ECE 278C Imaging Systems

8. Resolution limit (passive systems)

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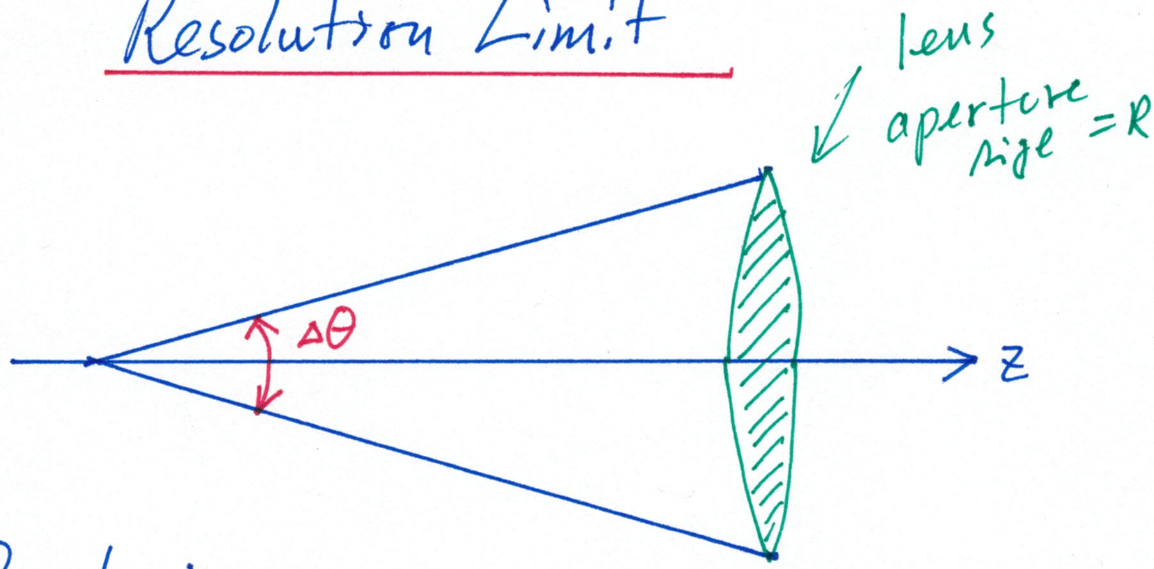
Resolution Limit



Rayleigh
resolution limit

$$\boxed{\Delta x = \frac{\lambda}{2\sin\left(\frac{\Delta\theta}{2}\right)}}$$

1. wavelength λ
2. propagation distance z
3. Aperture size R
4. Infinite aperture $\frac{\theta}{2} = 90^\circ$

$$\Delta x = \frac{\lambda}{2}$$

half-wavelength

approximation (small aperture)

$$\Delta x = \frac{\lambda}{2\sin\left(\frac{\Delta\theta}{2}\right)}$$

$$= \left(\frac{\lambda}{2}\right)\cdot \frac{1}{\sin\left(\frac{\Delta\theta}{2}\right)}$$

due to finite-size aperture

limit

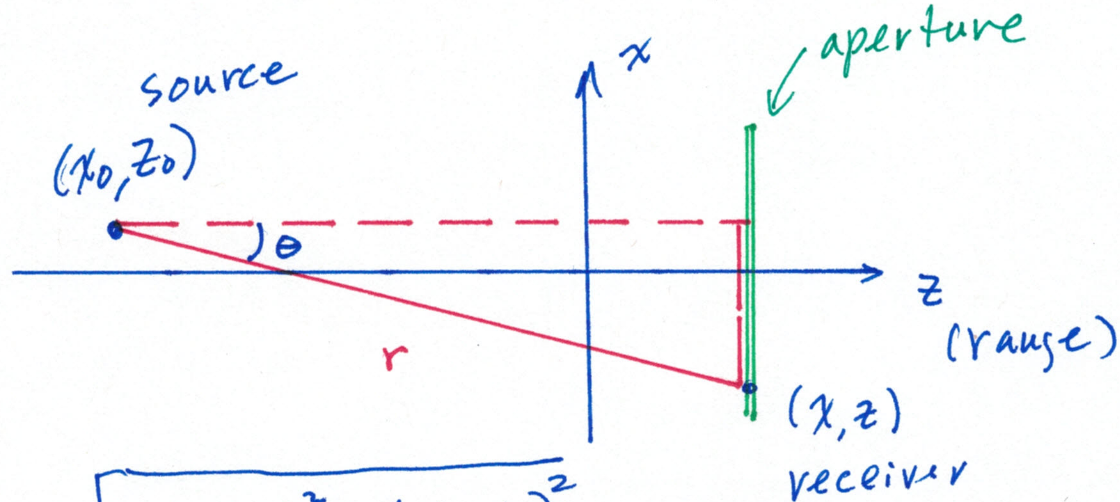
$$\sin\left(\frac{\Delta\theta}{2}\right) = \frac{(R/2)}{\sqrt{z^2+(R/2)^2}}$$

small aperture

$$\sin\left(\frac{\Delta\theta}{2}\right) \approx \tan\left(\frac{\Delta\theta}{2}\right) = \frac{R/2}{z}$$

$$\Delta x \approx \left(\frac{\lambda}{2}\right)\cdot\frac{z}{R/2} = \frac{\lambda z}{R}$$

Resolution Analysis



$$r = \sqrt{(z - z_0)^2 + (x - x_0)^2}$$

Resultant wavefield pattern

$$g(x, z) = \frac{1}{j\lambda r} \exp\left(j 2\pi r/\lambda\right)$$

$$= \frac{1}{j\lambda r} \exp(j\phi)$$

$$\phi = 2\pi r/\lambda$$

function of (x, z)

Frequency

$$\omega_x = 2\pi f_x = \frac{\partial}{\partial x}\phi(x,y)$$

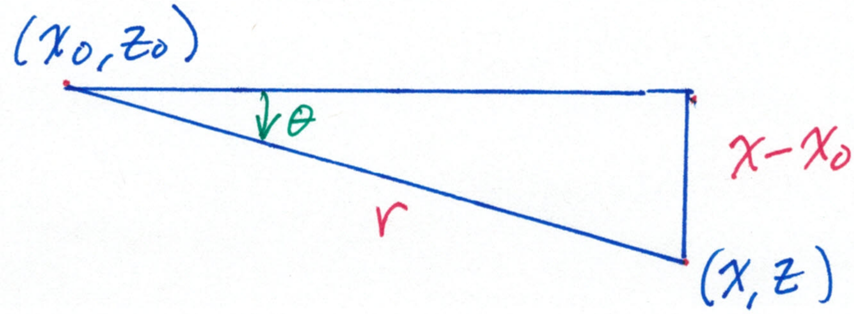
$$f_x = \frac{\partial}{\partial x}\left(\frac{r}{\lambda}\right)$$

$$= \frac{1}{\lambda}\frac{\partial}{\partial x}\sqrt{(x-x_0)^2+(z-z_0)^2}$$

$$= \frac{1}{\lambda}\cdot\frac{1/2}{\sqrt{(x-x_0)^2+(z-z_0)^2}}\cdot 2(x-x_0)$$

$$= \frac{1}{\lambda}\cdot\left(\frac{x-x_0}{r}\right)$$

$$= \frac{1}{\lambda}\sin\theta$$



range direction

$$\omega_z = 2\pi f_z = \frac{\partial}{\partial z}\phi$$

$$f_z = \frac{\partial}{\partial z}\left(\frac{r}{\lambda}\right)$$

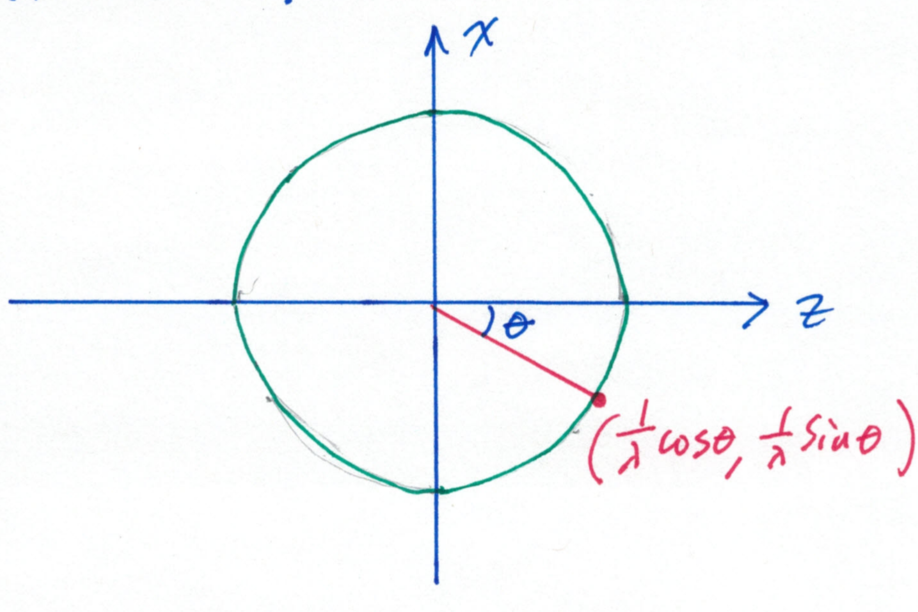
$$= \frac{1}{\lambda} \cdot \frac{z - z_0}{\sqrt{(x-x_0)^2 + (z-z_0)^2}}$$

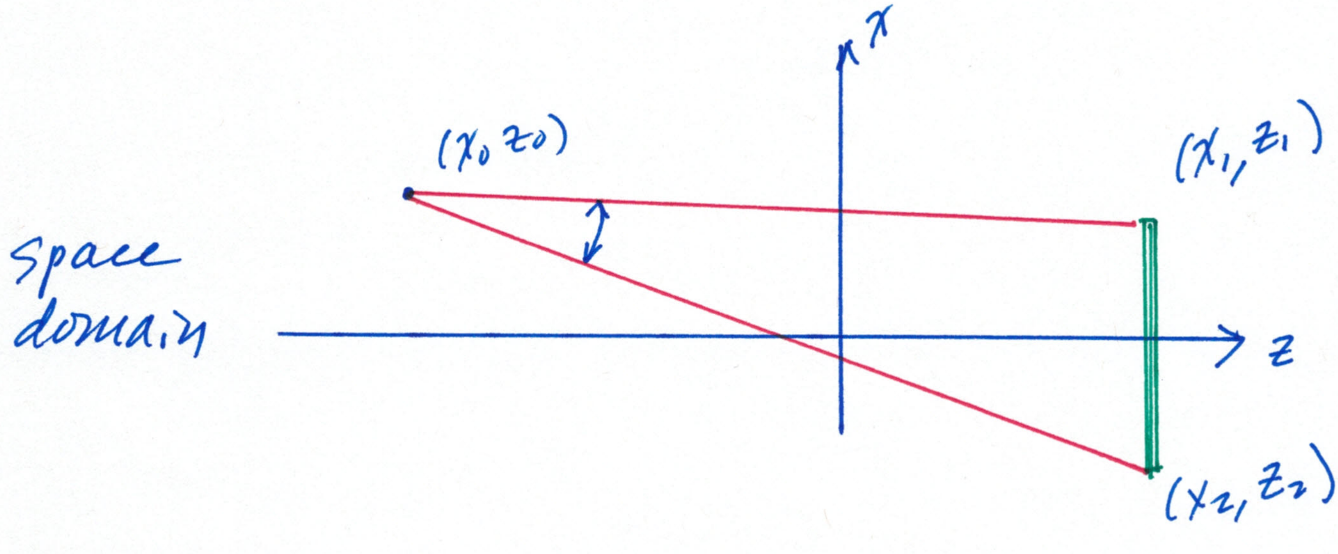
$$= \frac{1}{\lambda} \cdot \cos\theta$$

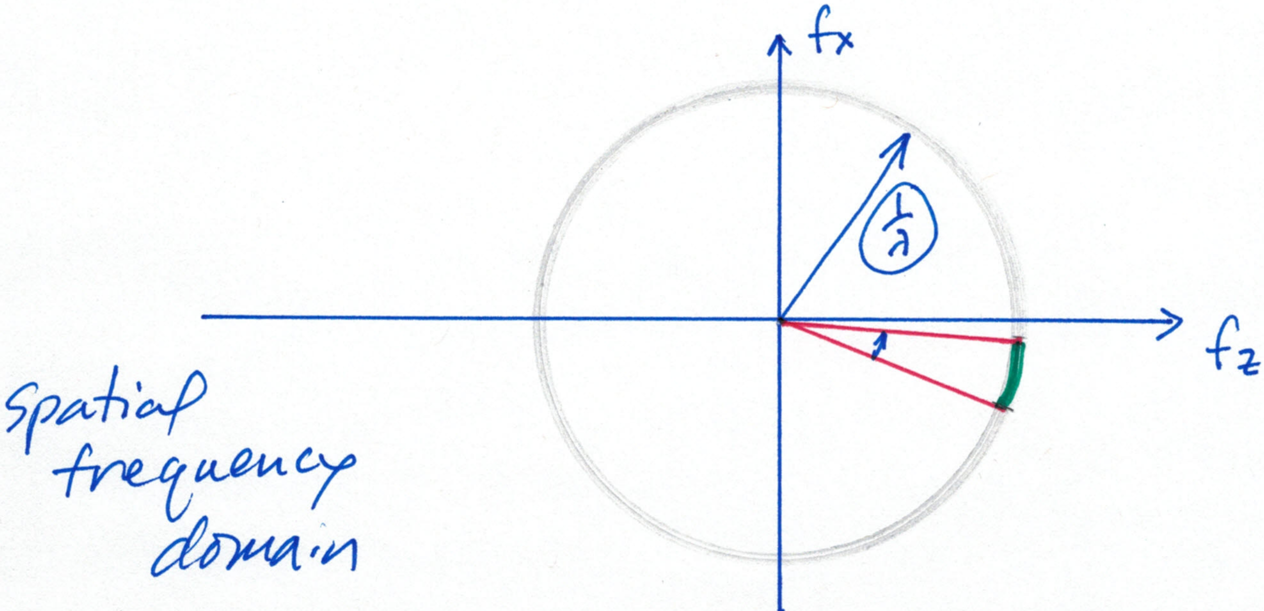
$$(f_x, f_z) = \left(\frac{1}{\lambda}\sin\theta, \frac{1}{\lambda}\cos\theta\right)$$

or $$(f_z, f_x) = \left(\frac{1}{\lambda}\cos\theta, \frac{1}{\lambda}\sin\theta\right)$$

$$f_x^2 + f_z^2 = \frac{1}{\lambda^2}$$







$$f_{x_1} = \frac{1}{\lambda}\sin\theta_1$$

$$f_{z_1} = \frac{1}{\lambda}\cos\theta_2$$

$$f_{x_2} = \frac{1}{\lambda}\sin\theta_2$$

$$f_{z_2} = \frac{1}{\lambda}\cos\theta_2$$

Spectral range

source at (x_0, z_0)

aperture span from (x_1, z_1) to (x_2, z_2)

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x direction (cross-range)

$$\text{Bandwidth} = \frac{1}{\lambda}(\sin\theta_2 - \sin\theta_1)$$

$$\text{Resolution} = \frac{\lambda}{\sin\theta_2 - \sin\theta_1}$$

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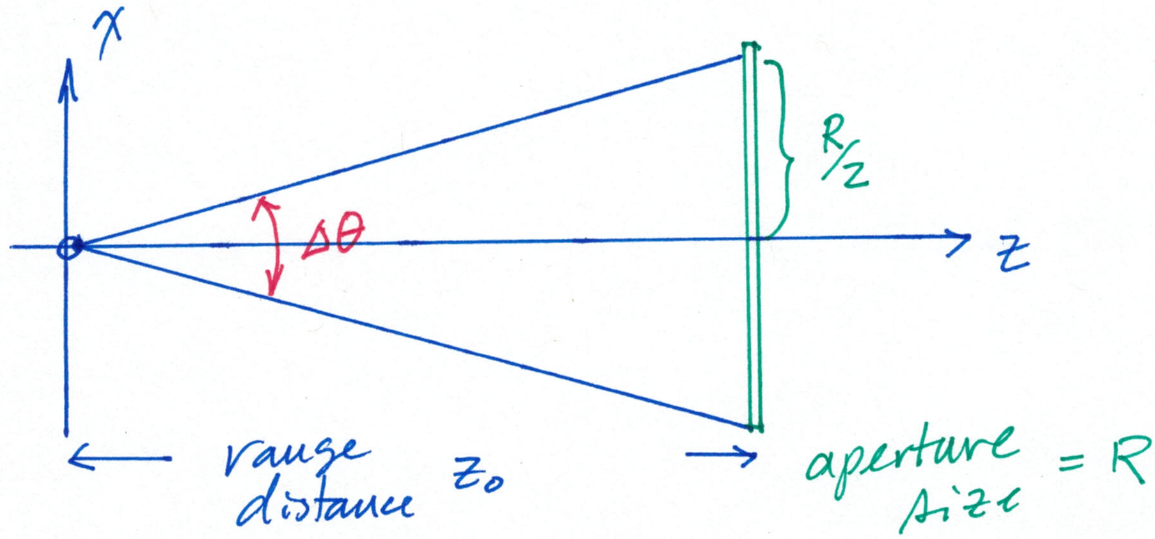
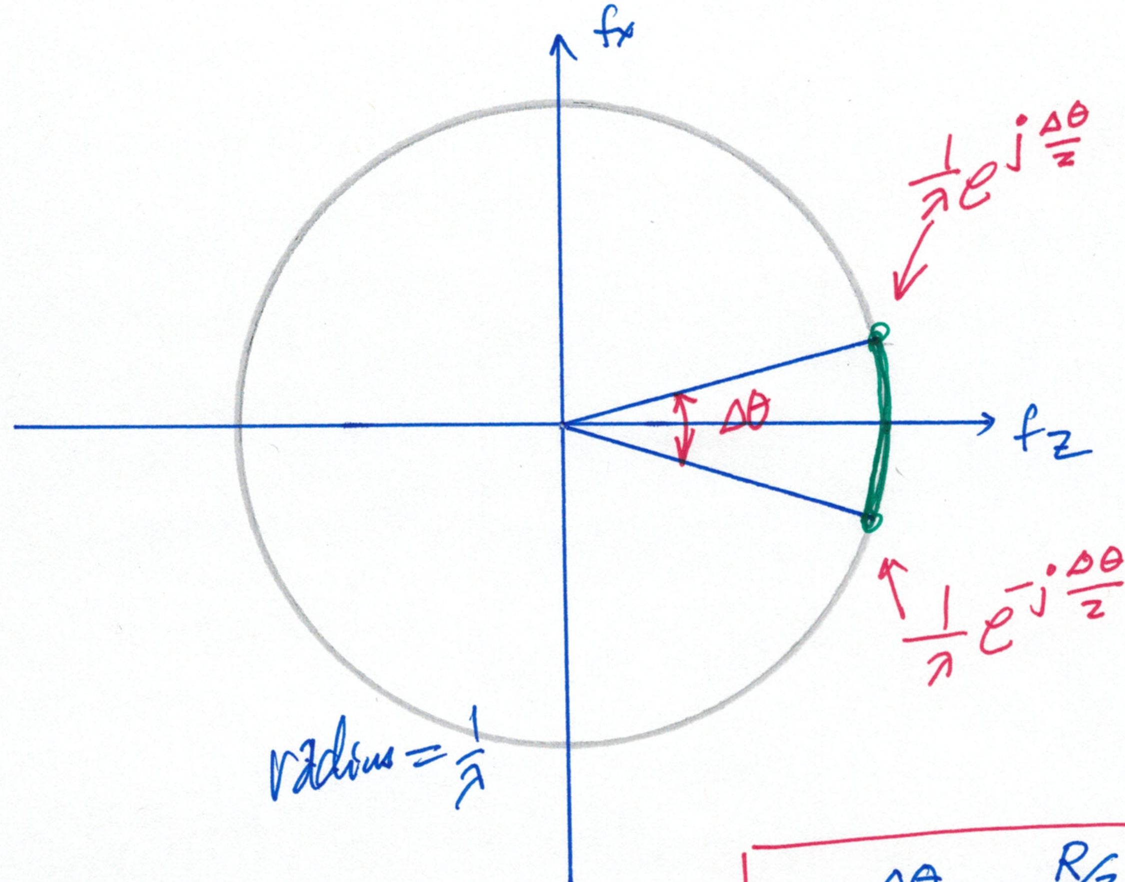
z direction (range)

$$\text{Bandwidth} = \frac{1}{\lambda}(\cos\theta_{max} - \cos\theta_{min})$$

$$\text{Resolution} = \frac{\lambda}{\cos\theta_{max} - \cos\theta_{min}}$$





$$\boxed{\tan \frac{\Delta\theta}{2} = \frac{R/2}{z_0}}$$

From $\frac{1}{\lambda} e^{-j\frac{\Delta\theta}{2}}$ to $\frac{1}{\lambda} e^{+j\frac{\Delta\theta}{2}}$

$$\frac{1}{\lambda} e^{-j\frac{\Delta\theta}{2}} \rightarrow \left[\frac{1}{\lambda}\cos\left(\frac{\Delta\theta}{2}\right), \frac{-1}{\lambda}\sin\left(\frac{\Delta\theta}{2}\right) \right]$$

$$\frac{1}{\lambda} e^{+j\frac{\Delta\theta}{2}} \rightarrow \left[\frac{1}{\lambda}\cos\left(\frac{\Delta\theta}{2}\right), \frac{+1}{\lambda}\sin\left(\frac{\Delta\theta}{2}\right) \right]$$

Bandwidth (cross-range)

$$\Delta f_x = \frac{1}{\lambda}\sin\left(\frac{\Delta\theta}{2}\right) - \left(-\frac{1}{\lambda}\sin\left(\frac{\Delta\theta}{2}\right)\right)$$

$$\boxed{= \frac{2}{\lambda}\sin\left(\frac{\Delta\theta}{2}\right)}$$

Resolution (cross-range)

$$\boxed{\Delta x = \frac{\lambda}{2\sin\left(\frac{\Delta\theta}{2}\right)}}$$

Bandwidth (range)

$$\Delta f_z = \frac{1}{\lambda}\left(1 - \cos\frac{\Delta\theta}{2}\right)$$

Resolution

$$\Delta z = \frac{\lambda}{1 - \cos\left(\frac{\Delta\theta}{2}\right)}$$