# Electrocardiograph Band-Pass Filter Phase Equalization

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***Abstract*—Electrocardiograph (ECG) systems measure electrical activity of muscle contractions via carefully attached electrodes. Analog band pass filters are a standard method of signal processing to eliminate noise components from the ECG signal. This paper assessed the magnitude and phase response of an analog band pass filter used in an industrial ECG design to ascertain the extent of the signal distortion. A series of all pass filters were used to decrease the phase distortion by %50 at a trade off of slightly more signal delay. Flatness of the group delay served as an optimization parameter to configure the all pass system.**

***Index Terms*—phase distortion, all-pass filter, electrocardiography, group delay, optimization**

## I. Introduction

### A. Context

Electrocardiography(ECG) is a standard tool for diagnosis of an individual's heart health and essential for identification of anomalous heart conditions. The ECG operates by measuring the electrical activity associated with each contraction of the heart muscles. Each "beat" of the heart consists of a pattern of contractions that produces a consistent ECG pattern(Fig. 1).

Before this measured signal is displayed to physicians, it must be filtered to remove noise that could potentially alter the waveform shape. A standard method is to first apply an analog band pass filter before analog to digital conversion [1]. It is extremely important that filtering does not alter the shape of the waveform as this could lead to a misdiagnosis. The magnitude response of the analog band pass filters has been carefully designed to minimize attenuation of the ECG waveform. Previous research has investigated the phase response of these filters and demonstrated that non-negligible phase distortion occurred in the bandwidth of ECG frequencies. [1] All pass systems provide an effective method to minimize phase distortion [2].

### B. Significance

Equalization of non-linear phase response with an all pass system can be implemented in the analog or digital domain. All that is required is to characterize the phase response of the analog band pass filter and then cascade a carefully selected phase equalizing all pass system to linearize the phase response. This phase correction could increase physician's rate of correct diagnosis and help to catch life threatening heart beat irregularities.

## II. Methodology

### A. Signal Processing Background

All pass filters are described by the following equation found in [2]:

$$H_{ap}(z) = A \prod_{k=1}^{M_r} \frac{z^{-1} - d_k}{1 - d_k z^{-1}} \prod_{k=1}^{M_c} \frac{(z^{-1} - e_k^*)(z^{-1} - e_k)}{(1 - e_k z^{-1})(1 - e_k^* z^{-1})} \tag{1}$$

Where $M_r$ is the number of real valued poles and zeros and $M_c$ is the number of complex valued poles and zeros. These filters have a magnitude response of one for all frequencies and a phase response:

$$\angle H_{AP} = \left[ \frac{(e^{-j\omega} - re^{-j\theta})(e^{-j\omega} - re^{j\theta})}{1 - re^{j\theta}e^{-j\omega})(1 - re^{-j\theta}e^{-j\omega})} \right] \tag{2}$$

Observing the group delay, it is apparent that the phase will always be non positive and the group delay will always be positive if the all pass is causal.

$$\text{grd}\left[ \frac{e^{-j\omega} - re^{j\theta})}{1 - re^{j\theta}e^{-j\omega}} \right] = \frac{1 - r^2}{|1 - re^{j\theta}e^{-j\omega}|^2} \tag{3}$$

As $r < 1$ in a causal system, group delay will always be positive [2]. With these properties, it is possible to manipulate the phase response of a system without effecting the magnitude response.

### B. Signal and Filtering

An ECG Lead II waveform was synthesized using a MATLAB script created by Karthik Raviprakash [3]. This lead location corresponds to the inferior surface of the heart shown on the left side of Fig. 1 denoted as "Lead II". An idealized version of a healthy ECG signal is seen on the right side of Fig. 1. An example of a synthesized ECG signal is seen in Fig 2. The simulated ECG signal was shifted down to zero to eliminate any potential for edge effects while filtering. The root mean squared difference between the input signal and filtered signal was used to quantify the extent of ECG phase distortion.

A schematic for a passive band pass filter used in an ECG device from NXP Semiconductors (https://www.nxp.com/) served as a rough template of industry design practice. The cutoff frequencies are .5 Hz to 153 Hz and the specific component values can be found in Fig. 3. These cutoff frequencies

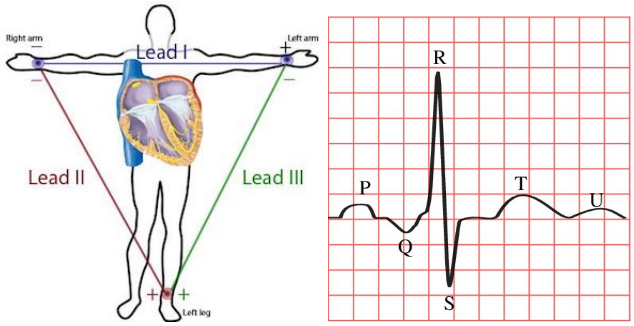


Fig. 1. Einthoven Triangle and Lead II ECG signal (https://www.radcliffecardiology.com/image-gallery/).

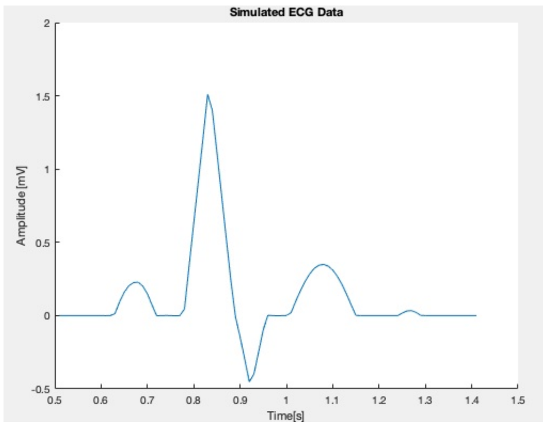


Fig. 2. Simulated Lead II ECG signal.

are common practice in ECG devices [1]. The next step was to derive the continuous time transfer function.

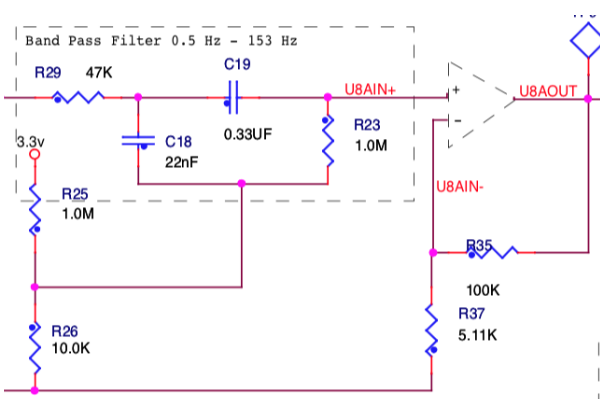


Fig. 3. Passive Band pass filter used in NXP MED-EKG device.

The continuous time transfer function is a 2nd order system seen in (4) labelled with the same notation as Fig. 3 as follows:

$$\begin{aligned} H(s) &= \frac{s\beta}{s^2\tau + s\alpha + 1} \\ \text{where} \quad \beta &= R_{29}C_{19} \\ \tau &= C_{19}C_{18}R_{29}R_{23}, \\ \alpha &= C_{19}R_{23} + C_{18}R_{29} - C_{19}R_{29} \end{aligned} \tag{4}$$

This transfer function was converted to the Z-domain using zero order holds with a sampling rate of 950 Hz. The Z-domain transfer function is shown in equation (5) and the magnitude and frequency response are show in Fig. 4 as well as the pole zero plot in Fig. 6.

$$H_{BP}(z) = \frac{.6504 - .6504z^{-1}}{1 - 1.376z^{-1} + .3778z^{-2}} \tag{5}$$

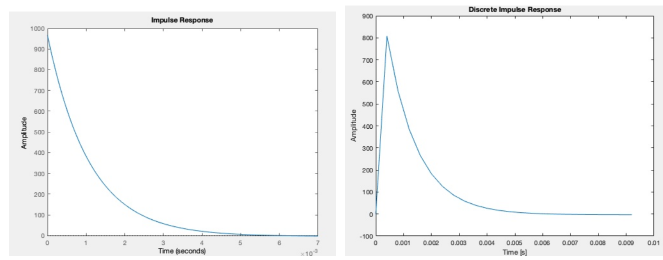


Fig. 4. Continuous time impulse response(left) and the discrete time impulse response(right).

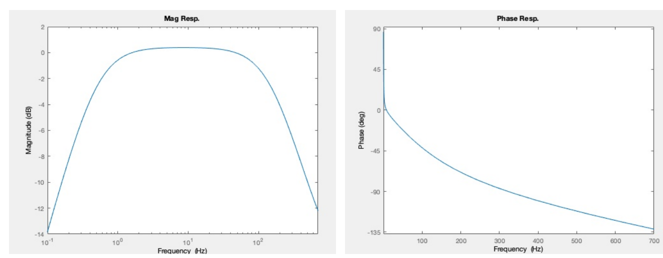


Fig. 5. Mag. and phase of the transformed discrete filter.

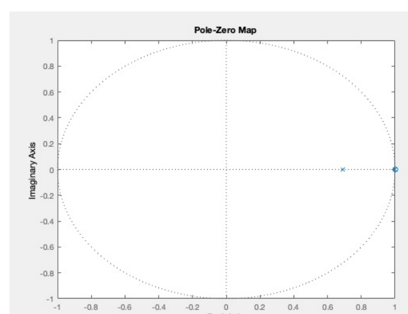


Fig. 6. Pole zero plot of the discrete band pass filter.

## *C. Phase Correction via All Pass Filters*

As described in equation (1), all pass filters can be cascaded to produce a more desirable phase response. A parameter sweep over phase angle and radius $r$ was carried out to create a range of pole/reciprocal zero locations. Phase angle ranged from 0 to $\pi$ with an increment of .05 and $r$ from 0 to 1 with an increment of .01. The criterion for the optimal all pass locations were:

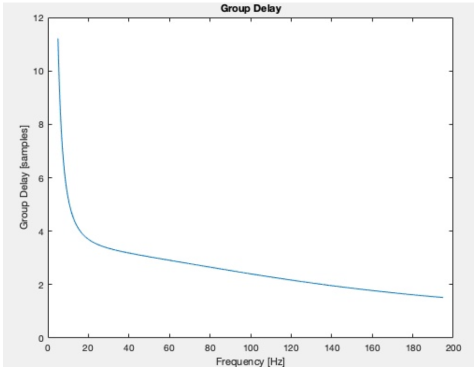


Fig. 7. Group delay of the discrete band pass filter.

$$\min \left\{ \mu = \left( \prod_{k=1}^{N} \left| \frac{d^2}{d\theta^2} \angle H_{BP} H_{AP}(e^{j\omega k}) \right| \right)^{\frac{1}{N}} \right\} \tag{6}$$

Where $N$ is equivalent to 4000 points over a frequency band of 5 - 195 Hz. The geometric mean provided better results during optimization testing than the arithmetic mean. For each iteration of the parameter sweep, $\mu$ is calculated for the cascade of the original band pass with this candidate all pass. After each sweep, the all pass filter that produced the minimum $\mu$ and is below a certain threshold value is added as a stage to the all pass system. The process is then repeated with this new band pass all pass transfer function. The goal is to progressively make the group delay more constant in the frequency band of interest until it meets a sufficient exit condition.

### D. Results

Results of the optimization are found in Fig. 8. After six iterations, the addition of all pass stages ceased to make the group delay more constant. Fig. 9 demonstrates how the group delay becomes more constant with more all pass stages. In Fig. 10, it is clear that the magnitude response is unity and the phase is linear over the range 5 Hz - 195 Hz.

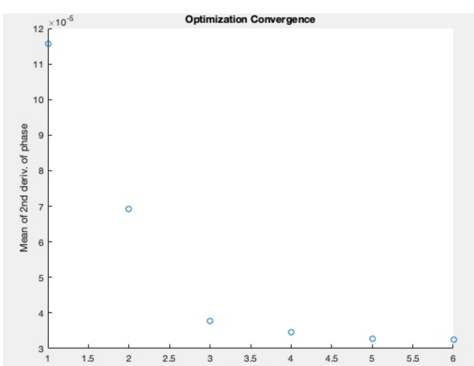


Fig. 8. Convergence of the optimization of band pass filter group delay for number of stages added to the all pass system.

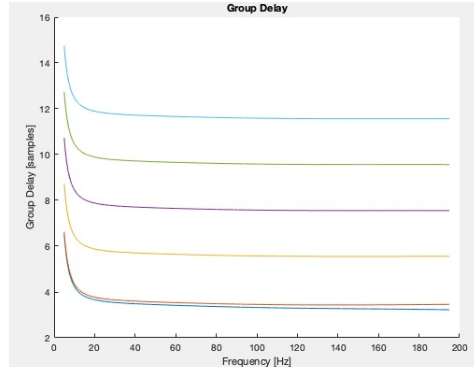


Fig. 9. From bottom to top: the group delay as successive all pass stages are cascaded with the band pass filter.

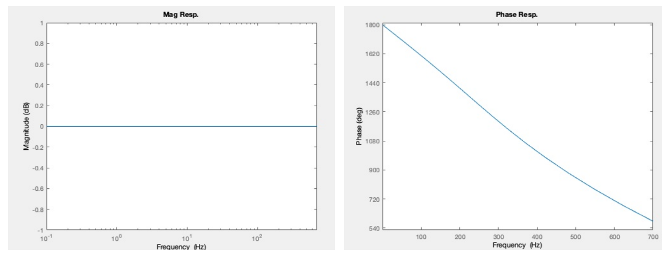


Fig. 10. All pass system magnitude response and phase response.

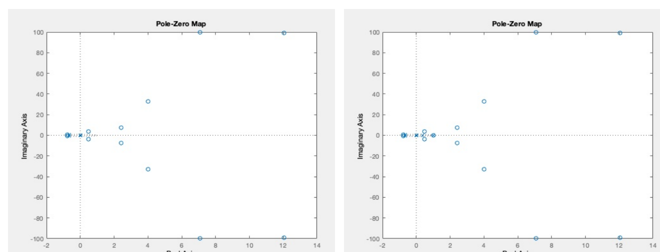


Fig. 11. On the left, pole zero plot of the all pass system. On the right, pole zero plot of the cascaded band pass and all pass system.

In Fig. 12, the phase distortion of the band pass filter is evident when compared with a zero phase implementation of the filter achieved with the "filtfilt" operation provided in MATLAB. The tail segment of the signal labeled "filt" is below that of the "filt-filt" signal. It appears that frequency components in the S wave are outside the pass band as these show a level of attenuation in both the "filt-filt" and "filt" signals. In Fig. 13, the band pass filter cascaded with the all pass system system plots does not visually show much improvement in the filtering performance. Table I gives a quantitative analysis of similarity with the root mean square error (RMSE) between the input signal and the filtered output signal. The input and output signals were aligned before taking the RMSE for the single direction filtering. As seen in Table I, The RMSE was reduced by 50% for the all pass band pass cascaded filter in comparison to just the band pass filter. The zero phase band pass implementation is still far superior in terms of RMSE.

### E. Discussion

The phase equalized band pass filter demonstrates the possibility of improved signal integrity in a signal processing chain. Trade offs for phase correction are a delay of the output signal by .013 s and potentially more components if the all pass system is implemented as a circuit. In the domain of biomedical devices, high quality signal representation should be a top priority and therefore the all pass system should be used. Future work could examine alternative optimization algorithms with finer increments or different objective functions. Testing of phase equalized band pass filters with various ECG signals representative of heart conditions would also be an important area to investigate.

### F. Conclusion

All pass systems provide a viable method to equalize the non linear phase response of various systems. While the all pass system does not entirely alleviate phase distortion issues, it can make a significant difference in some applications. In the area of electrocardiography, obfuscating phase distortion can be the difference between life and death for patients with certain arrhythmia's. I have demonstrated that all pass filters have potential to improve performance in these ECG systems.

TABLE I
ROOT MEAN SQUARE ERROR BETWEEN INPUT AND FILTERED SIGNAL

| *Filter* | *RMSE* |
|---|---|
| *BPF Single Filt* | .0034 |
| *BPF Filt-Filt* | 8.232e-04 |
| *BPF-AP Filt* | .0018 |
| *BPF-AP Filt-Filt* | .0050 |

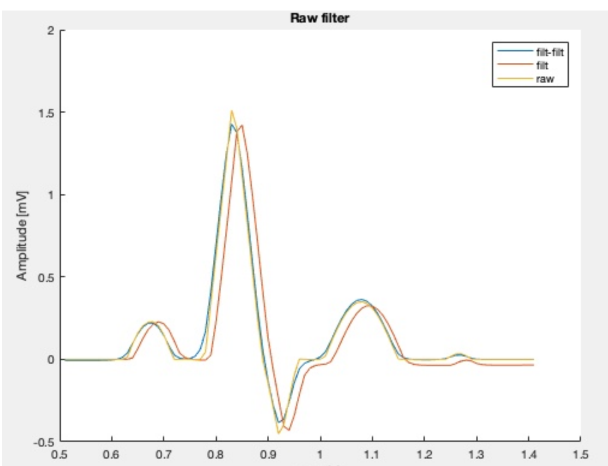


Fig. 12. Raw ECG signal with uni-directional band pass filtering and bi-directional("filtfilt") band pass filtering signal.

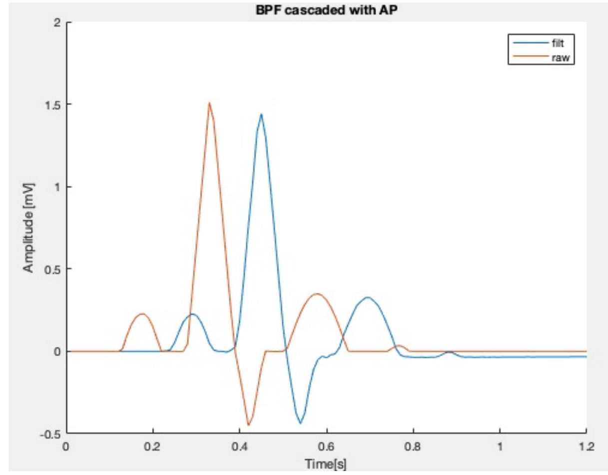


Fig. 13. Band pass filter cascaded with all pass system filtering results.

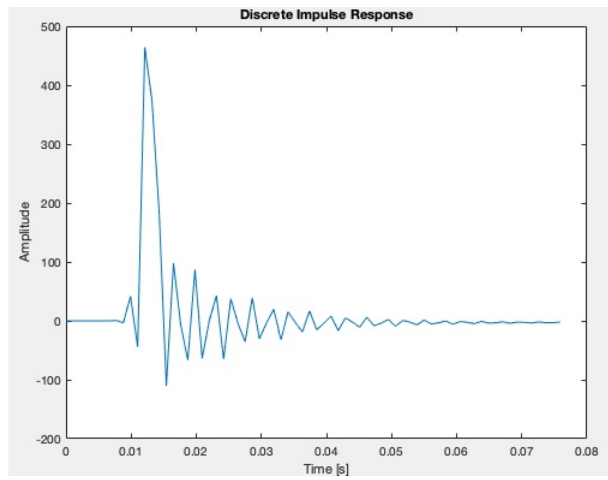


Fig. 14. Equalized band pass filter impulse response.


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