One Dimensional Fully developed Turbulent Channel Flow

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1 Introduction

In the present task a fully developed turbulent channel flow based on a chosen turbulence model is solved. The flow is one dimensional and the transport equations are simplified into three coupled 1-D diffusion equations for the velocity U and the two turbulent quantities. Since the flow is fully developed the quantities $V = \frac{\partial U}{\partial x} = \frac{\partial k}{\partial x} = \frac{\partial \epsilon}{\partial x} = 0$. The simplified transport equations are:

$$0 = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \tag{1}$$

$$0 = \frac{\partial}{\partial y}\left[(\nu + \frac{\nu_t}{\sigma_k})\frac{\partial k}{\partial y}\right] + P_k + \epsilon \tag{2}$$

$$0 = \frac{\partial}{\partial y}\left[(\nu + \frac{\nu_t}{\sigma_\epsilon})\frac{\partial \epsilon}{\partial y}\right] + \frac{\epsilon}{k}(c_1 P_k - c_2\epsilon) \tag{3}$$

Here the turbulent viscosity and the production term have the form,

$$\nu_t = c_\mu \tfrac{k^2}{\epsilon}$$

$$P_k = \nu_t \big(\tfrac{\partial U}{\partial y}\big)^2$$

Case

The given channel is a horizontal channel having a height of $y_{max} = 2\delta$ between the bottom flat plate (at y=0) and the top flat plate (at $y = y_{max}$). The flow is driven by the pressure gradient $\frac{\partial P}{\partial x}$, as shown in the above momentum equation. since the flow is fully developed, $\frac{\partial P}{\partial x}$ should be a constant and is balanced by the shear stresses. The flow is symmetric about the centerline of the channel. As consequence, only half of the channel is considered for the computational domain.

1.1 Introduction to $k-\epsilon$ Model and Mixing Length Model

In common task mixing length model and $k - \epsilon$ model are used. Purpose of mixing lenght was mainly to initiate the simulation and avoid divergence. After 2000 iterations turbulence model swithces to $k - \epsilon$ model.

Mixing lenght is a one equation model which assumes that kinematic viscosity ν_t can be expressed as product of turbulent velocity scale ϑ and turbulent lenght scale ℓ .

$$\nu_t = C\vartheta\ell$$

C is a dimensionless constant of proportionality. The connection between the mean flow and the largest eddies is important since most of the kinetic energy is in the largest eddies. In a simple two dimansional turbulent flow velocty scale can be stated as

$$\vartheta = c\ell \left| \frac{\partial U}{\partial y} \right|$$

Combining these two equations and two constants into a new length scale ℓ_m the turbulent viscosity equation becomes as below and called Prandtl's mixing length model.

$$\nu_t = \ell_m \left| \frac{\partial U}{\partial y} \right|^2$$

Mixing length ℓ_m is calculated thought the formula for boundary layer, viscous sub-layer

$$\ell_m = \kappa y[1 - exp(-y^+/26)]$$

The standard $k - \epsilon$ model has two model equations. One for turbulence kinetic energy and one for dissipation. Applying dimensional analysis will yield the turbulent viscosity. Then using the Boussinesq assumption, production term is gathered.

$$\nu_t = C_\mu \frac{k^2}{\epsilon}$$

$$P_k = \nu_t \frac{\partial U}{\partial y}^2$$

Since the flow is fully developed modelled k and ϵ equations can be simplified as in the equation (2) and (3) on page 2 because $V = \frac{\partial U}{\partial x} = \frac{\partial k}{\partial x} = \frac{\partial \epsilon}{\partial x} = 0$.

1.2 Introduction to Low Reynolds Number Model

In this part of task the Improved low reynolds number $k - \tilde{\epsilon}$ Model by Rahman and Siikonen was implemented. In the proposed model the turbulent kinetic energy k and the dissipation rate $\tilde{\epsilon}$ are determined by the following equations:

$$0 = \frac{\partial}{\partial y} \left[(\nu + \nu_T) \frac{\partial k}{\partial y} \right] + P_k - \epsilon + E_k$$

$$0 = \frac{\partial}{\partial y} \left[(\nu + \nu_T) \frac{\partial \tilde{\epsilon}}{\partial y} \right] + (c_{\epsilon 1} P_k - c_{\epsilon 2} \tilde{\epsilon} - De^{-(R_y/80)^2})/T_t + E_\epsilon$$

where $\epsilon = \tilde{\epsilon} + D$. The eddy viscosity and other variables are evaluated as

$$\nu_T = C_\mu f_\mu k T_t$$

$$T_t = max(\frac{k}{\tilde{\epsilon}}, C_T \sqrt{\frac{\nu}{\epsilon}})$$

$$D = \frac{2\nu k}{y_n^2}$$

$$R_y = \frac{\sqrt{k} y_n}{\nu}$$

where y_n is the wall normal distance from the wall. The turbulence timescale T_t prevents the singularity at $y_n = 0$ in the dissipation equation. The associated constants are $C_\mu = 0.09$, $c_{\epsilon 1} = 1.44$, $c_{\epsilon 2} = 1.92$, $C_T = \sqrt{2}$, $\sigma_k = 1.0$ and $\sigma_\epsilon = 1.3$.
The near wall damping function f_μ is taken to be function of R_λ defined by

$$f_\mu = 1 - exp(-0.01 R_\lambda - 0.0068 R_\lambda^3)$$

$$R_\lambda = y_n / \sqrt{\nu k / \tilde{\epsilon}} = y_n / \sqrt{\nu T_t}$$

The use of R_λ confronts the sigularity neighther the seperating nor the reattatching point in contrast to adoption of $y^+ = \frac{u_\tau y}{\nu}$.
The cross diffusion terms in the above equations E_k and E_ϵ are designed as

$$E_k = C_k C_\mu min\left[\frac{\partial(k/\epsilon)}{\partial y} \frac{\partial \tilde{\epsilon}}{\partial y}, 0\right]$$

$$E_\epsilon = C_\epsilon \frac{\nu_T}{T_t^2} \left[\frac{\partial(k/\epsilon)}{\partial y} \frac{\partial k}{\partial y}\right]$$

here $C_k = 0.5$ and $C_\epsilon = -2C_k$.

2 k-ϵ Model

In this section results of turbulent channel flow will be presented. As introduced before, mixing length and standard $k - \epsilon$ models were used in common task. A new mesh is not created but y coordinates provided for DNS data are used. Under relaxation factor is taken as 1 since there we no divergence experienced during calculations.

Boundary conditions are treated implicitly by manipulating the coefficient matrices. At the wall mean velocity U and turbulent kinetic energy k have Dirichlet boundary condition while dissipation ϵ has Neumann boundary condition. On the symmetry line of the channel; k , U and ϵ have Neumann boundary condition. Dirichlet B.C at the wall is below

$$U_{wall} = k_{wall} = 0$$

As explained in problem description source terms were complicated and negative terms are treated specially as can be seen above. S_P terms for k and ϵ respectively are as below

$$S_P = -\epsilon \Delta x / \mathrm{k}$$

$$S_P = -C_{2\epsilon}\epsilon\Delta x/\mathrm{k}$$

By usingGauss-Seidel solver convergence is reached for standard $k-\epsilon$ model after 4766 iterations where the first 2000 iterations were mixing length model. Residuals are calculated as right hand side minus left hand side of the standard form of discretization equation. Flux and maximum error used for convergence are as above

$$F = \sum U^2 \Delta x$$

$$error_{max} = 0.001$$

Calculated mean flow U can be seen figure 1. Obviously there is a great deviation from DNS data because of incapability of the standard $k-\epsilon$ model. In order to see what caused this result, it is important to remember the equation (1) on page 2. Only varying value in momentum equation is ν_t. Therefore the velocity profile is wrong because ν_t values are not proper.

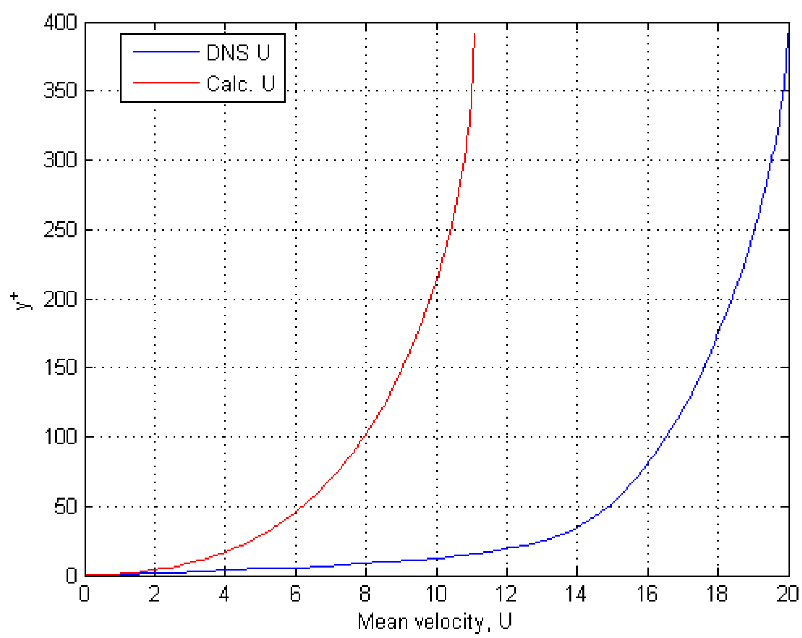

Figure 1: Comparison of U with the DNS data for the standard $k-\epsilon$ model

In standard $k-\epsilon$ model, ν_t is based on k and ϵ values. Therefore the deviation of U is pointing k and ϵ. Figure 2 and figure 3 are showing the calculated values for k and ϵ with the DNS data. First thing to realize for k is deviation from the DNS at the log-law region around $30 < y^+ < 100$. The asymptotic behaviour of

k couldn't captured at the described region while outer region is captured fairly good. For the dissipation, viscous region and some parts of log-law region was not captured successfully.

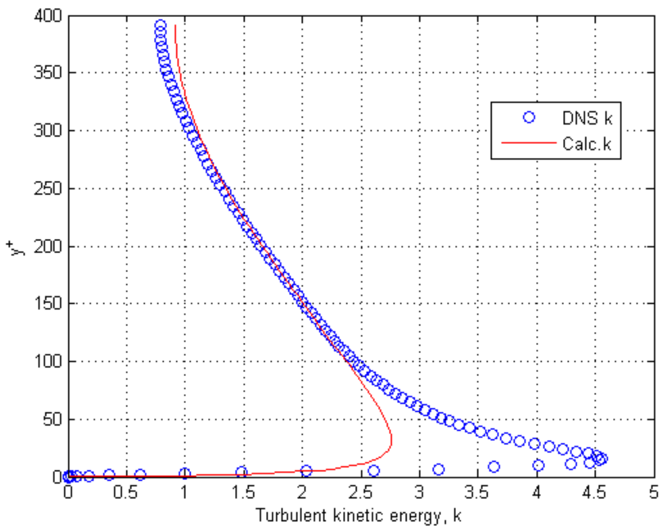

Figure 2: Comparison of k with the DNS data for the standard $k-\epsilon$ model

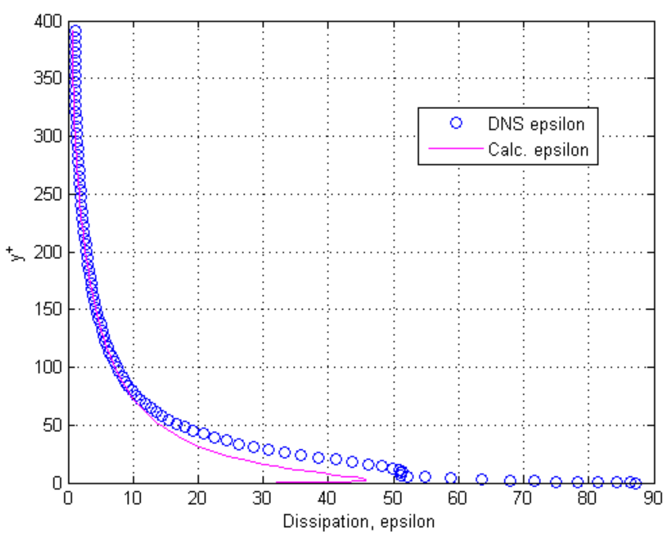

Figure 3: Comparison of ϵ with the DNS data for the standard $k-\epsilon$ model

In order to see how wrong ν_t is gathered from these values presented at figure 2 and figure 3, we can plot the calculated $(\nu + \nu_t)$ as can be seen at figure 4

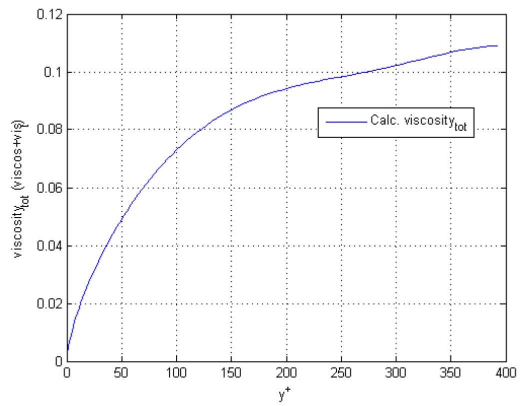

Figure 4: $(\nu + \nu_t)$ for the standard $k - \epsilon$ model

Since we blamed ν_t for deviation of mean velocity U in the standard $k-\epsilon$ model. We tried to use only the mixing length model. Results can be seen at figure 5 and figure 6.U improved considerably since ν_t) is better.

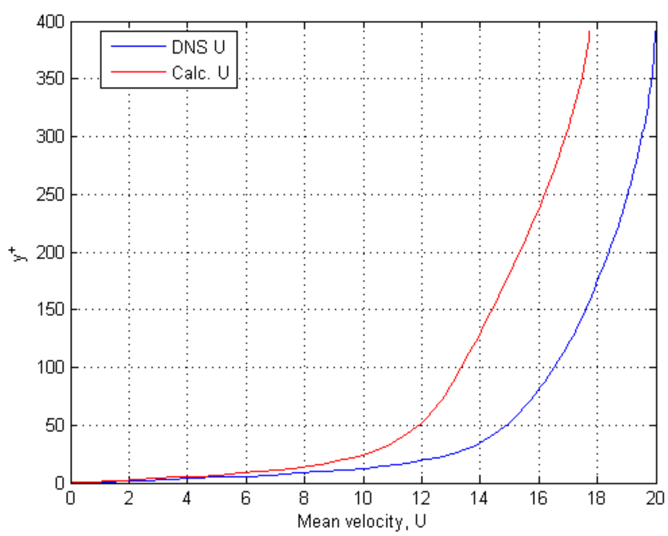

Figure 5: Comparison of U with the DNS data for the mixing lenght model

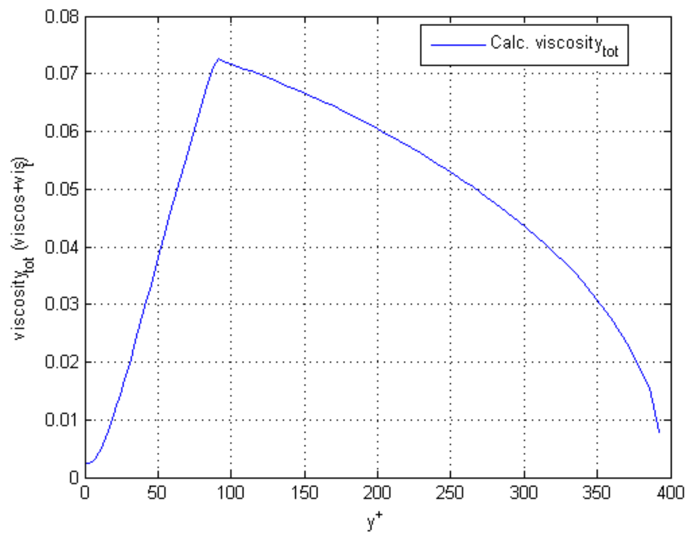

Figure 6: $(\nu + \nu_t)$ for the mixing lenght model

3 Low Reynolds Number k-ϵ Model

In this section results of the turbulent channel flow with an improved Low Reynolds Number $k-\tilde{\epsilon}$ model will be discussed. Like the previous model mixing length and improved $k-\tilde{\epsilon}$ models were used in this task. The mesh for this task is adopted based on the DNS data. The under relaxation is not considered in this case as there is no divergence experienced. The boundary conditions are adopted from the common task.

Unlike the previous model the present task is a $k-\tilde{\epsilon}$ model, where $\epsilon = D + \tilde{\epsilon}$. Here the value of D is equal to the wall value of ϵ and this model contains extra source terms like the cross diffusion terms (E_k, E_ϵ), D, turbulent time scale T_t etc. The mixing length model was implemented for the first 2000 iterations and then proceeded with the improved Low Reynolds Number $k-\tilde{\epsilon}$ model. The description of the extra source terms and the values of the constants are as described in section 1.2.

In the present implementation the source terms are split into S_U and S_P as usual. The following equations show how the source terms are split in k and $\tilde{\epsilon}$ equations respectively.

k-equation

$$S_U = P_k \delta y$$

$$S_P = \frac{(\epsilon - E_k)\delta y}{k}$$

$\tilde{\epsilon}$-equation

$$S_U = \frac{c_{\epsilon 1} P_k}{T_t} \delta y$$

$$S_P = ((c_{\epsilon 2}\tilde{\epsilon} - De^{-(R_y/80)^2})/T_t + E_\epsilon)\ \delta y/\tilde{\epsilon}$$

The equation system is solved using Guass-Seidel method and the convergence is reached after 23381 iterations. It is obvious that the equations converge very slowly because of the additional source terms and coupling terms.

3.1 Comparison of U, k and ϵ with DNS

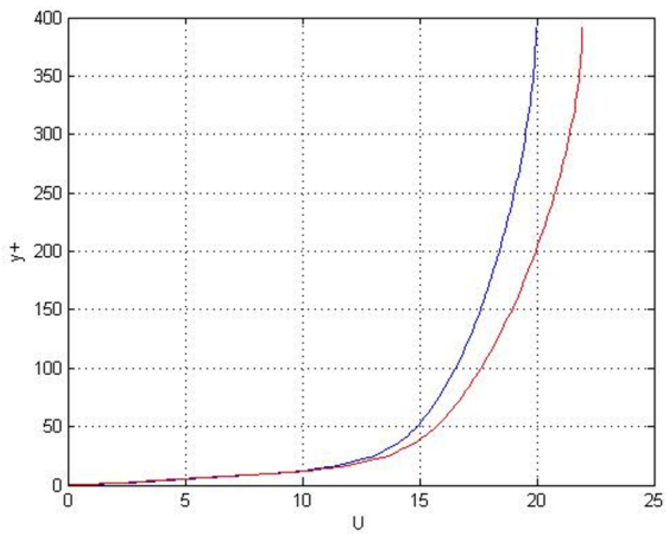

Figure 7: Comparison of U with the DNS data

Figure 7 shows the comparison of calculated U with DNS U. The calculated U follows similar trend as the DNS, however there is a little divergence as the y+ proceeds. The present model showed a great improvement compared with the common task. Here we can clearly obseve the influence of the viscous damping functions based on which the turbulent viscosity is calculated.

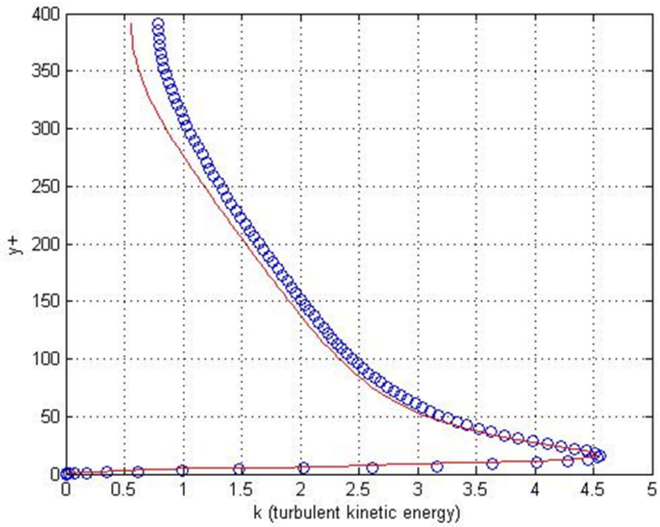

Figure 8: Comparison of k with the DNS data

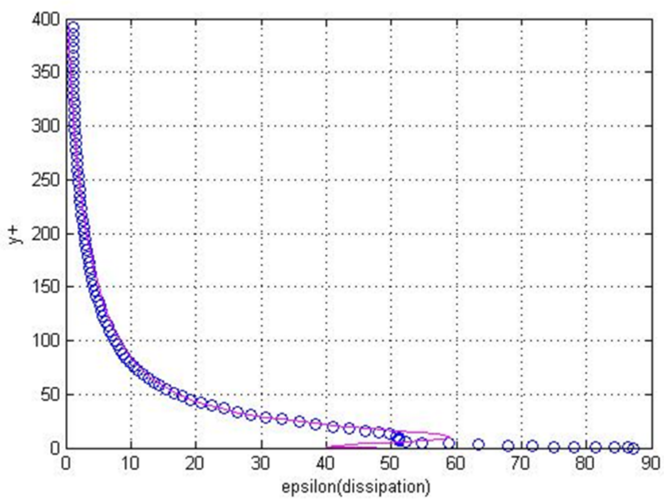

Figure 9: Comparison of ϵ with the DNS data

Figures (8) and (9) show the k and ϵ compared with the corresponding DNS data. The values of k and ϵ almost converged with the values of the corresponding DNS data. The asymptotoc behaviour could not be captured for k but the asymptotic behaviour is observed for ϵ near the wall in Figure (9).

3.2 Shear stress $\bar{uv}$

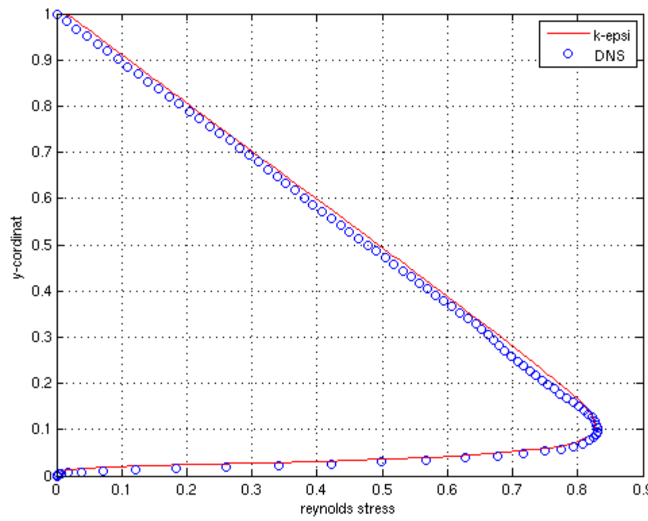

Figure 10: Comparison of shear stress($\bar{uv}$) with the DNS data

Figure 10 shows the calculated shear stress along y-coordinate compared with the corresponding DNS data. The values of calculated shear stress are converged with the DNS data which signify the accuracy of the model. The asymptotic behaviour couldn't be captured near the wall region in this case.

3.3 Comparison between Kolmogorov and Large turbulent scales

The Large turbulent scales were calculated based on the turbulent Reynolds number R_λ and are compared with the Kolmogorov scales as shown in Figures (11) and (12). Figure 11 shows the actual comparison between the Kolmogorov and large turbulent scales. The values of Large turbulent are very large compared with the Kolmogorov scales. In Figure 11, the large scales are not completely captured in the grid of the plot. The values of the Large scales are scaled down 10 times of their actual value in order to observe the trend (Figure 12). By this comparison it can be presumed that the values of the Large scales are approximately 100 times greater than the Kolmogorov scales.

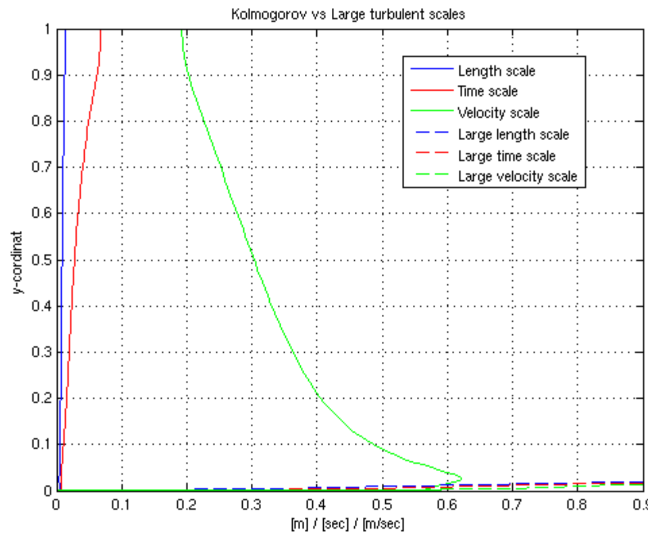

Figure 11: Comparison between Kolmogorov and Large turbulent scales

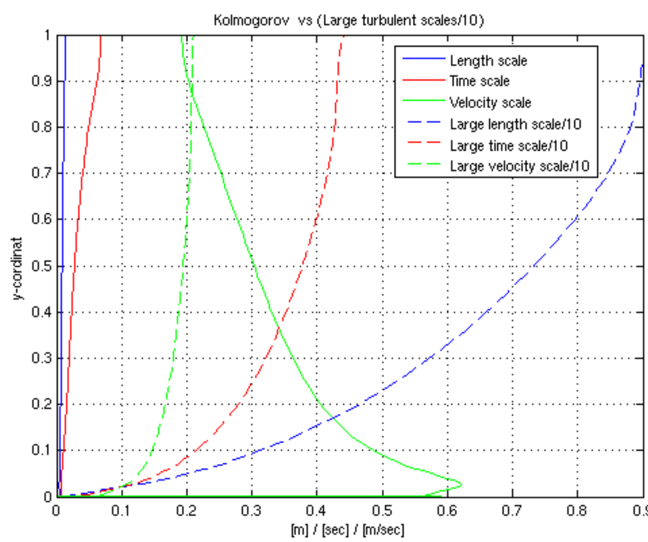

Figure 12: Comparison between Kolmogorov and (Large turbulent scales/10)

3.4 Budget of k equation

In the budget of the k-equation the dissipation ϵ, production P_k, diffusion due to pressure velocity fluctuations, diffusion due to triple velocity fluctuations and viscous diffusion are compared with the corresponding DNS data.

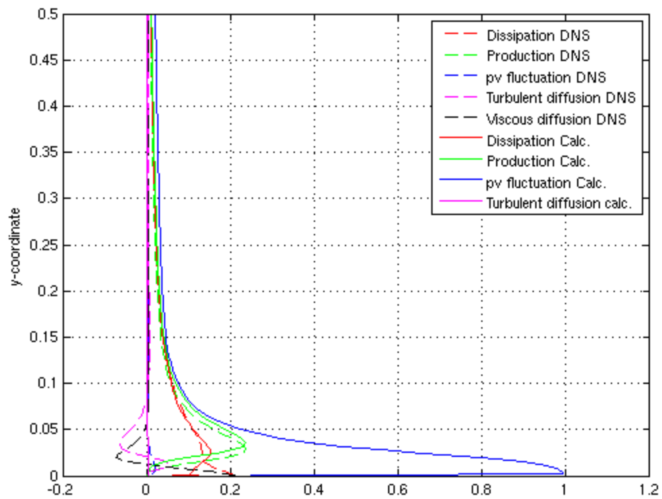

Figure 13: Comparison of calculated turbulent quatities with DNS data

Figure 13 shows the budget of k excluding the calculated viscous diffusion term. The viscous diffusion showed as very large divergence at the near wall region and couldn't be captured in the present grid of the plot. The pressure velocity fluctuation term showed a slight large divergence near the wall.

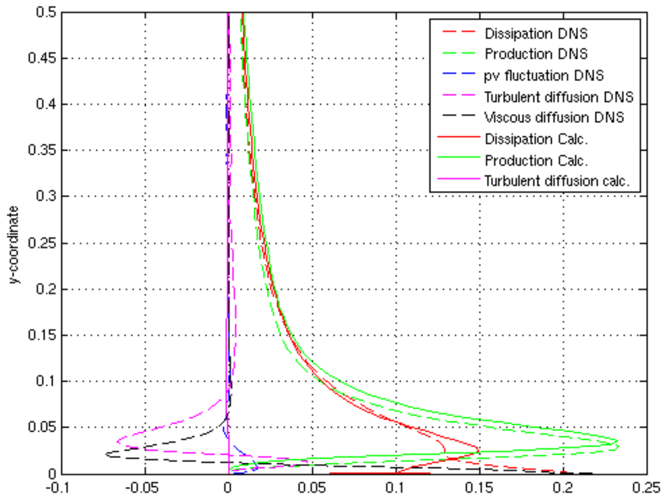

Figure 14: Comparison of calculated turbulent quatities with DNS data zoomed

Figure 14 shows the budget of k excluding the calculated viscous diffusion term and the pressure velocity fluctuation term. It can be observed that the model predicts the dissipation, production and turbulent diffusion quite well. However there is very slight divergence in the turbulent diffusion term near the wall.

3.5 Conclusion

The improved Low Reynolds Number $k - \tilde{\epsilon}$ model predictions are qualitatively good compared with the ordinary $k - \epsilon$ model except the pressure velocity fluctuation term and the viscous diffusion term. However there is a scope of development of model near the wall.