CFD Task: Two Dimensional Diffusion Equation

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1 Introduction

In this task we studied the two dimensional diffusion for temperature T which is,

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+b=0 \tag{1}$$

The above equation is discretised using finite volume method applied on a given computational domain and solved using Guass-Seidel method based on the given bondary conditions.

1.1 Case

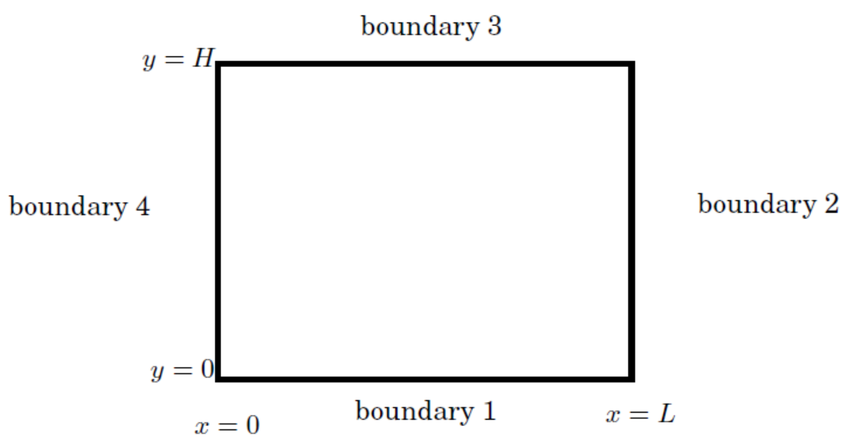


Figure 1: Configuration

Figure.1 shows the configuration for which we have to apply our method. The length of this assigned configuration L is 1.5 units and has a height H of 0.5 units. The boundaries 1 and 4 has a temperature value of 10, the variation of temperature across boundary 2 is given by the expression $10+20sin(\pi y/H)$ and the temperature gradient at boundary 3, $\frac{\partial T}{\partial x}=0$. Coefficient of conductivity k is equal to 0.01 in the region of $0.7<x<1.1, 0.3<y<0.4$ and in the remaining computational domain k=20. The source term b in our case is 0.

1.2 Discretisation

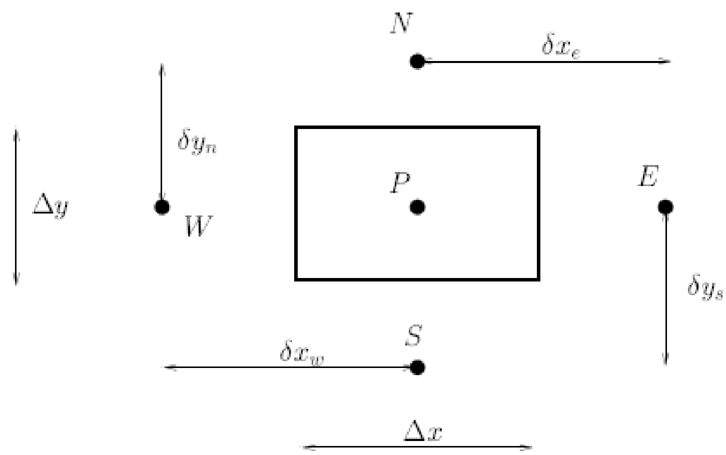


Figure 2: 2D control volume, node P represented at center of the cell

For discretisation, Equation1. is integrated over the 2D control volume with the help of central differencing we arrived at the following equation;

$$a_P T_p = a_E T_E + a_W T_W + a_N T_N + a_S T_S + b\Delta x \Delta y$$

where, $a_E = \frac{k_e \Delta y}{\delta x_e}$, $a_W = \frac{k_w \Delta y}{\delta x_w}$, $a_N = \frac{k_n \Delta x}{\delta y_n}$, $a_S = \frac{k_s \Delta x}{\delta y_s}$, $a_P = a_E + a_W + a_N + a_S$. Here k_e, k_w, k_n and k_s are the heat conductivities at the faces which are estimated by the linear interpolation between the adjacent nodes.

1.3 Implementing Boundary condition

The code calculates the solution from the input values at the boundaries. In our case we have three Dirichlet boundary conditions at boundaries 1,2,4 and one homogeneous Neumann boundary condition at boundary 3. The Dirichet boundary conditions are implemented explicitly in our case where as the Neumann boundary condition is implemented implicitly by making a_N at the last node in y direction to be 0.

1.4 Convergence

In order to have an adequate number of iterations the convergence criterion i.e $\frac{R}{F} \leq \epsilon$ is used, where $0.0001 < \epsilon < 0.01$. Here F is the flux of Temperature and R is the residual given by $R = \sum_{allcells} \mid a_E T_E + a_W T_W + a_N T_N + a_S T_S - a_P T_P \mid$.

2 Meshing

In this problem we used different meshes to solve the problem, some of them are

- 10 x 10 equidistant mesh
- 20 x 20 equidistant mesh
- 40 x 40 equidistant mesh
- other refined and manually defined size meshes which are sized according to the gradients.

Figure3. shows the temperature contour for a 10 x 10 equidistant mesh. we can see a pattern how the heat conduction take place in the domain. In our case there is a particular region where k is different as mentioned in section 1.1 which we are unable to observe its influence. Let us see the pattern in a 40 x 40 equidistant grid, Figure4. shows the temperature contour 40 x 40 equidistant grid. In this contour we can clearly observe there are some higher gradient regions especially in the region where k is different from other regions of the domain. Now the mesh in this particular region is made much finer in order to extract the values in the region more adequately. Figure5. show the refined mesh which is constructed according to these regions of higher gradients(the mesh is finer where we have high gradients). Here we can clearly observe the change in pattern only at that particular region of k. In Figure4 due to this change in k there is an effect on surrounding regions of this particular area (i.e for $x > 1.1$), as we make mesh finer and finer this effect can vanish.

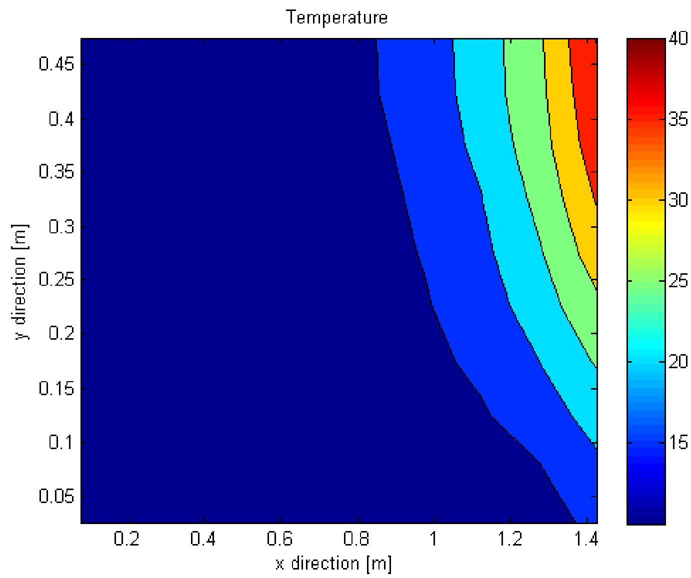


Figure 3: Contour plot of T with 10 x 10 equidistant mesh

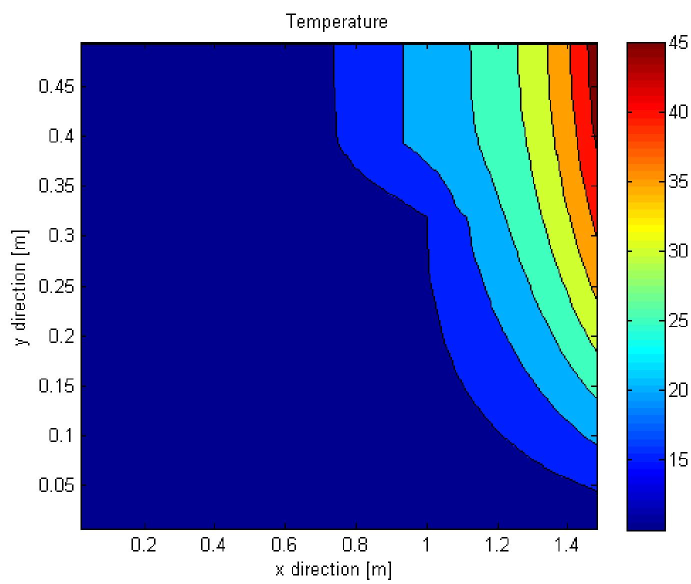


Figure 4: Contour plot of T with 40 x 40 equidistant mesh

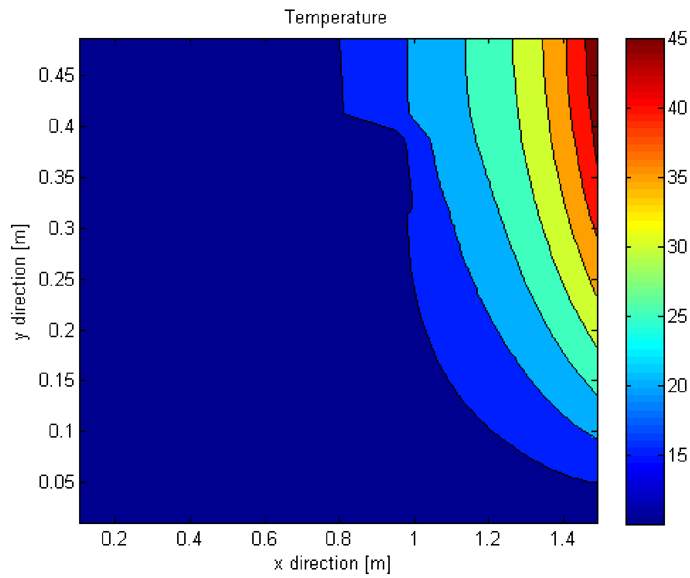


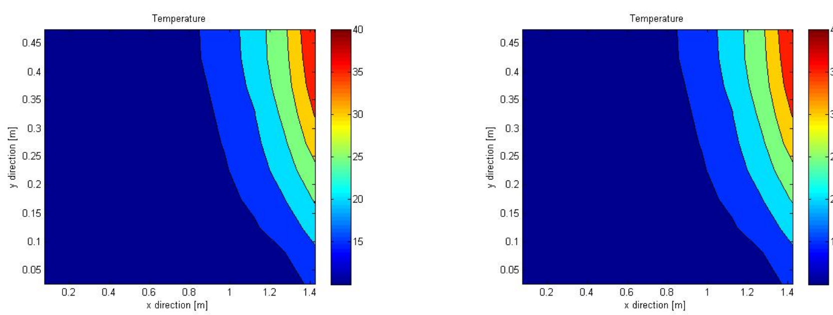
Figure 5: Contour plot of T with mesh refined at areas of larger gradients

3 Effect of coefficient of heat conductivity, k

Investigation of sensitiveness of solution to the coefficient conductivity,k, is done by increasing and decreasing k by factor of 100. It is important to note that k is given as constant values at certain areas in whole domain. Calculation domain simply includes two different k values which are not dependent on heat or some other variable. Another important point is the source term, b, which is given as zero at all domain. As statements done here allow us to re-write the Figure 6 as below because k can be moved out of the derivative since it is a constant and source term is zero. This equation obviously should not be sensitive to any change unless the proportion of the k values in two different region changes.

$$k\left(\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right)\right)=0$$

In order to see if there is any difference in calculated temperatures for different k values two extreme scaling factors, $k_{scale}=100$ and $k_{scale}=1/100$ is plotted in Figure 6

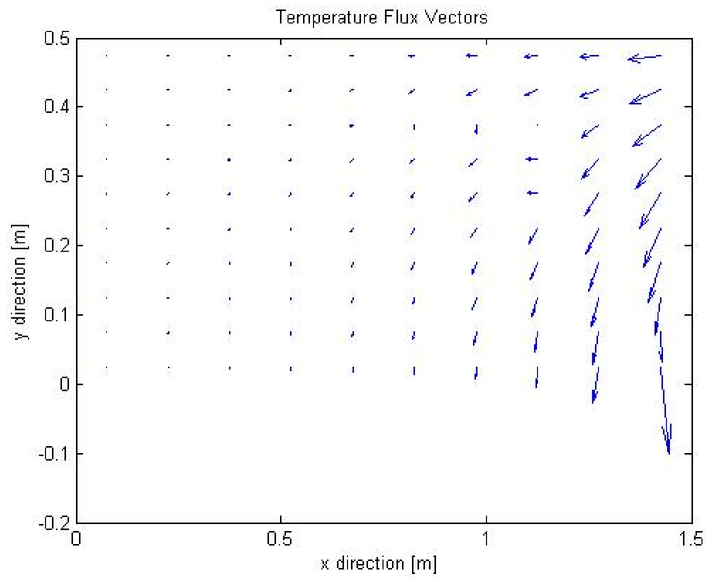


(a) k is multiplied by 100

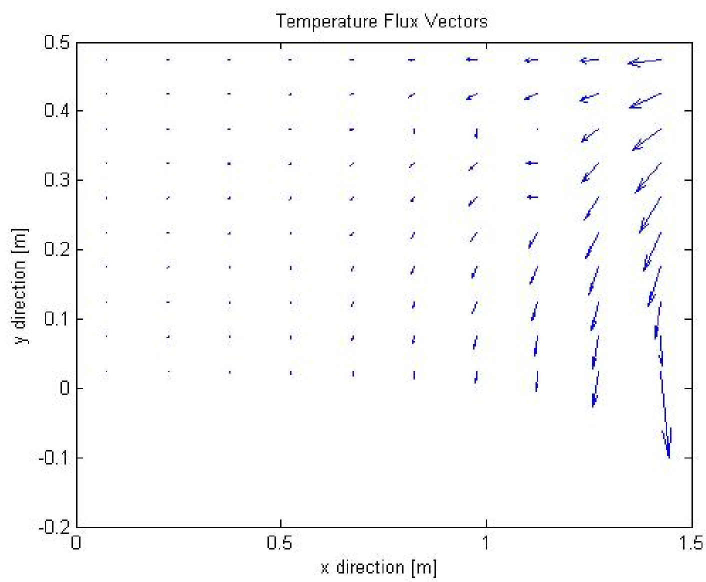
(b) k is multiplied by 1/100

Figure 6: Contour plots of equidistant 10x10 mesh for initial B.C a)k is multiplied by 100 b) k is multiplied by 1/100

Another comparison can be done by using heat flux vector plots as done in Figure 7. As it is explained earlier no differentiation is observed between two different k values.



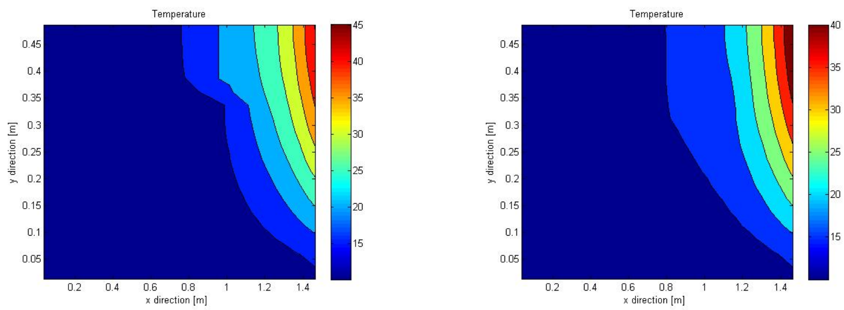
(a) k is multiplied by 100



(b) k is multiplied by 1/100

Figure 7: Heat flux vectors of equidistant 10x10 mesh for initial B.C a)k is multiplied by 100 b) k is multiplied by 1/100

It has shown that if k is increased or reduced proportionally in two different areas stated in the assignment description, solution will not be effected. However if k values in two areas changed in without a proportion solutions will differ considerably. It can be seen from Figure 8 that solutions are different due to increase of k 10000 times only in one region where was $k = 0.01$. In Figure 8 on left $k = 0.01$ is acting as a insulator while figure on the right having $k = 100$ in the particular region is helping high temperatures to reach even further from the wall.



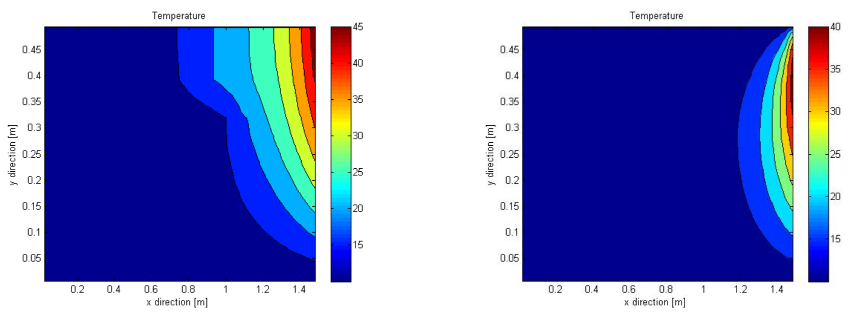
(a) k is as given

(b) k is multiplied by 10000 only in one region

Figure 8: Contour plots of equidistant 20x20 mesh for initial B.C a)k is as given b) k is multiplied by 10000 only in one region

4 Boundary conditions

Given boundary conditions are described earlier as 3 Dirichlet and one Neumann at the boundaries. In this chapter Neumann condition at Boundary 3 will be changed to Dirichlet condition.



(a) Neumann BC at top wall

(b) Dirichlet BC at top wall

Figure 9: Contour plots of equidistant 40x40 mesh a)Neumann BC at top wall b) 10^0 temperature at top wall as Dirichlet BC

As can be seen from Figure 9, temperatures are drastically changing between two different boundary condition. Neumann boundary condition is acting as simply an insulator. It allows temperature gradients to be on x direction as can be seen in Figure 7. That is why warmer temperatures can penetrate more into whole field. However at the Drichlet boundary condition when the temperature at Boundary 3 is $10^0 degree$, heat flux vectors are almost normal to the wall.

In order to make the case more interesting, lets change the temperature of the boundary 3 from $10^0 degree$ to $20^0 degree$ as Drichlet condition. As it can be seen from Figure 9, temperature field changed drastically. Area that is having lower heat conductivity,k, is observed clearly because since that area is having lower gradient of temperature, it is so to speak resisting to transfer warmer temperatures to lower temperatures compared to surroundings.

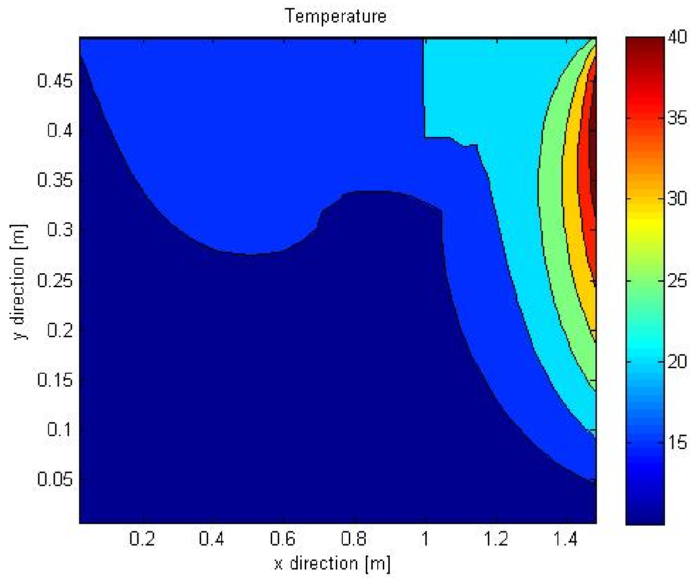
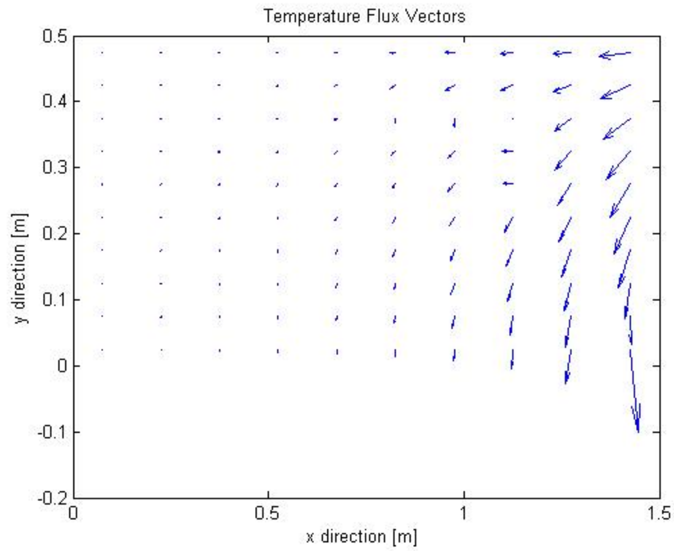


Figure 10: Contour plot of T with 40 x 40 equidistant mesh

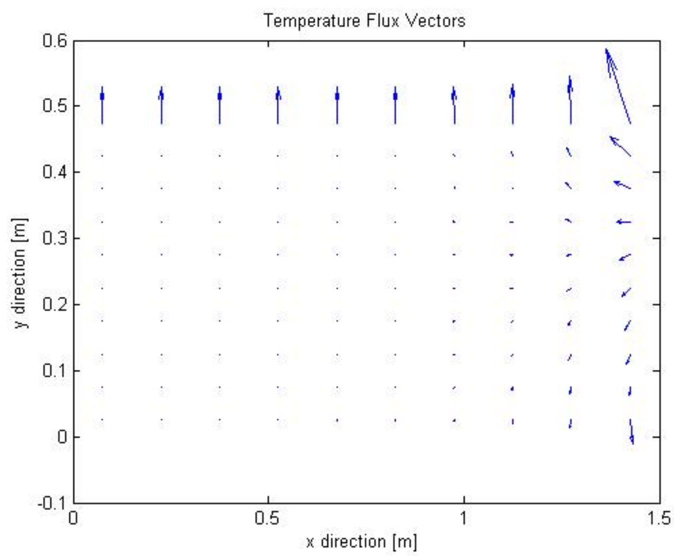
5 Heat flux Vectors

The heat flux vectors in x and y direction are calculated in order to illustrate heat flow. Another important use of these vectors is convergence study. Therefore for a convergence study, heat flux vectors are calculated at each iteration of temperature matrix.

A comparison of heat flux vectors is done as can be seen at Figure 11. In order to see the vectors clearly, 10x10 mesh is selected. A common comment can be done for each case on the bottom left part of the domain. Since Boundary 1 and 4 are defined as Dirichlet B.C and $T = 10^0$ at both walls, one can expect no major heat transfer happens at that area. However regions close to boundary 2, having the highest temperatures, are having the biggest gradients. Vectors also pointing that flow is from boundary 2 to other regions. Both phenomenons are reasonable because as we know from thermodynamic laws, flow should be from warmer temperatures to colder temperatures.



(a) B.C at Boundary 3 is Neumann



(b) B.C at Boundary 3 is Drichlet

Figure 11: Heat flux vectors of equidistant 10x10 mesh for different Boundary Conditions