The notion of “likelihood” is intrinsic to our worldview, so I will not belabor its definition. Onto this intuition we are going to impose some formality. This will enable us to formulate odds exactly, to agree on what we mean by odds, and to construct large, complex, and powerful odds machines that can contend with models and decisions which exceed our ability to reason intuitively.

To those who think the ratio of English to equations in my lessons is too high, rejoice, for there are many formulas here to learn; to those who fear walls of equations, simply regard equations as unusual English and read them as such.

## Sample Spaces and Events

What are the possible outcomes of a coin flip? Are they “heads” and “tails”? What if the coin lands on its edge? What if a bird snatches it out of the air? Even in situations where certain outcomes are by far the most likely, the messy universe provides infinitely many other outcomes.

Hence, in formulating a problem we define the outcomes that we are going to consider. This set of outcomes is called the sample space.

\*\*defn\*\*

Recall that the elements of a set can be pretty much anything. So the sample space of a coin toss could be defined as S = {H, T}, while the outcome of an experiment to measure height could be the positive real numbers, an infinite set.

In this course and in standardized tests you will come across questions like “what are the odds of rolling an even number” or “what are the odds of drawing two queens and two hearts when choosing four cards.” We need a systematic way to deal with these complex collections of outcomes.

We call any collection of outcomes an event, and rely on set theory to combine and compose events.

\*\*defn\*\*

Comprehension Q’s:

1. Define a reasonable sample space for the following experiments:
   1. Rolling two dice
   2. Measuring stellar masses
   3. Recording someone’s purchases and sales in a market
2. For the sample space S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, describe the following events using set theory and English:
   1. Event A = An odd number occurs
   2. Event B = A number greater than 3 occurs
   3. Events A and B occur
   4. Events A and not(b) occur

## Probability Functions

A probability function assigns real numbers to events in the sample space. Put another way, it is a map or function f:B -> [0,1], where B is the collection of all possible events. Note that this is not the same as the sample space. If S = {H, T}, B = {{}, {H}, {T}, {H, T}}.

A probability function must obey the following axioms:

\*\* defn \*\*

Here are some useful identities for the probabilities of two events. They are all derivable from the Kolmogorov Axioms.