ECE 278C Imaging Systems

6. Phase-only techniques

Department of Electrical and Computer Engineering
University of California, Santa Barbara

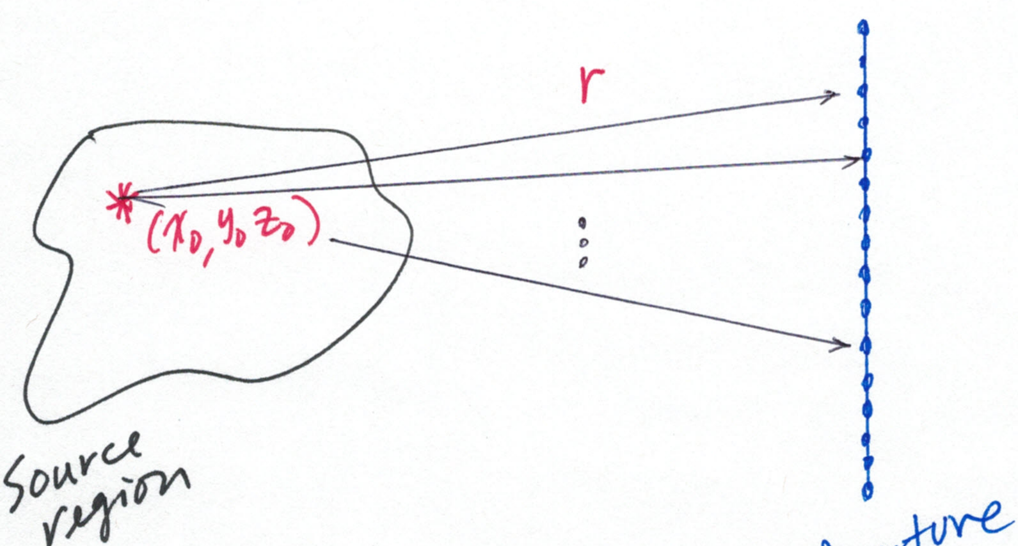
Single point source at (x_0, y_0, z_0)

$$g(x,y,z) = s(x,y,z) * h(x,y,z)$$

$$= \delta(x-x_0, y-y_0, z-z_0) * h(x,y,z)$$

$$= \frac{1}{j\lambda r} \exp(j 2\pi r/\lambda)$$

$$r = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}$$



Backward propagation

$$g(x,y,z) \ast h^{*}(x,y,z) = \hat{s}(x,y,z)$$

$$= \iiint_A \left[\frac{1}{j\lambda r} \exp(j 2\pi r/\lambda)\right] \cdot \left[\frac{-1}{j\lambda r'} \exp(-j 2\pi r'/\lambda)\right] dx' dy' dz'$$

over the aperture area

$$= \iiint_A \frac{1}{\lambda^2 r r'} \exp(j 2\pi (r-r')/\lambda) \, dx' dy' dz'$$

$$r = \sqrt{(x'-x_0)^2 + (y'-y_0)^2 + (z-z_0)^2}$$

$$r' = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

Superposition of vectors

$$\iiint \frac{1}{\lambda^2 r r'} \exp\left(j 2\pi (r-r')/\lambda\right) dx' dy' dz'$$

at the source location $(x, y, z) = (x_0, y_0, z_0)$

$\rightarrow \quad r = r'$

when $\hat{s}(x_0, y_0, z_0) = \iiint_A \frac{1}{\lambda^2 r^2} dx' dy' dz'$

real and positive

① Standard backward propagation

reconstruction

$$\hat{s}(x,y,z) = \iiint \frac{1}{\lambda^2 r r'} \exp\left(j2\pi(r-r')/\lambda\right) dx'dy'dz'$$

(4) $$\hat{s}(x_0,y_0,z_0) = \iiint \frac{1}{\lambda^2 r^2} dx'dy'dz'$$

---

② phase-only data

$$\hat{s}(x,y,z) = \iiint \left(\frac{-1}{j\lambda r'}\right) \exp\left(j2\pi(r-r')/\lambda\right) dx'dy'dz'$$

$$\hat{s}(x_0,y_0,z_0) = \iiint \left(\frac{-1}{j\lambda r}\right) dx'dy'dz'$$

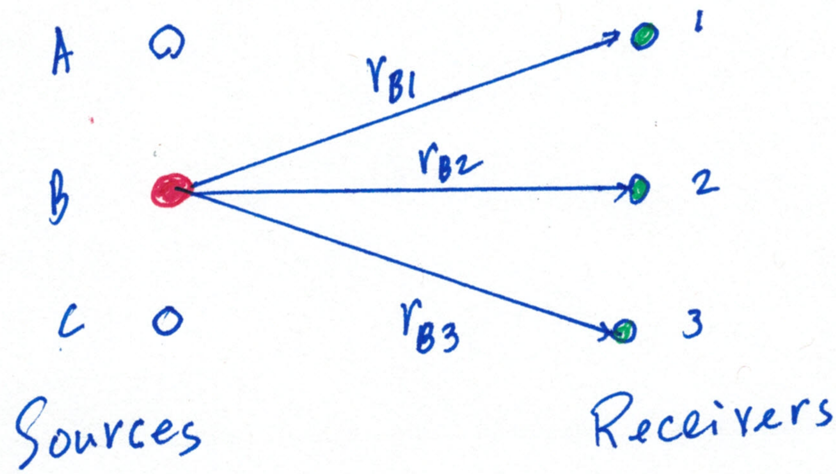
---

③ Full phase-only method

$$\hat{s}(x,y,z) = \iiint \exp\left(j2\pi(r-r')/\lambda\right) dx'dy'dz'$$

$$\hat{s}(x_0,y_0,z_0) = \iiint dx'dy'dz'$$

Example:



At receiver #1: $$g_1 = \frac{1}{j\lambda r_{B1}} \exp(j2\pi r_{B1}/\lambda)$$

#2: $$g_2 = \frac{1}{j\lambda r_{B2}} \exp(j2\pi r_{B2}/\lambda)$$

#3: $$g_3 = \frac{1}{j\lambda r_{B3}} \exp(j2\pi r_{B3}/\lambda)$$

Standard backward propagation

At source position A:

$$\hat{S}_A = \left(\frac{1}{\lambda^2 r_{B1} r_{A1}}\right) \exp\left(j2\pi (r_{B1} - r_{A1})/\lambda\right) + \left(\frac{1}{\lambda^2 r_{B2} r_{A2}}\right) \exp\left(j2\pi (r_{B2} - r_{A2})/\lambda\right) + \left(\frac{1}{\lambda^2 r_{B3} r_{A3}}\right) \exp\left(j2\pi (r_{B3} - r_{A3})/\lambda\right)$$

Position C:

$$\hat{S}_C = \left(\frac{1}{\lambda^2 r_{B1} r_{C1}}\right) \cdot \exp\left(j2\pi (r_{B1} - r_{C1})/\lambda\right) + \left(\frac{1}{\lambda^2 r_{B2} r_{C2}}\right) \cdot \exp\left(j2\pi (r_{B2} - r_{C2})/\lambda\right) + \left(\frac{1}{\lambda^2 r_{B3} r_{C3}}\right) \cdot \exp\left(j2\pi (r_{B3} - r_{C3})/\lambda\right)$$

Position B:

$$\hat{S}_B = \frac{1}{\lambda^2 r_{B1} r_{B1}} + \frac{1}{\lambda^2 r_{B2} r_{B2}} + \frac{1}{\lambda^2 r_{B3} r_{B3}}$$

$$= \frac{1}{\lambda^2}\left(\frac{1}{r_{B1}^2} + \frac{1}{r_{B2}^2} + \frac{1}{r_{B3}^2}\right)$$

real + positive

phase terms from data

At position A:

$$\hat{s}_A = \left(\frac{1}{-j\lambda r_{A1}}\right)\exp\left(j2\pi(r_{B1} - r_{A1})/\lambda\right)$$
$$+ \left(\frac{1}{-j\lambda r_{A2}}\right)\exp\left(j2\pi(r_{B2} - r_{A2})/\lambda\right)$$
$$+ \left(\frac{1}{-j\lambda r_{A3}}\right)\exp\left(j2\pi(r_{B3} - r_{A3})/\lambda\right)$$

---

point C: $$\hat{s}_C = \left(\frac{1}{-j\lambda r_{C1}}\right)\exp\left(j2\pi(r_{B1} - r_{C1})/\lambda\right)$$
$$+ \left(\frac{1}{-j\lambda r_{C2}}\right)\exp\left(j2\pi(r_{B2} - r_{C2})/\lambda\right)$$
$$+ \left(\frac{1}{-j\lambda r_{C3}}\right)\exp\left(j2\pi(r_{B3} - r_{C3})/\lambda\right)$$

---

point B:

$$\hat{s}_B = \frac{1}{-j\lambda r_{B1}} + \frac{1}{-j\lambda r_{B2}} + \frac{1}{-j\lambda r_{B3}}$$

$$= \left(\frac{1}{-j\lambda}\right)\left[\frac{1}{r_{B1}} + \frac{1}{r_{B2}} + \frac{1}{r_{B3}}\right]$$

real and positive

Full phase-only technique

At point A:

$$\hat{s}_A = \exp(j2\pi r_{B1}/\lambda)\cdot\exp(-j2\pi r_{A1}/\lambda)$$
$$+\exp(j2\pi r_{B2}/\lambda)\cdot\exp(-j2\pi r_{A2}/\lambda)$$
$$+\exp(j2\pi r_{B3}/\lambda)\cdot\exp(-j2\pi r_{A3}/\lambda)$$

At C: $$\hat{s}_C = \exp(j2\pi r_{B1}/\lambda)\cdot\exp(-j2\pi r_{C1}/\lambda)$$
$$+\exp(j2\pi r_{B2}/\lambda)\cdot\exp(-j2\pi r_{C2}/\lambda)$$
$$+\exp(j2\pi r_{B3}/\lambda)\cdot\exp(-j2\pi r_{C3}/\lambda)$$

At source location B:

$$\hat{s}_B = \exp(j2\pi r_{B1}/\lambda)\cdot\exp(-j2\pi r_{B1}/\lambda)$$
$$+\exp(j2\pi r_{B2}/\lambda)\cdot\exp(-j2\pi r_{B2}/\lambda)$$
$$+\exp(j2\pi r_{B3}/\lambda)\cdot\exp(-j2\pi r_{B3}/\lambda)$$
$$= 3$$