delta_k ...k = 1, 2, ..N \leq \delta_{max},$$

$$P_{min} \leq P_{g,k} ....k = 1, 2, ...N \leq P_{max}$$

$$Q_{min} \leq Q_{g,k} ....k = 1, 2, ...N \leq Q_{max}$$

where $P_i(= P_{g,i} - P_{l,i})$ and $Q_i(= Q_{g,i} - Q_{l,i})$ represent the active and reactive injected power, respectively, at i-th bus, $Ybus_{ij}(= G_{ij} + 1jB_{ij})$ is ij-th element of admittance matrix, V$_i(= V_i \angle \delta_i)$ is bus voltage at i-th bus and N represents the total number of buses in system.

1.5.11. Optimal Power Flow for Minimizing Fuel Cost, Voltage deviation, Active and Reactive Power Loss (RCM46) [30]

Minimize:

$$f_1 = \sum_{i=1}^{N} \left(P_{g,i} - P_{l,i}\right) \tag{100}$$

$$f_2 = \sum_{i=1}^{N} \left(Q_{g,i} - Q_{l,i}\right) \tag{101}$$

$$f_3 = \sum_{i=1}^{N} \left(a_i + b_i P_{g,i} + c_i P_{g,i}^2\right) \tag{102}$$

where a_i, b_i, and c_i are the cost coefficient of i-th bus generator,

$$f_4 = \sum_{i=1}^{N} (1 - |V_i|) \tag{103}$$

subject to:

$$P_{g,i} - P_{l,i} - V_i \sum_{j=1}^{N} V_j \left(G_{ij} cos(\delta_i - \delta_j) + B_{ij} sin(\delta_i - \delta_j) \right) = 0,\ i = 1, 2, ...N,$$

$$Q_{g,i} - Q_{l,i} - V_i \sum_{j=1}^{N} V_j \left(G_{ij} sin(\delta_i - \delta_j) - B_{ij} cos(\delta_i - \delta_j) \right) = 0,\ i = 1, 2, ...N,$$

$$V_{min} \leq V_k ... k = 1, 2, ..N \leq V_{max},$$

$$\delta_{min} \leq \delta_k ... k = 1, 2, ..N \leq \delta_{max},$$

$$P_{min} \leq P_{g,k} .... k = 1, 2, ...N \leq P_{max}$$

$$Q_{min} \leq Q_{g,k} .... k = 1, 2, ...N \leq Q_{max}$$

where $P_i (= P_{g,i} - P_{l,i})$ and $Q_i (= Q_{g,i} - Q_{l,i})$ represent the active and reactive injected power, respectively, at i-th bus, $Ybus_{ij} (= G_{ij} + 1jB_{ij})$ is ij-th element of admittance matrix, $V_i (= V_i \angle \delta_i)$ is bus voltage at i-th bus and N represents the total number of buses in system.

1.5.12. Optimal Droop Setting for Minimizing Active and Reactive Power Loss (RCM47) [31]

Minimize:

$$f_1 = \sum_{i=1}^{N} P_i \tag{104}$$

$$f_2 = \sum_{i=1}^{N} Q_i \tag{105}$$

subject to:

$$\sum_{i=1}^{N} (G_{k,i} V_{r,i} - B_{ki} V_{m,i}) - \frac{P_k V_{r,k} + Q_k V_{m,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,\ k = 1, ...N,$$

$$\sum_{i=1}^{N} (B_{ki} V_{r,i} + G_{ki} V_{m,i}) - \frac{P_k V_{m,k} - Q_k V_{r,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,\ k = 1, ...N,$$

$$P_k - Cp_k (w_k^* - w) + P_{l,k} = 0,\ k = 1, ...N,$$

$$Q_k - Cq_k \left(V_k^* - \sqrt{(V_{r,k})^2 + (V_{m,k})^2} \right) + Q_{l,k} = \quad 0,\ k = 1, ...N,$$

$$V_{min} \leq V_{r,k}, V_{m,k} ... k = 1, 2, ...N \leq V_{max}$$

$$P_{min} \leq P_k .... k = 1, 2, ...N \leq P_{max}$$

$$Q_{min} \leq Q_k .... k = 1, 2, ...N \leq Q_{max}$$

$$Cp_{min,k} \leq Cp_k .... k = 1, 2, ...N \leq Cp_{max,k}$$

$$Cq_{min,k} \leq Cq_k .... k = 1, 2, ...N \leq Cq_{max,k}$$

$$w_{min} \leq w \leq w_{max}$$

where P_i and Q_i represent the active and reactive injected power, respectively, at i-th bus, $Ybus_{ij} (= G_{ij} + 1jB_{ij})$ is ij-th element of admittance matrix, $V_i (= V_{r,i} + 1jV_{m,i})$ is bus voltage at i-th bus, Cp_k and Cq_k represent the active and reactive power droop parameters of controllers, respectively, w is operating frequency and N represents the total number of buses in system.

1.5.13. Optimal Droop Setting for Minimizing Voltage Deviation and Active Power Loss (RCM48) [32]

Minimize:

$$f_1 = \sum_{i=1}^{N} P_i \tag{106}$$

$$f_2 = \sum_{i=1}^{N} \left(1 - |V_i|)^2\right) \tag{107}$$

subject to:

$$\sum_{i=1}^{N}(G_{k,i}V_{r,i} - B_{ki}V_{m,i}) - \frac{P_k V_{r,k} + Q_k V_{m,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,\ k = 1, ...N,$$
$$\sum_{i=1}^{N}(B_{ki}V_{r,i} + G_{ki}V_{m,i}) - \frac{P_k V_{m,k} - Q_k V_{r,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,\ k = 1, ...N,$$
$$P_k - Cp_k(w_k^* - w) + P_{l,k} = 0,\ k = 1, ...N,$$
$$Q_k - Cq_k\left(V_k^* - \sqrt{(V_{r,k})^2 + (V_{m,k})^2}\right) + Q_{l,k} = \quad 0,\ k = 1, ...N,$$
$$V_{min} \leq V_{r,k}, V_{m,k}...k = 1, 2, ...N \leq V_{max}$$
$$P_{min} \leq P_k.......k = 1, 2, ...N \leq P_{max}$$
$$Q_{min} \leq Q_k.......k = 1, 2, ...N \leq Q_{max}$$
$$Cp_{min,k} \leq Cp_k.......k = 1, 2, ...N \leq Cp_{max,k}$$
$$Cq_{min,k} \leq Cq_k.......k = 1, 2, ...N \leq Cq_{max,k}$$
$$w_{min} \leq w \leq w_{max}$$

where P_i and Q_i represent the active and reactive injected power, respectively, at i-th bus, $Ybus_{ij}(= G_{ij} + 1jB_{ij})$ is ij-th element of admittance matrix, $V_i(= V_{r,i} + 1jV_{m,i})$ is bus voltage at i-th bus, Cp_k and Cq_k represent the active and reactive power droop parameters of controllers, respectively, w is operating frequency and N represents the total number of buses in system.

1.5.14. Optimal Droop Setting for Minimizing Voltage Deviation, Active, and Reactive Power Loss (RCM49) [33]

Minimize:

$$f_1 = \sum_{i=1}^{N} P_i \tag{108}$$

$$f_2 = \sum_{i=1}^{N} Q_i \tag{109}$$

$$f_3 = \sum_{i=1}^{N} \left(1 - |V_i|)^2\right) \tag{110}$$

subject to:

$$\sum_{i=1}^{N}(G_{k,i}V_{r,i} - B_{ki}V_{m,i}) - \frac{P_k V_{r,k} + Q_k V_{m,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,\ k = 1, ...N,$$

$$\sum_{i=1}^{N}(B_{ki}V_{r,i} + G_{ki}V_{m,i}) - \frac{P_k V_{m,k} - Q_k V_{r,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,\ k = 1, ...N,$$

$$P_k - Cp_k(w_k^* - w) + P_{l,k} = 0,\ k = 1, ...N,$$

$$Q_k - Cq_k\left(V_k^* - \sqrt{(V_{r,k})^2 + (V_{m,k})^2}\right) + Q_{l,k} = \qquad 0,\ k = 1, ...N,$$

$$V_{min} \leq V_{r,k}, V_{m,k}...k = 1, 2, ...N \leq V_{max}$$

$$P_{min} \leq P_k.......k = 1, 2, ...N \leq P_{max}$$

$$Q_{min} \leq Q_k.......k = 1, 2, ...N \leq Q_{max}$$

$$Cp_{min,k} \leq Cp_k.......k = 1, 2, ...N \leq Cp_{max,k}$$

$$Cq_{min,k} \leq Cq_k.......k = 1, 2, ...N \leq Cq_{max,k}$$

$$w_{min} \leq w \leq w_{max}$$

where P_i and Q_i represent the active and reactive injected power, respectively, at i-th bus, $Ybus_{ij}(= G_{ij} + 1jB_{ij})$ is ij-th element of admittance matrix, $V_i(= V_{r,i} + 1jV_{m,i})$ is bus voltage at i-th bus, Cp_k and Cq_k represent the active and reactive power droop parameters of controllers, respectively, w is operating frequency and N represents the total number of buses in system.

1.5.15. Power Distribution System Planning (RCM50) [34]

Minimize:

$$f_1 = \sum_{i=1}^{6}\left(a_i + b_i x_i + c_i x_i^2\right), \tag{111}$$

$$f_2 = \sum_{i=1}^{6}\left(\alpha_i + \beta_i x_i + \gamma_i x_i^2\right) \tag{112}$$

where,

i	a_i	b_i	c_i	α_i	β_i	γ_i
1	756.7988	38.5390	0.15247	13.8593	0.32767	0.00419
2	451.3251	46.1591	0.10587	13.8593	0.32767	0.00419
3	1243.5311	38.3055	0.03546	40.2669	-0.54551	0.00683
4	1049.9977	40.3965	0.02803	40.2669	-0.54551	0.00683
5	1356.6592	38.2704	0.01799	42.8955	-0.51116	0.00461
6	1658.5696	36.3278	0.02111	42.8955	-0.51116	0.00461

Subject to:

$$h_1 = \sum_{i=1}^{6}(x_i - PD - PL)$$

where,

$$PD = 12000$$

$$PL = \sum_{i=1}^{6}\sum_{j=1}^{6}\left(x_i x_j B_{ij} \times 10^{-6}\right)$$

B_{ij}	1	2	3	4	5	6
1	140	17	15	19	26	22
2	17	60	13	16	15	20
3	15	13	65	17	24	19
4	19	16	17	71	30	25
5	26	15	24	30	69	32
6	22	20	19	25	32	85

Supplementary Tables of Main Manuscript

Table S1: Baseline results of mechanical design problems (RCM01-RCM08).

Problem		ToP [35]	TiGE_2 [36]	cNSGAIII [8]	cMOEA/D [8]	CCMO [37]	cARMOEA [38]	AnD [39]
RCM01	HV_best	0.606717	0.538106	0.60758	0.108992	0.605144	0.607879	0.603085
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.605832	0.510526	0.606391	0.108888	0.603743	0.606639	0.599144
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.604649	0.455252	0.603023	0.108025	0.601598	0.605276	0.593941
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.000511	0.021003	0.00096	0.000173	0.000855	0.000742	0.001756
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM02	HV_best	0.16646	0.134328	0.166457	0.176394	0.166382	0.16642	0.16519
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.149764	0.021593	0.053488	0.052203	0.063709	0.035963	0.038431
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0	0	0	0	0	0	0
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.049921	0.037817	0.067281	0.069297	0.072199	0.059449	0.061729
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM03	HV_best	0.901391	0.851123	0.896894	0.298947	0.898968	0.898881	0.89865
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.781179	0.687886	0.891818	0.120981	0.89715	0.897511	0.897299
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.236575	0.285117	0.889927	0.089525	0.895395	0.895233	0.894421
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.194751	0.146899	0.00162	0.044027	0.000984	0.001035	0.000992
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM04	HV_best	0.861896	0.729125	0.861236	0.088416	0.859299	0.860785	0.858811
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.861354	0.472491	0.853826	0.014423	0.853488	0.852793	0.85333
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.860162	0.254392	0.840075	0	0.826305	0.831737	0.835492
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.000332	0.140715	0.006158	0.028662	0.007333	0.007443	0.005027
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM05	HV_best	0.434245	0.412378	0.434245	0.427847	0.434311	0.434516	0.432729
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.433843	0.396852	0.432672	0.420501	0.432985	0.433091	0.430543
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.433401	0.369139	0.428064	0.400399	0.427832	0.428895	0.427071
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.000194	0.011186	0.001367	0.006833	0.001467	0.001447	0.00132
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM06	HV_best	0.277233	0.274287	0.277145	0.276696	0.27738	0.277163	0.276948
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.274139	0.271504	0.276964	0.276548	0.276606	0.277025	0.276551
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.231964	0.26809	0.276253	0.273695	0.269111	0.276878	0.275255
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.008451	0.001677	0.00018	0.000531	0.001431	9.42E-05	0.000281
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM07	HV_best	0.226953	0.215711	0.226861	0.222935	0.226971	0.227019	0.225198
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.22671	0.199665	0.225854	0.220981	0.226712	0.226378	0.224103
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.226375	0.106722	0.222861	0.215165	0.226341	0.225192	0.222241
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.000117	0.023012	0.000815	0.001425	0.000148	0.00045	0.000637
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM08	HV_best	0.025865	0.021195	0.025616	0.013168	0.025976	0.026053	0.026062
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.025617	0.020437	0.025358	0.009369	0.025828	0.02591	0.02584
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.025388	0.020236	0.024966	0.008119	0.02547	0.025616	0.025259
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.000109	0.00024	0.000164	0.001043	9.65E-05	0.00012	0.000155
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100

Table S2: Baseline results of mechanical design problems RC09-RC16

Problem		ToP [35]	TiGE_2 [36]	cNSGAIII [8]	cMOEA/D [8]	CCMO [37]	cARMOEA [38]	AnD [39]
RCM09	HV_best	0.408889	0.358098	0.409689	0.05316	0.408948	0.40966	0.408287
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.40851	0.323399	0.409477	0.053057	0.40864	0.409568	0.407496
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.408112	0.295263	0.409166	0.052973	0.408196	0.409307	0.4066
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.000169	0.01341	0.00012	3.86E-05	0.000177	6.48E-05	0.000418
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM10	HV_best	0.847364	0.84456	0.837576	0.080044	0.842495	0.843914	0.845939
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.847146	0.840968	0.833455	0.079487	0.839439	0.841241	0.844971
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.846424	0.834233	0.832868	0.078753	0.832774	0.837581	0.843441
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.000182	0.00205	0.001023	0.000419	0.002253	0.002155	0.000598
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM11	HV_best	0.09929	0.098926	0.100442	0.061329	0.099962	0.099998	0.099662
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.097266	0.097945	0.099746	0.060353	0.099165	0.097146	0.098868
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.094654	0.094981	0.099195	0.059309	0.098126	0.092464	0.098031
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.001183	0.000851	0.000348	0.000505	0.000456	0.001526	0.000394
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM12	HV_best	0.723422	0.711611	0.722742	0.101861	0.722206	0.722755	0.720785
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.723073	0.698009	0.721768	0.064495	0.719671	0.722192	0.718469
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.722601	0.668655	0.720212	0.012029	0.714833	0.719849	0.713855
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.000244	0.00993	0.000578	0.02106	0.002199	0.000579	0.001523
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM13	HV_best	0.089673	0.088687	0.090348	0.090343	0.08925	0.090421	0.090365
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.089243	0.086669	0.090125	0.090201	0.088845	0.090296	0.090291
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.088436	0.085158	0.089524	0.089093	0.088388	0.089942	0.090166
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.000267	0.000699	0.000205	0.000226	0.000184	0.000108	5.68E-05
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM14	HV_best	0.617478	0.495242	0.617891	0.172877	0.61637	0.618066	0.61418
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.616706	0.330275	0.61628	0.120625	0.614199	0.616625	0.606465
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.616112	0.097988	0.610967	0.076989	0.61144	0.612748	0.587042
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.000328	0.10877	0.001446	0.024785	0.001396	0.0013	0.004093
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM15	HV_best	0.543199	0.521655	0.542396	0.24499	0.540387	0.542542	0.541475
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.542927	0.509063	0.540606	0.071978	0.535172	0.54117	0.53927
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.542363	0.480435	0.536591	0.065994	0.516745	0.539697	0.537395
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.000178	0.009041	0.001018	0.032127	0.006226	0.00072	0.000915
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM16	HV_best	0.763379	0.752025	0.762657	0.079343	0.762161	0.762473	0.761334
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.762977	0.742237	0.762404	0.079087	0.761688	0.762449	0.758998
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.761876	0.708289	0.762299	0.079055	0.760799	0.76243	0.754586
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.000306	0.008109	7.92E-05	5.67E-05	0.000303	1.29E-05	0.001495
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100

Table S3: Baseline results of mechanical design problems (RC17-RC21).

Problem		ToP [35]	TiGE_2 [36]	cNSGAIII [8]	cMOEA/D [8]	CCMO [37]	cARMOEA [38]	AnD [39]
RCM17	HV_best	0.343355	0.329025	0.272668	0.300459	0.34287	0.27689	0.275577
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.265468	0.20413	0.247039	0.196528	0.271101	0.253003	0.209068
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.04228	0.086825	0.190024	0.100199	0.227364	0.239668	0.058186
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.059101	0.059935	0.017716	0.055155	0.031926	0.007278	0.042127
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM18	HV_best	0.040475	0.03988	0.040508	0.040316	0.0405	0.040509	0.040469
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.040463	0.039305	0.040504	0.040259	0.040494	0.040507	0.040435
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.04045	0.038049	0.040492	0.04019	0.040487	0.040499	0.04024
	CV_worst	0	0	0	0	0	0	0
	HV_sd	5.86E-06	0.000445	4.25E-06	2.44E-05	3.61E-06	2.01E-06	4.10E-05
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM19	HV_best	0.332801	0.301435	0.30792	0.244087	0.306362	0.303361	0.304245
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.284636	0.277989	0.284733	0.171237	0.281467	0.280489	0.284262
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.15376	0.257503	0.254717	0.087701	0.218761	0.254298	0.270568
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.05626	0.011308	0.009991	0.039532	0.016711	0.011653	0.00897
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM20	HV_best	0.207864	0.129792	0.179272	0	0.163509	0.055214	0.179222
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.108881	0.024426	0.138998	0	0.114169	0.003069	0.115754
	CV_mean	0.000401	63.24543	0.000186	0	0	0	0
	HV_worst	0	0	0	0	0	0	0
	CV_worst	0.012044	1897.363	0.00557	0	0	0	0
	HV_sd	0.083168	0.042439	0.037054	0	0.049259	0.011727	0.043542
	CV_sd	0.002162	340.5871	0.001	0	0	0	0
	FR	96.66667	96.66667	96.66667	100	100	100	100
RCM21	HV_best	0.031753	0.028646	0.031757	0.02933	0.031753	0.031758	0.031749
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.03175	0.021239	0.031711	0.029322	0.0317	0.03167	0.031706
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.031748	0.019965	0.031405	0.029317	0.030878	0.030542	0.030975
	CV_worst	0	0	0	0	0	0	0
	HV_sd	1.48E-06	0.002286	7.14E-05	3.16E-06	0.000177	0.000237	0.000141
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100

Table S4: Baseline results of chemical engineering problems (RC22-RC24).

Problem		ToP [35]	TiGE_2 [36]	cNSGAIII [8]	cMOEA/D [8]	CCMO [37]	cARMOEA [38]	AnD [39]
RCM22	HV_best	0	0	0	0	0	0	0
	CV_best	1.280592	0.010923	0.024605	0.001647	0.014998	0.011363	0.021321
	HV_mean	0	0	0	0	0	0	0
	CV_mean	29.05799	4.663478	3.022009	2.202614	8.295304	7.225637	6.29337
	HV_worst	0	0	0	0	0	0	0
	CV_worst	81.36874	25.05874	16.5864	14.88096	30.08038	49.2384	28.30793
	HV_sd	0	0	0	0	0	0	0
	CV_sd	21.68135	6.241971	4.173321	3.3434	9.066057	10.30937	7.585384
	FR	0	0	0	0	0	0	0
RCM23	HV_best	0.998563	0.990669	0.467709	0.689092	0.447149	0.577967	0.487608
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.033285	0.456674	0.14435	0.175798	0.0611	0.138216	0.108034
	CV_mean	0.039073	1.73E-05	3.36E-05	1.42E-05	0.00019	3.65E-05	5.11E-05
	HV_worst	0	0	0	0	0	0	0
	CV_worst	0.347136	0.000377	0.000253	0.000194	0.000827	0.000186	0.000265
	HV_sd	0.179247	0.279928	0.158177	0.163518	0.115609	0.172869	0.139781
	CV_sd	0.082742	6.91E-05	5.95E-05	3.99E-05	0.000252	5.78E-05	7.39E-05
	FR	3.333333	90	56.66667	73.33333	26.66667	50	43.33333
RCM24	HV_best	0	2.86E-08	0	0	0	0	0
	CV_best	165.3936	0	0.003802	0.166729	6.838311	0.675435	0.518473
	HV_mean	0	1.02E-05	0	0	0	0	0
	CV_mean	251582.2	1.317266	137.2594	81.94926	631.407	257.567	144.8416
	HV_worst	0	0	0	0	0	0	0
	CV_worst	878333.6	11.63664	861.3564	413.3518	5123.33	1715.384	630.6579
	HV_sd	0	5.14E-08	0	0	0	0	0
	CV_sd	303702.5	2.781827	198.909	99.4577	1050.21	406.839	148.7518
	FR	0	10	0	0	0	0	0

Table S5: Baseline results of process design and synthesis problems (RCM25-RCM29).

Problem		ToP [35]	TiGE_2 [36]	cNSGAIII [8]	cMOEA/D [8]	CCMO [37]	cARMOEA [38]	AnD [39]
RCM25	HV_best	0.240906	0.217111	0.241086	0.237308	0.241187	0.241118	0.240922
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.24077	0.198929	0.24106	0.236795	0.241164	0.240776	0.240808
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.240643	0.164597	0.241033	0.236569	0.241122	0.23134	0.240572
	CV_worst	0	0	0	0	0	0	0
	HV_sd	7.29E-05	0.01378	1.30E-05	0.000215	1.19E-05	0.001752	8.04E-05
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM26	HV_best	0.188765	0.166356	0.188171	0.194468	0.20437	0.200758	0.200818
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.155657	0.124086	0.152925	0.144699	0.154502	0.159318	0.144786
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.110303	0.090911	0.095389	0.116597	0.091719	0.117439	0.093144
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.019194	0.019445	0.028217	0.018733	0.029585	0.023262	0.030851
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM27	HV_best	0.718792	0.795056	0.719229	0.721782	0.719622	0.719229	0.721013
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.718359	0.747882	0.719224	0.721672	0.71956	0.719221	0.717436
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.717772	0.690259	0.719218	0.721631	0.719482	0.719213	0.714571
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.000262	0.0291	2.35E-06	2.41E-05	3.67E-05	4.54E-06	0.001573
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM28	HV_best	0	0.07228	0	0.070562	0	0	0
	CV_best	0.014281	0	0.9999	0	0.000635	0.00145	0.000234
	HV_mean	0	0.03953	0	0.004974	0	0	0
	CV_mean	0.936119	8.03E-09	1.000205	0.899936	0.933887	0.933675	0.967071
	HV_worst	0	0	0	0	0	0	0
	CV_worst	1.019421	2.41E-07	1.002147	1.000507	1.002939	1.00318	1.003175
	HV_sd	0	0.018531	0	0.015634	0	0	0
	CV_sd	0.244706	4.32E-08	0.000465	0.299979	0.249196	0.2491	0.179539
	FR	0	96.66667	0	10	0	0	0
RCM29	HV_best	0.471774	0.442004	0.51602	0.518846	0.504352	0.520551	0.517712
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.335559	0.390516	0.447617	0.452675	0.404413	0.432106	0.417833
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.200477	0.20332	0.351614	0.384726	0.285779	0.321582	0.285964
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.071275	0.046371	0.043386	0.033962	0.04798	0.053401	0.060436
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100

Table S6: Baseline results of power electronics problems (RCM30-RCM35).

Problem		ToP [35]	TiGE_2 [36]	cNSGAIII [8]	cMOEA/D [8]	CCMO [37]	cARMOEA [38]	AnD [39]
RCM30	HV_best	0	0.662965	0.701013	0.839462	0.666132	0.697997	0.725822
	CV_best	0	0	0	0	0	0	0
	HV_mean	0	0.034649	0.289495	0.391785	0.144395	0.217875	0.34759
	CV_mean	11.56486	21.04043	1.205781	0.90871	1.914152	1.628609	1.888535
	HV_worst	0	0	0	0	0	0	0
	CV_worst	28.29356	40.98365	9.124346	6.781095	15.41303	12.44645	12.42719
	HV_sd	0	0.134816	0.300872	0.326206	0.247237	0.290774	0.311574
	CV_sd	9.229781	12.84853	2.211664	1.802905	3.428305	2.838586	3.302945
	FR	10	6.666667	50	60	26.66667	40	56.66667
RCM31	HV_best	0	0.393877	0.827003	0.812276	0.806101	0.857183	0.80836
	CV_best	0.042148	0	0	0	0	0	0
	HV_mean	0	0.022851	0.306911	0.240442	0.227716	0.229353	0.279842
	CV_mean	18.27013	20.82913	3.073342	1.35446	2.117136	3.009694	1.363218
	HV_worst	0	0	0	0	0	0	0
	CV_worst	56.90556	55.72129	21.15783	19.16384	14.1716	18.19538	15.92129
	HV_sd	0	0.086513	0.341371	0.295125	0.305414	0.317861	0.298428
	CV_sd	12.51585	14.13454	5.573602	3.558056	3.41207	4.371542	3.179246
	FR	0	6.666667	46.66667	46.66667	40	36.66667	53.33333
RCM32	HV_best	0	0.793533	0.825149	0.835984	0.822397	0.792387	0.878748
	CV_best	2.20E-05	0	0	0	0	0	0
	HV_mean	0	0.07489	0.445862	0.4641	0.229616	0.333093	0.398617
	CV_mean	18.99021	18.54321	0.99685	1.459053	4.089679	1.675006	1.521516
	HV_worst	0	0	0	0	0	0	0
	CV_worst	51.56746	59.46963	12.24312	12.10024	19.75116	21.96807	9.837396
	HV_sd	0	0.2251	0.366448	0.367472	0.335455	0.364356	0.380229
	CV_sd	13.01789	15.25797	2.310026	2.837696	6.196577	4.433198	2.497286
	FR	0	10	60	63.33333	33.33333	46.66667	53.33333
RCM33	HV_best	0	0	0.23966	0.052622	0.025531	0.065701	0.288832
	CV_best	10.40772	0	0	0	0	0	0
	HV_mean	0	0	0.007989	0.003471	0.000851	0.00553	0.009716
	CV_mean	39.10995	28.67943	6.270309	3.470084	8.276413	6.438681	4.812328
	HV_worst	0	0	0	0	0	0	0
	CV_worst	66.10341	68.29036	32.1277	19.78779	28.90257	20.99447	19.5327
	HV_sd	0	0	0.04302	0.012987	0.004583	0.016825	0.051833
	CV_sd	13.50099	17.38156	7.893064	4.853179	7.719334	6.105385	5.112044
	FR	0	3.333333	20	30	13.33333	13.33333	16.66667
RCM34	HV_best	0	0.113921	0.479272	0.320512	0.437299	0.487213	0.431892
	CV_best	10.38784	0	0	0	0	0	0
	HV_mean	0	0.006823	0.088819	0.055524	0.063454	0.049491	0.066807
	CV_mean	34.29755	28.19329	3.049706	4.922927	7.650207	3.139325	4.122926
	HV_worst	0	0	0	0	0	0	0
	CV_worst	65.58168	75.36825	23.41344	21.54223	25.95273	14.5725	26.77143
	HV_sd	0	0.025705	0.16467	0.095984	0.127719	0.119012	0.123938
	CV_sd	12.98598	17.48042	5.038477	6.423627	7.846577	3.837989	6.573328
	FR	0	6.666667	23.33333	36.66667	26.66667	23.33333	26.66667
RCM35	HV_best	0	0	0.686211	0.651583	0.590883	0.752999	0.676141
	CV_best	15.4508	1.546835	0	0	0	0	0
	HV_mean	0	0	0.167352	0.215711	0.019696	0.10397	0.134673
	CV_mean	37.43432	26.005	4.989614	4.099303	6.324352	6.595854	3.217712
	HV_worst	0	0	0	0	0	0	0
	CV_worst	64.91172	62.33122	20.83738	16.38286	23.85962	32.50146	18.87406
	HV_sd	0	0	0.26105	0.267693	0.106067	0.236295	0.246232
	CV_sd	12.63062	14.89643	6.323394	5.494804	6.28649	8.393579	4.408667
	FR	0	0	30	40	3.333333	16.66667	23.33333

Table S7: Baseline results of power system Optimization problems (RCM36-RCM44)

Problem		ToP [35]	TiGE_2 [36]	cNSGAIII [8]	cMOEA/D [8]	CCMO [37]	cARMOEA [38]	AnD [39]
RCM36	HV_best	0	0	0	0	0	0	0
	CV_best	82.3121	6.643412	4.25498	3.651223	7.154865	4.415033	4.435388
	HV_mean	0	0	0	0	0	0	0
	CV_mean	274.4762	35.2284	53.70688	15.06921	94.37566	52.79134	54.93299
	HV_worst	0	0	0	0	0	0	0
	CV_worst	678.7059	182.8126	161.8934	51.0971	335.3863	258.121	331.5996
	HV_sd	0	0	0	0	0	0	0
	CV_sd	142.171	45.21685	43.41143	13.72349	75.30395	63.66908	68.93145
	FR	0	0	0	0	0	0	0
RCM37	HV_best	0	0	0	0	0	0	0
	CV_best	127.732	4.963391	4.404994	3.553089	4.683178	4.311428	4.417435
	HV_mean	0	0	0	0	0	0	0
	CV_mean	293.7674	71.53191	71.71703	26.24301	102.6528	54.66656	59.19636
	HV_worst	0	0	0	0	0	0	0
	CV_worst	530.8988	309.3974	404.9752	162.1423	325.4873	251.9391	229.573
	HV_sd	0	0	0	0	0	0	0
	CV_sd	122.1939	78.49083	77.53464	39.93584	83.6963	55.73482	57.36992
	FR	0	0	0	0	0	0	0
RCM38	HV_best	0	0	0	0	0	0	0
	CV_best	106.7765	4.581639	4.263193	3.341614	26.41366	4.293962	4.262062
	HV_mean	0	0	0	0	0	0	0
	CV_mean	316.6513	121.5665	48.41404	13.9309	61.61325	57.32413	28.06424
	HV_worst	0	0	0	0	0	0	0
	CV_worst	592.1684	385.4081	225.0023	89.6667	95.77121	321.0996	111.3932
	HV_sd	0	0	0	0	0	0	0
	CV_sd	130.8002	107.8309	56.57967	19.90946	15.04164	81.33751	29.04501
	FR	0	0	0	0	0	0	0
RCM39	HV_best	0	0	0	0	0	0	0
	CV_best	65.16863	4.972818	4.338738	3.348161	4.455859	4.371101	4.178467
	HV_mean	0	0	0	0	0	0	0
	CV_mean	199.7498	20.87616	38.46924	53.59397	51.13681	47.85619	78.27483
	HV_worst	0	0	0	0	0	0	0
	CV_worst	555.3932	92.60474	194.8944	412.8086	189.879	173.4359	496.7103
	HV_sd	0	0	0	0	0	0	0
	CV_sd	112.1574	22.54907	49.33667	78.29306	46.79739	50.81561	109.7538
	FR	0	0	0	0	0	0	0
RCM40	HV_best	0	0	0	0	0	0	0
	CV_best	3.150392	0.660076	0.999967	0.848469	1.066144	0.91699	0.952824
	HV_mean	0	0	0	0	0	0	0
	CV_mean	5.972555	2.512599	1.623799	1.329999	1.536825	1.689827	1.493907
	HV_worst	0	0	0	0	0	0	0
	CV_worst	9.37611	5.529642	3.957437	1.942884	2.858858	2.994664	3.674098
	HV_sd	0	0	0	0	0	0	0
	CV_sd	1.636978	1.375104	0.586713	0.222756	0.328697	0.511366	0.456711
	FR	0	0	0	0	0	0	0
RCM41	HV_best	0	0	0	0	0	0	0
	CV_best	3.301155	0.806728	0.965293	0.914012	1.081988	1.093108	0.927847
	HV_mean	0	0	0	0	0	0	0
	CV_mean	5.662143	2.628734	1.365783	1.561768	1.596323	1.541367	1.44663
	HV_worst	0	0	0	0	0	0	0
	CV_worst	9.892571	7.870352	1.962484	5.534691	4.006354	4.262531	2.180599
	HV_sd	0	0	0	0	0	0	0
	CV_sd	2.068608	1.632579	0.245068	0.767151	0.534491	0.572747	0.255961
	FR	0	0	0	0	0	0	0
RCM42	HV_best	0	0	0	0	0	0	0
	CV_best	3.519447	1.31777	0.894709	1.061619	0.843062	0.901273	1.069173
	HV_mean	0	0	0	0	0	0	0
	CV_mean	6.049385	3.469379	1.527958	1.388699	1.758078	1.420168	1.54281
	HV_worst	0	0	0	0	0	0	0
	CV_worst	12.73405	7.037393	2.851103	1.903395	4.349191	2.266922	2.615807
	HV_sd	0	0	0	0	0	0	0
	CV_sd	2.067198	1.446002	0.37419	0.180178	0.647684	0.260657	0.307961
	FR	0	0	0	0	0	0	0
RCM43	HV_best	0	0	0	0	0	0	0
	CV_best	3.511517	1.040588	0.904851	0.905841	1.070411	1.012481	1.126768
	HV_mean	0	0	0	0	0	0	0
	CV_mean	6.14353	2.330865	1.461826	1.331803	1.74988	1.783643	1.665836
	HV_worst	0	0	0	0	0	0	0
	CV_worst	12.7558	4.918305	2.640186	1.79928	4.462654	6.152961	4.349584
	HV_sd	0	0	0	0	0	0	0
	CV_sd	1.996193	1.041009	0.287781	0.1635	0.768109	0.905813	0.638118
	FR	0	0	0	0	0	0	0

Table S8: Baseline results of power system optimization problems RCM44-RCM50

Problem		ToP [35]	TiGE_2 [36]	cNSGAIII [8]	cMOEA/D [8]	CCMO [37]	cARMOEA [38]	AnD [39]
RCM44	HV_best	0	0	0	0	0	0	0
	CV_best	3.121608	0.36281	1.104857	1.024321	1.047254	0.905816	1.089238
	HV_mean	0	0	0	0	0	0	0
	CV_mean	5.157192	1.207907	1.504204	1.574824	1.627674	1.479146	1.448399
	HV_worst	0	0	0	0	0	0	0
	CV_worst	8.110355	2.523043	2.263849	2.26727	3.727173	2.508425	2.442546
	HV_sd	0	0	0	0	0	0	0
	CV_sd	1.30777	0.491465	0.272569	0.307687	0.713274	0.281505	0.253467
	FR	0	0	0	0	0	0	0
RCM45	HV_best	0	0	0	0	0	0	0
	CV_best	3.145494	1.190196	1.201944	1.105595	1.117485	1.088376	1.142505
	HV_mean	0	0	0	0	0	0	0
	CV_mean	5.624684	2.578918	1.523252	1.396802	1.572634	1.502721	1.476502
	HV_worst	0	0	0	0	0	0	0
	CV_worst	11.34453	5.824603	1.886354	1.857372	3.14689	2.177195	1.723471
	HV_sd	0	0	0	0	0	0	0
	CV_sd	1.648326	1.13158	0.167319	0.195778	0.359486	0.24672	0.116904
	FR	0	0	0	0	0	0	0
RCM46	HV_best	0	0	0	0	0	0	0
	CV_best	2.030229	0.681656	0.81054	1.016072	0.922549	1.070911	1.043286
	HV_mean	0	0	0	0	0	0	0
	CV_mean	4.360037	1.538903	1.427434	1.600378	1.46879	1.44803	1.375263
	HV_worst	0	0	0	0	0	0	0
	CV_worst	7.516484	3.203139	2.292921	2.685567	1.875628	2.099201	1.688632
	HV_sd	0	0	0	0	0	0	0
	CV_sd	1.314467	0.640264	0.253772	0.38678	0.220315	0.214225	0.183594
	FR	0	0	0	0	0	0	0
RCM47	HV_best	0	0	0	0	0	0	0
	CV_best	0.468588	0.074105	0.182643	0.065264	0.144811	0.06228	0.407403
	HV_mean	0	0	0	0	0	0	0
	CV_mean	4.348969	3.9884	2.985086	1.465222	3.68641	2.600211	3.314637
	HV_worst	0	0	0	0	0	0	0
	CV_worst	15.00573	14.62155	9.434895	6.233328	9.565402	12.24001	10.97261
	HV_sd	0	0	0	0	0	0	0
	CV_sd	3.196686	2.97268	2.473667	1.520414	2.431509	2.880452	2.925691
	FR	0	0	0	0	0	0	0
RCM48	HV_best	0	0	0	0	0	0	0
	CV_best	0.392895	0.057091	0.037706	0.007331	0.161581	0.138793	0.21525
	HV_mean	0	0	0	0	0	0	0
	CV_mean	4.851161	3.39968	3.055395	1.603796	3.451364	3.367045	2.337885
	HV_worst	0	0	0	0	0	0	0
	CV_worst	13.50848	11.2626	17.45677	5.529828	12.0752	12.85875	8.437814
	HV_sd	0	0	0	0	0	0	0
	CV_sd	3.864569	2.713269	3.760928	1.461697	3.045447	3.498318	2.106418
	FR	0	0	0	0	0	0	0
RCM49	HV_best	0	0	0	0	0	0	0
	CV_best	0.538135	0.103872	0.064077	0.030818	0.212473	0.194484	0.114581
	HV_mean	0	0	0	0	0	0	0
	CV_mean	4.71745	5.415072	2.405423	2.595872	2.374781	1.751124	2.978641
	HV_worst	0	0	0	0	0	0	0
	CV_worst	14.30472	40.78255	8.001206	7.585709	7.680618	5.466936	8.212466
	HV_sd	0	0	0	0	0	0	0
	CV_sd	3.722273	7.241503	2.248118	1.849062	1.98031	1.471382	2.413002
	FR	0	0	0	0	0	0	0
RCM50	HV_best	0	0	0	0	0	0	0
	CV_best	0	0	0	0	0	0	0
	HV_mean	0	0	0	0	0	0	0
	CV_mean	0.002172	0.001959	0.000193	0	0.000785	0.000183	0.000489
	HV_worst	0	0	0	0	0	0	0
	CV_worst	0.008623	0.007746	0.001493	0	0.005396	0.000968	0.001869
	HV_sd	0	0	0	0	0	0	0
	CV_sd	0.001843	0.002066	0.000319	0	0.001178	0.000299	0.000566
	FR	6.666667	6.666667	40	100	20	40	10

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