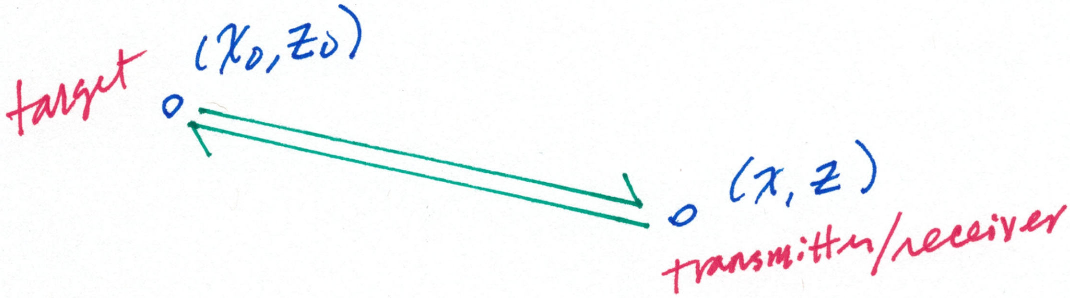
ECE 278C Imaging Systems

9. Resolution limit (active systems)

Department of Electrical and Computer Engineering
University of California, Santa Barbara

$\boxed{\text{Monostatic mode}}$



$$\frac{1}{j\lambda r} \cdot \exp(j 2\pi r/\lambda) \cdot \frac{1}{j\lambda r} \cdot \exp(j 2\pi r/\lambda)$$

$$= \left(\frac{1}{j\lambda r}\right)^2 \exp(j 2\pi \, 2r/\lambda)$$

$$r = \sqrt{(x-x_0)^2 + (z-z_0)^2}$$

$$f_x = \frac{\partial}{\partial x}\left(\frac{2r}{\lambda}\right)$$

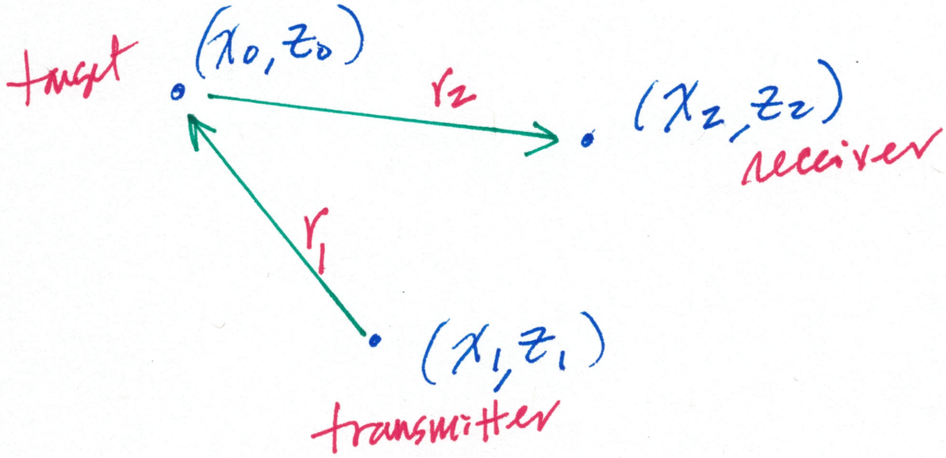
$$= \frac{2}{\lambda} \cdot \frac{\partial r}{\partial x} = \frac{2}{\lambda} \cdot \frac{\frac{1}{2} \cdot 2 \cdot (x-x_0)}{\sqrt{(x-x_0)^2 + (z-z_0)^2}}$$

$$= \frac{2}{\lambda} \cdot \sin\theta$$

$$f_z = \frac{\partial}{\partial z}\left(\frac{2r}{\lambda}\right) = \frac{2}{\lambda} \cos\theta$$

$$f = (f_z, f_x) = \frac{2}{\lambda}(\cos\theta, \sin\theta) \leftarrow$$

Bistatic mode



transmission

$$\frac{1}{j\lambda r_1} \exp(j 2\pi r_1/\lambda) \qquad r_1 = \sqrt{(x_1 - x_0)^2 + (z_1 - z_0)^2}$$

modulation
at (x_0, z_0)

$$f_x = \frac{\partial}{\partial x_0}(r_1/\lambda)$$

$$= \frac{1}{\lambda} \cdot \frac{-\frac{1}{2} \cdot 2 \cdot (x_1 - x_0)}{\sqrt{(x_1 - x_0)^2 + (z_1 - z_0)^2}}$$

$$= \frac{1}{\lambda} \cdot \frac{-(x_1 - x_0)}{r} = -\frac{1}{\lambda} \cdot \sin\theta_1$$

$$f_z = \frac{\partial}{\partial z_0}\left(\frac{r_1}{\lambda}\right)$$

$$= \frac{1}{\lambda} \cdot \frac{-\frac{1}{2}\cdot 2 \cdot (z_1 - z_0)}{\sqrt{(x_1-x_0)^2+(z_1-z_0)^2}}$$

$$= \frac{1}{\lambda} \cdot \frac{-(z_1-z_0)}{r}$$

$$= -\frac{1}{\lambda} \cdot \cos\theta_1$$

modulation
at (x_0, z_0)
by the wavefield
with spatial frequency

$$f_1 = \underline{-\frac{1}{\lambda}(\cos\theta_1, \sin\theta_1)}$$

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at (x_2, z_2)

$$f_x = \frac{\partial}{\partial x_2}\left(\frac{r_2}{\lambda}\right) = \frac{1}{\lambda}\sin\theta_2$$

$$f_z = \frac{\partial}{\partial z_2}\left(\frac{r_2}{\lambda}\right) = \frac{1}{\lambda}\cos\theta_2$$

$$f_2 = \frac{1}{\lambda}(\cos\theta_2, \sin\theta_2)$$

Overall

$$f = f_2 \underbrace{- f_1}_{\text{demodulation}} = \frac{1}{\lambda}(\cos\theta_1 + \cos\theta_2, \sin\theta_1 + \sin\theta_2)$$

Back to monostatic

$\theta_1 = \theta_2$

$$f_\cdot = \frac{1}{\lambda}(2\cos\theta, 2\sin\theta)$$

$$= \frac{2}{\lambda}(\cos\theta, \sin\theta)$$

5

Summary

1. The role of *a detected wavefield sample* is equivalent to *a sample of the spatial-frequency spectrum.* A set of N wavefield samples over the aperture contains the information of N spatial-frequency components.

2. With a passive coherent system, a complex wavefield data sample represents a sample of the spatial spectrum. The spatial frequency of this spectral sample is located along a circle of radius $1/\lambda$. The bearing angle of the location, θ, is the same angle defined by the relative bearing between the source location and the receiver. It can be represented in the complex form as $\frac{1}{\lambda} exp(j\theta)$.

3. For an active bistatic system, a complex wavefield data sample also represents a sample of the spatial spectrum. The spatial frequency of this spectral sample is now located at $\frac{1}{\lambda}\,(exp(j\theta_1)+exp(j\theta_2))$, where θ_1 and θ_2 are bearing angles of the transmitter and receiver, from the perspective of the target location.

4. The monostatic data-acquisition mode is a special case of the bistatic format, where the transmitter and receiver are combined into one transceiver unit. Thus, it is the case $\theta = \theta_1 = \theta_2$. Therefore, the spatial frequency of this data sample is located at $\frac{2}{\lambda} exp(j\theta)$. The resolution is improved by a factor of *2* as a result.

	Modality	*Spectral location*	*Complex form*
1	*passive*	$f = \frac{1}{\lambda}\,(\cos\theta, \sin\theta)$	$f = \frac{1}{\lambda}\,exp(j\theta)$
2	*monostatic*	$f = \frac{2}{\lambda}\,(\cos\theta, \sin\theta)$	$f = \frac{2}{\lambda}\,exp(j\theta)$
3	*bistatic*	$f = \frac{1}{\lambda}\,(\cos\theta_1 + \cos\theta_2,\ \sin\theta_1 + \sin\theta_2)$	$f = \frac{1}{\lambda}\,(exp(j\theta_1) + exp(j\theta_2))$