ECE 278C Imaging Systems

10. X-ray tomography

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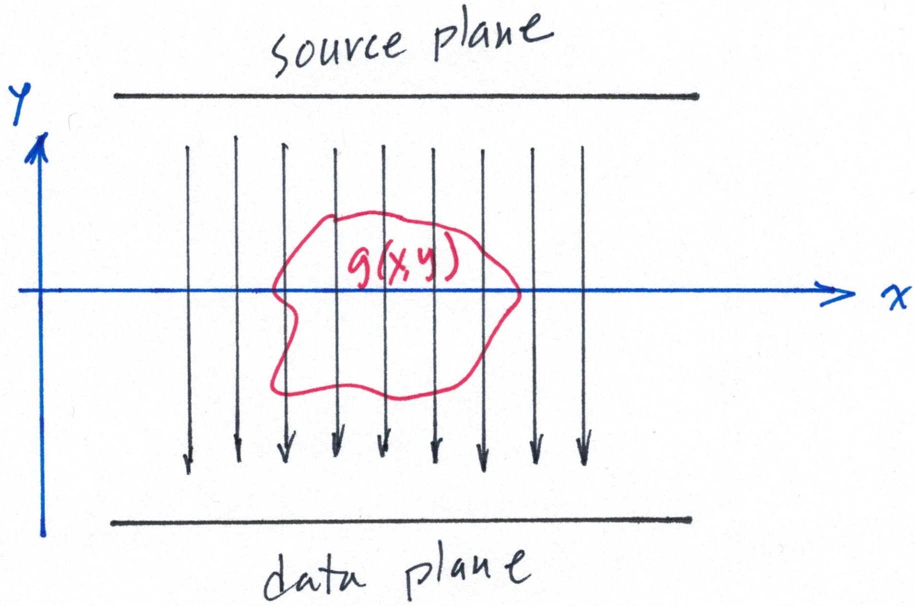
X-ray Tomography



1. Source intensity : I_0
2. object attenuation profile: $g(x,y)$ or $\exp(-g(x,y))$
3. received intensity : $I(x)$

$$I(x) = I_0 \cdot \exp\left\{-\int g(x,y)\,dy\right\}$$

$$\ln \frac{I(x)}{I_0} = -\int_{-\infty}^{\infty} g(x,y)\,dy$$

Define

$$p(x) = -\ln \frac{I(x)}{I_0} = \int_{-\infty}^{\infty} g(x,y)\,dy$$

projection

$$\mathcal{F}\{p(x)\} = \int_{-\infty}^{\infty} p(x) \cdot \exp(-j2\pi f_x \cdot x)\,dx$$

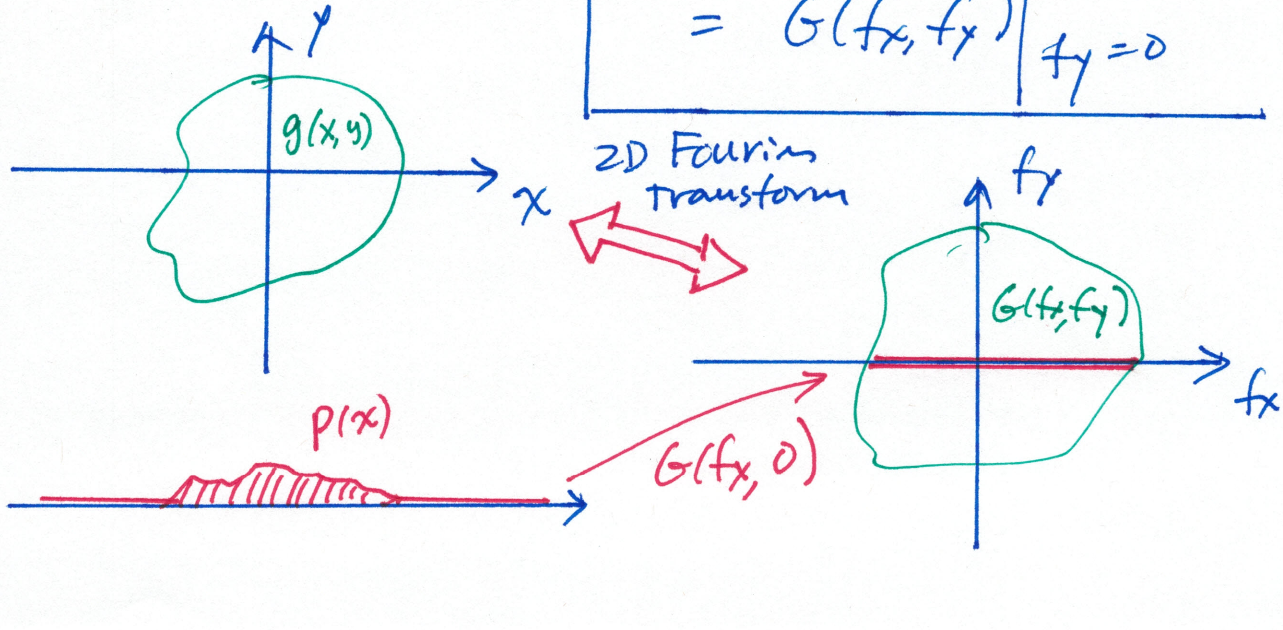
$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y)\,dy \cdot \exp(-j2\pi f_x \cdot x)\,dx$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) \cdot \exp(-j2\pi f_x \cdot x)\,dx\,dy$$

$$G(f_x, f_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) \cdot \exp(-j2\pi (f_x \cdot x + f_y \cdot y))\,dx\,dy$$

$$\mathcal{F}\{p(x)\} = P(f_x) = G(f_x, 0)$$

$$= G(f_x, f_y)\Big|_{f_y = 0}$$



2D Fourier Transform

$$\mathcal{F}\{g(x,y)\} = G(f_x, f_y)$$

$$= \iint_{-\infty}^{+\infty} g(x,y) \cdot \exp(-j2\pi(f_x \cdot x + f_y \cdot y))\, dx\, dy$$

vector form

$$\underline{f} = \begin{bmatrix} f_x \\ f_y \end{bmatrix} \qquad \underline{x} = \begin{bmatrix} x \\ y \end{bmatrix}$$

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vector version

$$G(\underline{f}) = \mathcal{F}\{g(\underline{x})\}$$

$$= \int_{-\infty}^{\infty} g(\underline{x}) \cdot \exp(-j2\pi \underbrace{\langle \underline{f}, \underline{x} \rangle}_{\text{inner product}})\, d\underline{x}$$

$$\langle \underline{f}, \underline{x} \rangle = f_x \cdot x + f_y \cdot y$$

2D rotation

rotation matrix $[A] = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

angular rotation $= \theta$

after rotation

$$g'(x,y) = g'(\mathbf{x}) = g([A]\mathbf{x})$$

Spectrum becomes

$$G'(f_x, f_y) = G'(\mathbf{f}) = \mathcal{F}\{g'(\mathbf{x})\}$$

$$= \mathcal{F}\{g([A]\mathbf{x})\}$$

$\mathbf{x}' = [A]\mathbf{x}$

$$= \int g([A]\mathbf{x}) \cdot \exp(-j2\pi \langle \mathbf{f}, \mathbf{x} \rangle)\, d\mathbf{x}$$

$$= \int g(\mathbf{x}') \cdot \exp(-j2\pi \langle \mathbf{f}, [A]^T\mathbf{x}' \rangle)\, d\mathbf{x}'$$

$$= \int g(\mathbf{x}') \cdot \exp(-j2\pi \langle [A]\mathbf{f}, \mathbf{x}' \rangle)\, d\mathbf{x}'$$

$$= G([A]\mathbf{f})$$

Fourier transform pairs

$$g(\mathbf{x}) \longleftrightarrow G(\mathbf{f})$$

$$g([A]\mathbf{x}) \longleftrightarrow G([A]\mathbf{f})$$

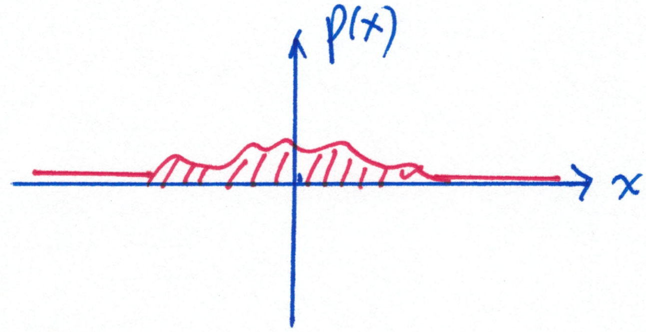
Rotation invariance property

of Multi-dimensional Fourier transform

Tomographic image formation

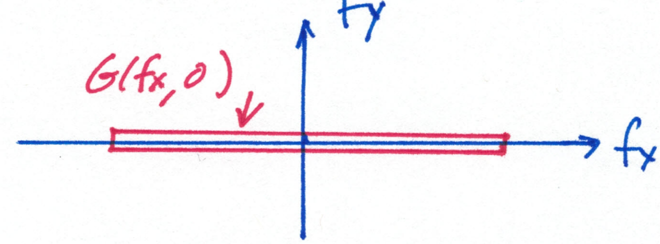
Step #1:

Obtain a projection through data acquisition



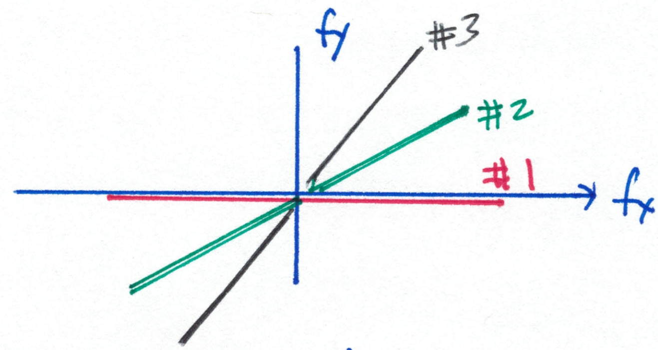
Step #2:

Fourier transform and place 1D spectrum onto (f_x, f_y) plane



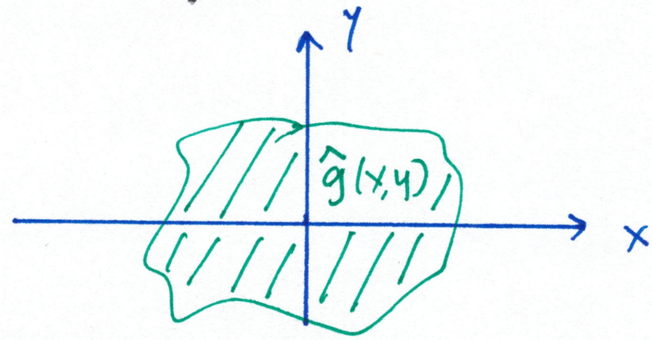
Step #3:

Rotate and repeat



Step #4:

Inverse Fourier transform



Point Spread Function

space domain

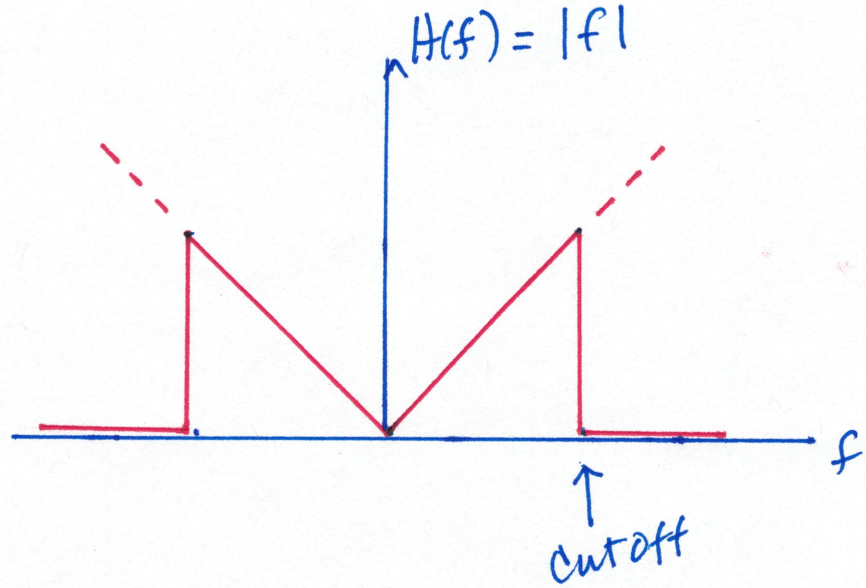
$$p.s.f = \frac{1}{r} \qquad r = \sqrt{x^2 + y^2}$$

spatial-frequency domain

$$F\left\{\frac{1}{r}\right\} = \frac{\text{constant}}{R}$$

$$R = \sqrt{f_x^2 + f_y^2}$$

Enhancement filter



governed by the noise level