

ASSIGNMENT 4: Multi-Frequency Backward Propagation

Due: Tuesday, February 20

Objective: The objective of this programming assignment is to experience image formation by multi-frequency backward propagation and to observe the effects on image resolution in the form of spectral coverage.

The system configuration is the same as that in *Assignment 2*, with 3 point sources at the same locations. The *receiver aperture* is a centered linear receiver array with a span of $60\lambda_0$ (from $x = -30\lambda_0$ to $x = +30\lambda_0$), located along $y = y_o = +60\lambda_0$, with quarter-wavelength ($\lambda_0/4$) spacing. For each coherent frequency, there are 241 wavefield samples in total over the $60\lambda_0$ - long aperture.

In *Assignment 2*, one single wavelength was applied. As we move up to the multi-frequency (wideband) operating mode, we collect wavefield samples over a wide range of spectrum corresponding to 64 different wavelengths, in the form

$$\lambda_n = 64 \lambda_0 / (n+32) \quad n = 1, 2, \dots, 64$$

Thus, this imaging modality operates with a sequence of wavelengths, from $0.67\lambda_0$ to $2\lambda_0$, corresponding to the spatial-frequency band from $0.5(1/\lambda_0)$ to $1.5(1/\lambda_0)$ resulting a spatial-frequency bandwidth of $(1/\lambda_0)$.

The complete data set is a (64×241) array, corresponding to 64 wavelengths and 241 receiver positions.

Superposition of coherent images:

This is to perform coherent image formation procedure 64 times corresponding to 64 different operating wavelengths and the same receiver array configuration. This process produces 64 coherent sub-images, $\hat{s}_n(x, y)$, for $n = 1, 2, \dots, 64$. For simplicity, the phase-only version of the Green's function is applied to image reconstruction.

The goal is to perform multi-frequency image reconstruction over the $(60\lambda_0 \times 60\lambda_0)$ 2D source region with the 64 sets of coherent wavefield samples. The source region is an area centered at $(0, 0)$ and bounded by $x = \pm 30\lambda_0$ and $y = \pm 30\lambda_0$. For consistency, use quarter-wavelength $(\lambda_0/4)$ spacing as the sample spacing in both directions for the display of the images.

- (1) By repeating the coherent backward propagation image formation procedure, produce the 64 coherent sub-images.
- (2) Superimpose the 64 coherent complex sub-images *sequentially* and observe the convergence to the overall image,

$$\hat{S}_n(x, y) = \sum_{k=1}^n \hat{s}_k(x, y) \quad n = 1, 2, \dots, 64.$$

- (3) Produce the spectra of the complex image sequence $\hat{S}_n(x, y)$, $n = 1, 2, \dots, 64$.
- (4) Compile a 64-frame video of the image sequence $\hat{S}_n(x, y)$.
- (5) Compile a 64-frame video of the spectra of the image sequence.

Report format:

- (A) Cover page.
- (B) Video sequences.
- (C) Summary: (comments based on your observations)
- (D) Appendix: (computer code)