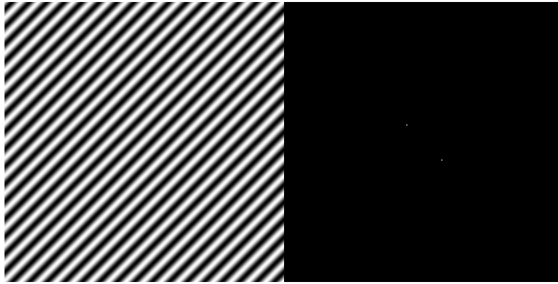
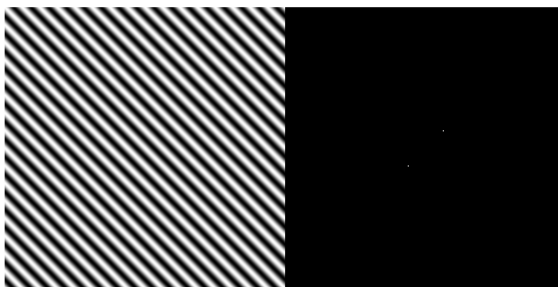


2D Sinusoids

A) The image of $\cos[2\pi(8x/128 + 8y/128)]$ and its spectrum:



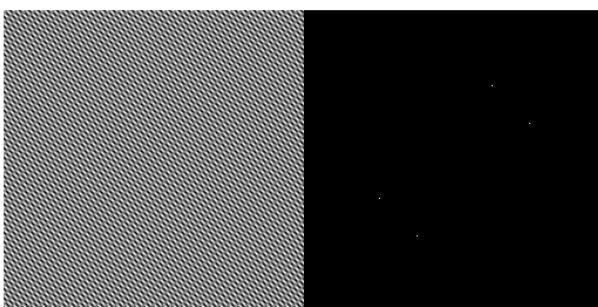
B) The image of $\cos[2\pi(8x/128 - 8y/128)]$ and its spectrum:



In 1D cos function is an even function with respect to x ($\cos(x) = \cos(-x)$) but this is not the case when it is in 2D as it is with the images.

A sign transformation in the variables affects the image by changing its time domain values and frequency domain value. Therefore in the left, the image is the symmetry of the first one with respect to y axis and its spectrum also the symmetry of the first one w.r.t. v axis.

C) The image of $\cos[2\pi(8x/128 - 8y/128)] \cos[2\pi(24x/128 - 24y/128)]$ and its spectrum:

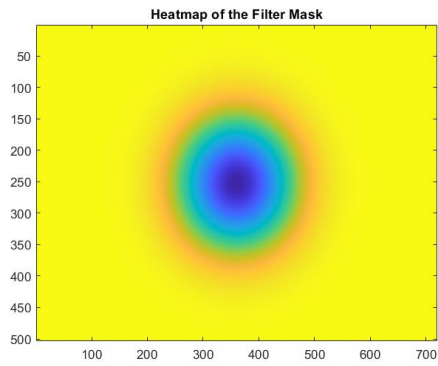


Since it is a multiplication in the time domain the image is the multiplication of the two cos function value and one of them has the -y component therefore the plots are intertwined.

Multiplication in one domain means convolution in the other. The value of the frequencies are larger in the $\cos[2\pi(24x/128 - 24y/128)]$ function therefore their convolution got far away from the center.

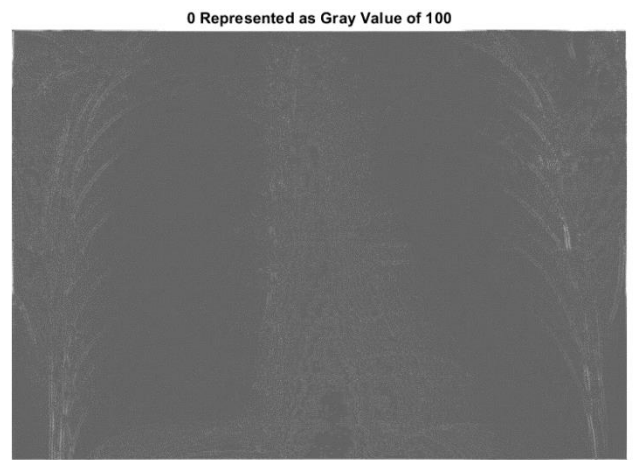
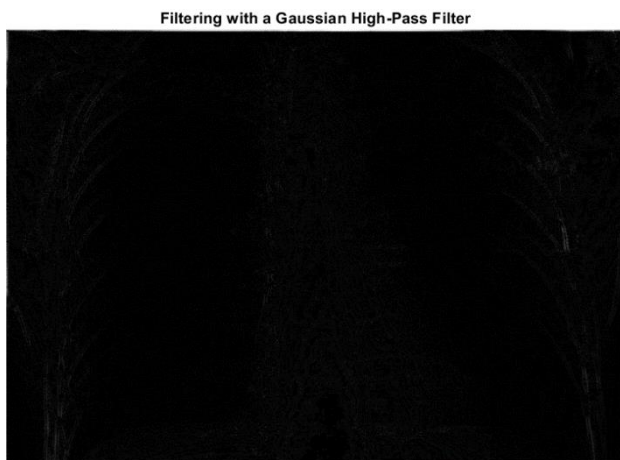
Unsharp Masking and High-Boost Filtering:

a)

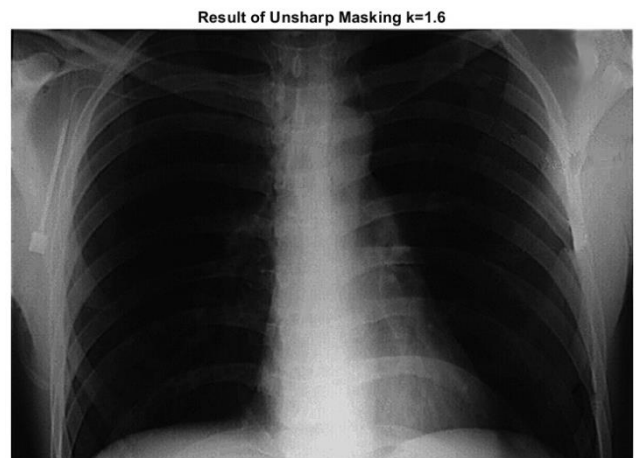
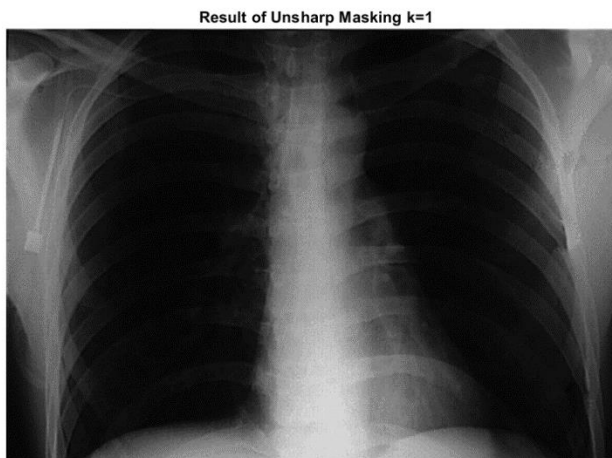


Since we can choose D_0 as the %5-10 of the long image dimension, I chose it to be $D_0 = \%10.720 = 72$.

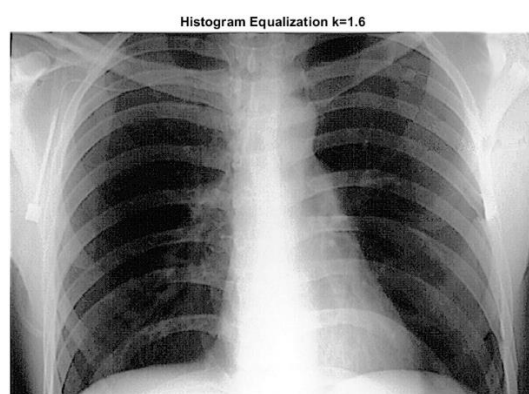
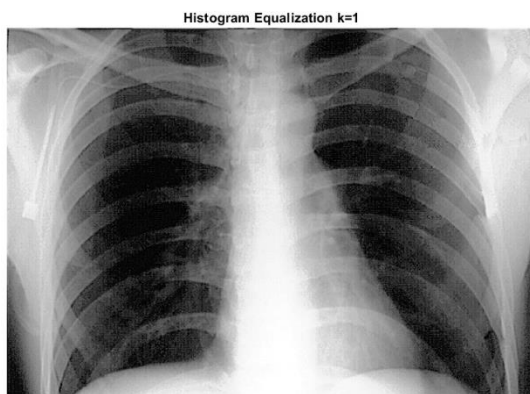
b)



c)



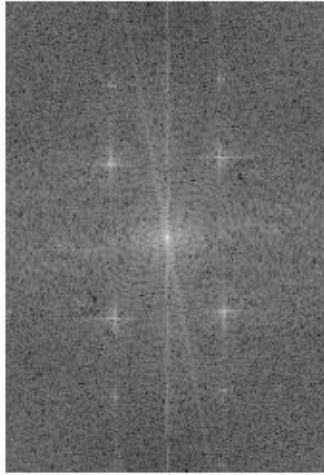
d)



Moiré Noise Removal

a)

Magnitude Spectrum of the Image



b) I chose the peaks and their diameters where the peak value is decreased to its $1/\sqrt{2}$ value interactively on the magnitude spectrum on the left. The peak points (x,y) and diameters (taking the image center as (0,0)):

H1: peak at (-28.5,37.5) , d0=18

H2: peak at (28.5,-37.5) , d0=18

H3: peak at (-26.5,-42.5) , d0=16

H4: peak at (26.5,42.5) , d0=16

H5: peak at (-26.5,-83.5) , d0=8

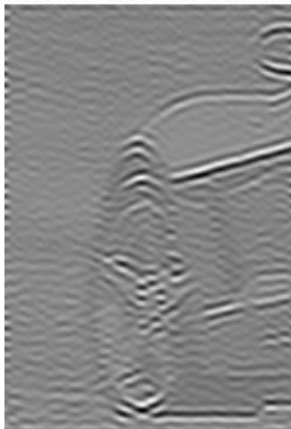
H6: peak at (26.5,83.5) , d0=8

H7: peak at (30.5,-79.5) , d0=10

H8: peak at (-30.5,79.5) , d0=10

c)

Extracted Moire Pattern



d)

Filter Spectrum With the Moire Noise Components Eliminated

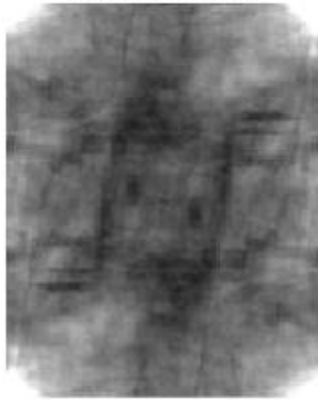


Image Without the Moire Noise



Importance of the Phase:

a) **Reconstruction of Trump from only $|Trump|$**

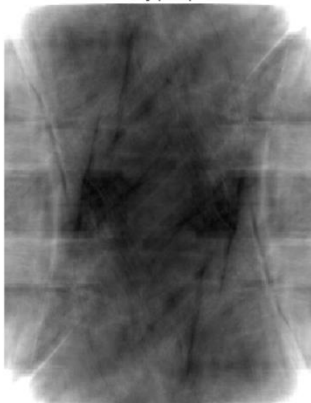


Reconstruction of Trump from only $\angle(Trump)$



Magnitude spectrum does say much about the shapes in the picture and the human eye is more sensitive to catching shape structures. Therefore phase spectrum gives more information about the images, when they are displayed.

b) **Reconstruction of RTE from only $|RTE|$**



Reconstruction of RTE from only $\angle(RTE)$



c) **Reconstruction of Trump from $|Trump|$ and $\angle(RTE)$**



d) **Reconstruction of Trump from $|RTE|$ and $\angle(Trump)$**



Since the phase spectrum is the determining factor of the shapes we see the shape of the face that belongs to the phase spectrum's source image.

e)

Reconstruct Trump from $|MTr(u,v)| \exp j\theta Tr(u,-v)$



Since we needed to find $-v$ of the $F(u,v)$ I took the symmetry with respect to u axis in order to work with $-v$ values. Since the phase spectrum is mirrored, also the shapes on the image are mirrored.

From Probs. 4.51, 4.52

a)

Due to properties of the DFT, we know that:

$$\frac{\partial^m}{\partial t^m} f(t,z) \Leftrightarrow (j2\pi\nu)^m F(\mu,\nu) \quad \text{and} \quad \frac{\partial^n f(t,z)}{\partial z^n} \Leftrightarrow (j2\pi u)^n F(u,\nu)$$

We also know that Laplacian of a function is defined as:

$$\nabla^2 f(x,y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

Then we can calculate the components as:

$$\frac{\partial^2 f(x,y)}{\partial x^2} \Leftrightarrow (j2\pi u)^2 F(u,\nu)$$

$$\frac{\partial^2 f(x,y)}{\partial y^2} \Leftrightarrow (j2\pi \nu)^2 F(u,\nu)$$

Therefore:

$$\begin{aligned} \mathcal{F}\{\nabla^2 f(x,y)\} &= (j2\pi)^2 F(u,\nu) (u^2 + \nu^2) \\ &= -4\pi^2 (u^2 + \nu^2) F(u,\nu) \end{aligned}$$

b)

1-

0	1	0
1	-4	1
0	1	0

This mask can be written as:

$$= f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

If we take the Fourier transforms:

$$\Rightarrow F(u, v) \left(e^{j\frac{2\pi u}{M}} + e^{-j\frac{2\pi u}{M}} + e^{j\frac{2\pi v}{N}} + e^{-j\frac{2\pi v}{N}} - 4 \right)$$

$$= 2 \left(\cos \frac{2\pi u}{M} + \cos \frac{2\pi v}{N} - 2 \right) = H_1(u, v)$$

2.

1	1	1
1	-8	1
1	1	1

$$= f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

$$+ f(x+1, y+1) + f(x+1, y-1) + f(x-1, y+1) + f(x-1, y-1) - 4f(x, y)$$

$$\Rightarrow \left[H_1(u, v) + \left[e^{j\frac{2\pi u}{M}} e^{j\frac{2\pi v}{N}} + e^{j\frac{2\pi u}{M}} e^{-j\frac{2\pi v}{N}} + e^{-j\frac{2\pi u}{M}} e^{j\frac{2\pi v}{N}} + e^{-j\frac{2\pi u}{M}} e^{-j\frac{2\pi v}{N}} - 4 \right] \right] F(u, v)$$

$$\Rightarrow F(u, v) \left[H_1(u, v) + 2e^{j\frac{2\pi u}{M}} \cos \frac{2\pi v}{N} + 2e^{-j\frac{2\pi u}{M}} \cos \frac{2\pi v}{N} - 4 \right]$$

$$\Rightarrow F(u, v) \left[H_1(u, v) + 4 \cos \frac{2\pi v}{N} \cos \frac{2\pi u}{M} - 4 \right]$$

$$\Rightarrow F(u, v) \left[H_1(u, v) + 2 \cos \left(\frac{2\pi v}{N} - \frac{2\pi u}{M} \right) + 2 \cos \left(\frac{2\pi v}{N} + \frac{2\pi u}{M} \right) - 4 \right]$$

$$H_2(u, v) = 2 \cos \frac{2\pi u}{M} + 2 \cos \frac{2\pi v}{N} - 8 + 2 \cos \left(\frac{2\pi v}{N} - \frac{2\pi u}{M} \right) + 2 \cos \left(\frac{2\pi v}{N} + \frac{2\pi u}{M} \right)$$