CFD Task: Two Dimensional Convection-Diffusion Equation

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1 Introduction

In the present task the two dimensional convection-diffusion equation/transport equation for temperature is studied on a particular domain with the given grids. The two-dimensional transport equation for temperature reads,

$$\frac{\partial(\rho UT)}{\partial x} + \frac{\partial(\rho VT)}{\partial y} = \frac{\partial}{\partial x}\left(\Gamma\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\Gamma\frac{\partial T}{\partial y}\right) + S \tag{1}$$

$$\Gamma = \frac{k}{c_p}$$

The above algebraic equation is discretised using the finite volume method method and solved using Guass-Siedel method and Tri-diagonal matrix algorithm based on the given boundary conditions.

1.1 Case and Implementation

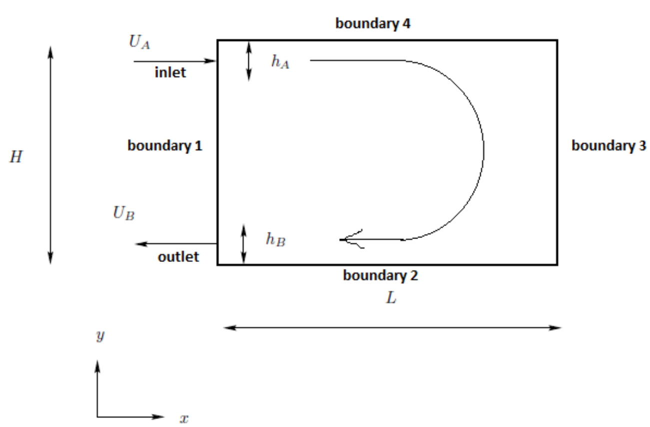


Figure 1: Configuration

Figure.1 shows the configuration for which the method is applied. The assigned configuration has a length of L and height of H. The with of inlet is h_A and the width of the outlet is h_B. Boundary 1 had a temperature value of 10^oC and the inlet temperature is 20^oC. The temperatur gradients at boundaries 2,4 are assumed as, $\frac{\partial T}{\partial x} = 0$ and the temperature gradients at boundary 3 and outlet is assumed to be $\frac{\partial T}{\partial y} = 0$. In this case the density of the fluid $\rho = 1$, conefficient of conductivity k=1 and the pressure coefficient $C_p = 500$.

The code calculates the solution from the input values from the boundaries. The inlet and boundary 1 have the Dirichlet boundary conditions which are implemented explicitly and boundaries 2,3,4, outlet have homogeneous Neumann boundary conditions which are implemented explicitly.

1.2 Convergence

In order to obtain a converged solution the convergence criterion is applied. The residual at each iteration is computed as:

$$\epsilon = \frac{1}{F}\left(\sum_{allcells} \mid a_E T_{i+1,j} + a_W T_{i-1,j} + a_N T_{i,j+1} + a_S T_{i,j-1} + S_U - a_P T_{i,j} \mid \right) \quad (2)$$

Here F is the temperature flux used for normalization of residual. The F is represented as the total flux in the domain. In this case F is taken as the product of inlet mass flux and temperature flux at inlet and outlet.

$$F = (\rho U h)_A \Delta T, \quad (3)$$

here, ΔT is the temperature difference between inlet and outlet. The continuity error ΔF is 0.

2 Sensitivity to Heat Conductivity

In this section effect of heat conductivity on relative strength of convection and diffusion will be investigated. The non-dimensional cell Peclet number is a proper tool to show the transportativeness in this sense. As stated in the course book $Pe = 0$ means no convection and pure diffusion while $Pe \to \infty$ means no diffusion and pure convection. In this assignment, hybrid differencing scheme is applied and $Pe = 2$ is used as a threshold switch between central differencing scheme and first order upwind scheme. Figure 2 shows the results of original boundary conditions provided by assignment description.

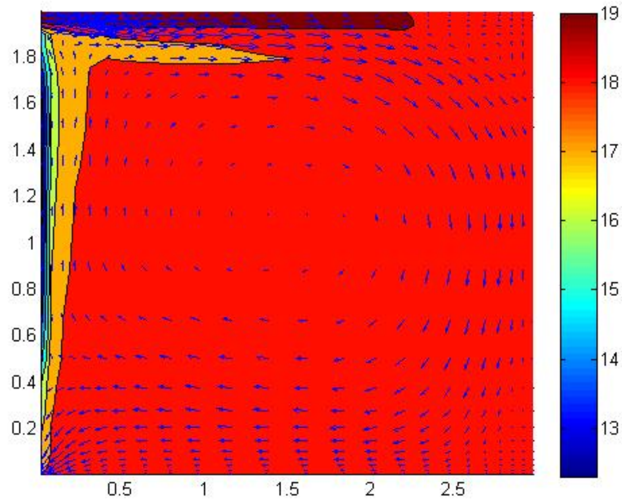
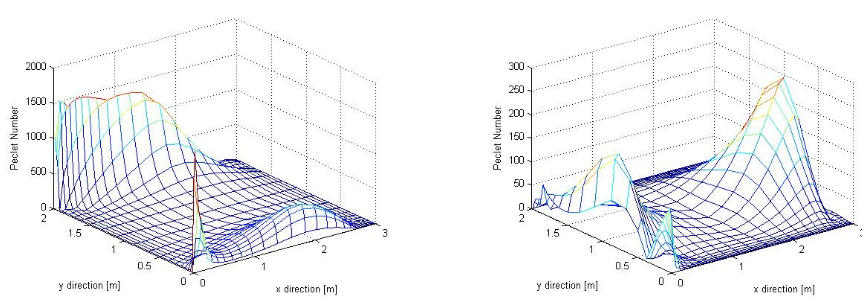


Figure 2: Contour plot of coarse mesh for given Γ

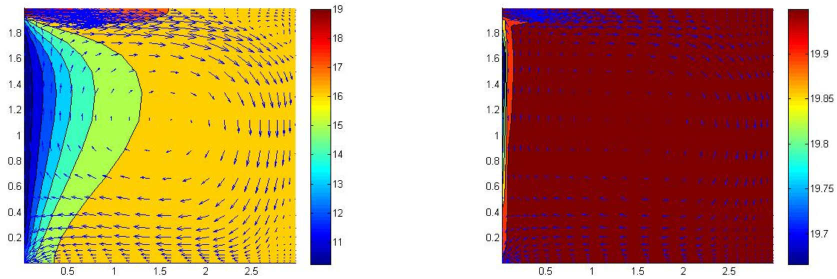
As it can be seen in Figure 2 diffusion is not dominant against convection. To verify this observation Peclet numbers is used. In u direction mean $Pe_u = 207.3$ while in v direction mean $Pe_v = 25.5$. These values are way above the threshold of central differencing scheme, therefore convection term is dominant. In order to understand the temperature contour plot more deeply, Figure 3 is used as a supplement. Lets focus the temperature change at the top wall region, inlet temperature $T = 20^0$ is reaching almost right wall however it stopped because of the velocity vectors at the corner is not pointing steam-wise anymore. The reason for this distinctly high temperature stuck before the top-right corner can be seen at Figure 3, Pe_v has very low values close to the top wall. Therefore inlet temperatures cannot be transported in v direction anymore.



(a) Peclet Number at cells in u direction

(b) Peclet Number at cells in v direction

Figure 3: Peclet Numbers at cells for coarse mesh



(a) Contour plot of coarse mesh for $\Gamma * 100$

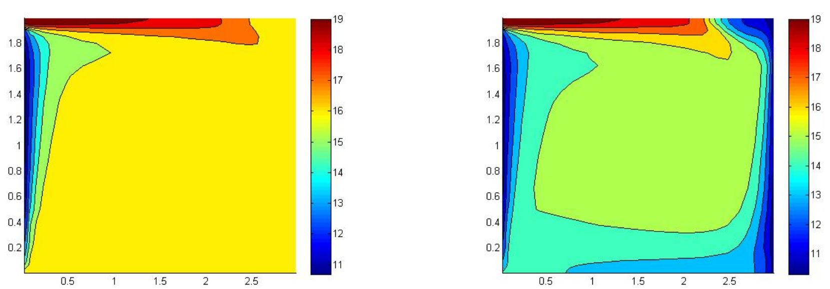
(b) Contour plot of coarse mesh for $\Gamma * 1/100$

Figure 4: Contour plot of coarse mesh

Figure 3 shows that resulting temperatures are greatly sensitive to heat conductivity. If we compare the Peclet numbers of two plots in Figure 3, contour plot a) mean $Pe_u = 2.0733$ while contour plot b) mean $Pe_u = 20733$. Therefore second plot is a proof that almost no diffusion happening and temperature at all domain is very close to inlet temperature.

3 Sensitivity to Boundary Conditions

In this section the boundary condition of right wall will be changed from Neumann B.C to Drichlet B.C. Γ is multiplied by the factor of 10 here for the purpose of having more equally balanced strength of convection and diffusion. Now in u direction mean $Pe_u = 11.4$ while in v direction mean $Pe_v = 1.5$. Left wall temperature is given as $T = 10°$. When B.C is changed to Drichlet at right wall, temperature is alse given as $T = 10°$. Results of fine mesh of two different boundary conditions through Gauss-Seidel solver can be seen at Figure 5.



(a) Contour plot of fine mesh with initial boundary conditions

(b) Contour plot of fine mesh with Drichlet B.C at right wall

Figure 5: Peclet Numbers at cells for coarse mesh

By zooming and adding the velocity vectors to the Figure 5 b), Figure 6 shows how convection term effects the temperatures. Warm temperatures from inlet is stopped in the u direction by the flow near to right top corner,however thank to velocity vectors in v direction high temperatures are canalized to negative v direction.

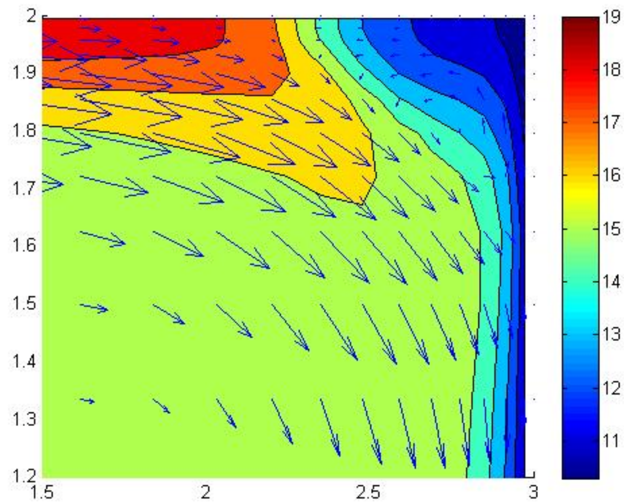


Figure 6: Zoomed to the right-top corner contour plot of coarse mesh for given $\Gamma * 10$

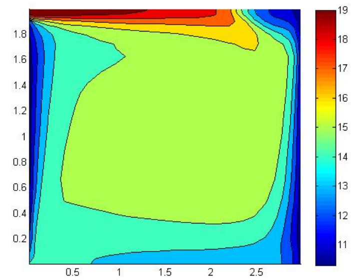
4 Sensitivity to convergence

Convergence criteria ϵ is varied to check whether the solution for temperatures are sensitive to convergence criteria or not. Number of iterations are very sensitive to relative strength of convection to diffusion. As can be seen at Table 1 the greater the convergence the more iterations are needed.

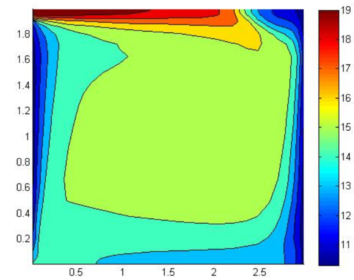
Number of iterations	$\Gamma * 10$	Γ	$\Gamma \div 10$
$\epsilon = 0.01$	462	1315	2339
$\epsilon = 0.001$	580	1692	2918
$\epsilon = 0.0001$	698	2068	3496

Table 1: Number of iterations at corresponding Γ and ϵ for coarse mesh

Even though number of iterations are changing drastically, for different convergence criteria results are very similar. Comparison of results can be seen at Figure 7, Figure 8 and Figure 9.

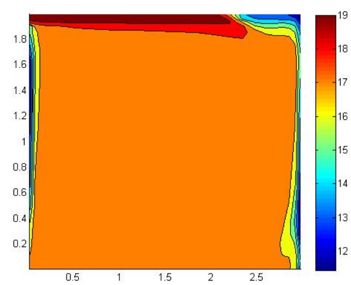


(a) $\epsilon = 0.01$

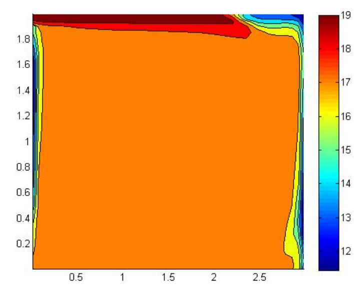


(b) $\epsilon = 0.0001$

Figure 7: Contour plot of coarse mesh with Drichlet B.C at right wall,$\Gamma * 10$

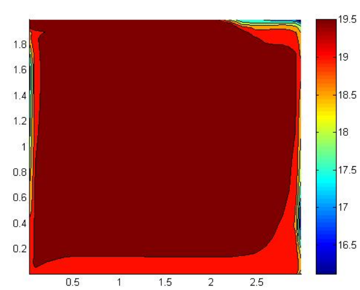


(a) $\epsilon = 0.01$

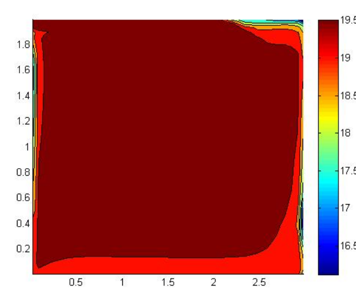


(b) $\epsilon = 0.0001$

Figure 8: Contour plot of coarse mesh with Drichlet B.C at right wall,Γ



(a) $\epsilon = 0.01$



(b) $\epsilon = 0.0001$

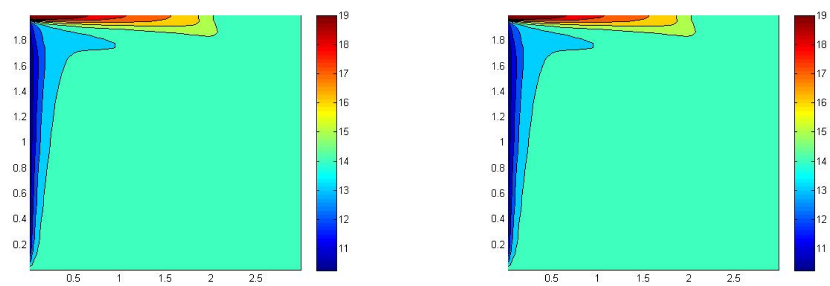
Figure 9: Contour plot of coarse mesh with Drichlet B.C at right wall,$\Gamma \div 10$

5 Heat Flux

Heat flux is calculated after interpolating the temperatures at faces. Boundary condition at right wall is turned Neumann Boundary condition as the original case. In order to clarify that the results from two different solvers are the same Figure 10 can be observed. Required iterations for convergence can be seen at Table 2.

Number of iterations	$\Gamma * 10$
TDMA solver	4841
Gauss-Seidel Solver	5674

Table 2: Number of iterations at $\epsilon = 0.001$ for fine mesh

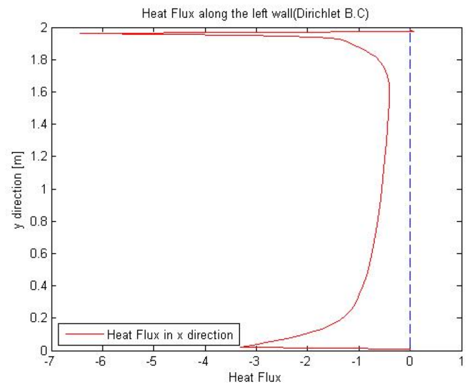


(a) Solution from TDMA solver

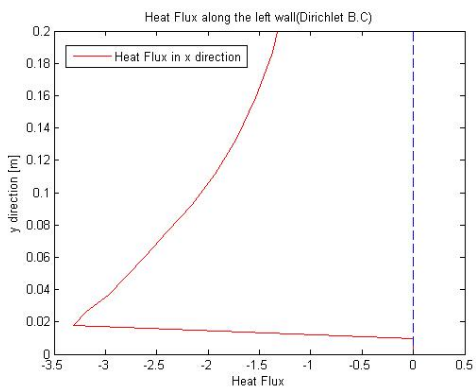
(b) Solution from Gauss-Seidel solver

Figure 10: Contour plot of fine mesh with Neumann B.C at right wall at $\Gamma * 10$

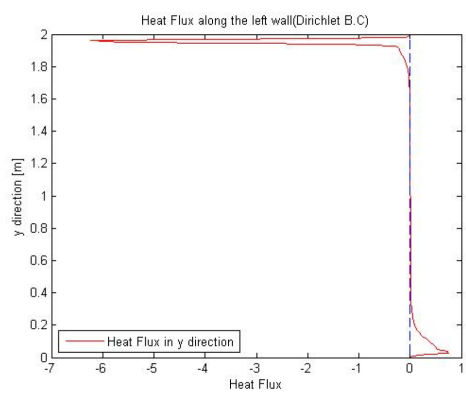
As it is expected, heat flux is almost always negative.That means heat moves from calculation domain to outwards through left wall. However left wall is special because it also has Neumann B.C on it due to outlet. Figure 11 a) and c) shows the general behaviour of Dirichlet B.C while Figure 11 b) shows the at outlet heat flux at x direction is zero due to Neumann B.C as described.



(a) Heat Flux in x direction



(b) Heat Flux in x direction zoomed to outlet



(c) Heat Flux in y direction

Figure 11: Heat Flux along the left wall(Dirichlet B.C) at $\Gamma * 10$

In order to check the global convergence heat fluxes are calculated at all walls and summed together. Conditions and B.C are as in the Figure 9, Figure 8,

Figure 7, right and left walls Dirichlet while top and bottom walls are Neumann boundary condition.
Table 3 shows that summation of heat fluxes are not zero. And when Γ gets smaller values, convection gets more important. As result summation is getting smaller. On the other hand when Γ gets bigger values diffusion dominates and summation deviates more. However as it is told, fluxes are calculated through temperature values at faces. But the the first face between two nodes (other than the boundary) is interpolated independent of boundary temperature. Even if there is Neumann B.C meaning first node (after boundary) temperature and wall temperature is the same, the first face doesn't necessarly has the temperature of first node. That difference makes the summation bigger or smaller than zero but not exact value of zero.

$\sum$ **of fluxes at Boundaries**	$\Gamma \div 10$	Γ	$\Gamma * 10$
Heat flux in x direction	0.1895	0.2172	-3.5246
Heat flux in x direction	0.0335	-0.0592	-4.4238

Table 3: Sum of flux at walls at $\epsilon = 0.0001$ for fine mesh