ECE 278C Imaging Systems

12. Synthetic aperture radar

Department of Electrical and Computer Engineering
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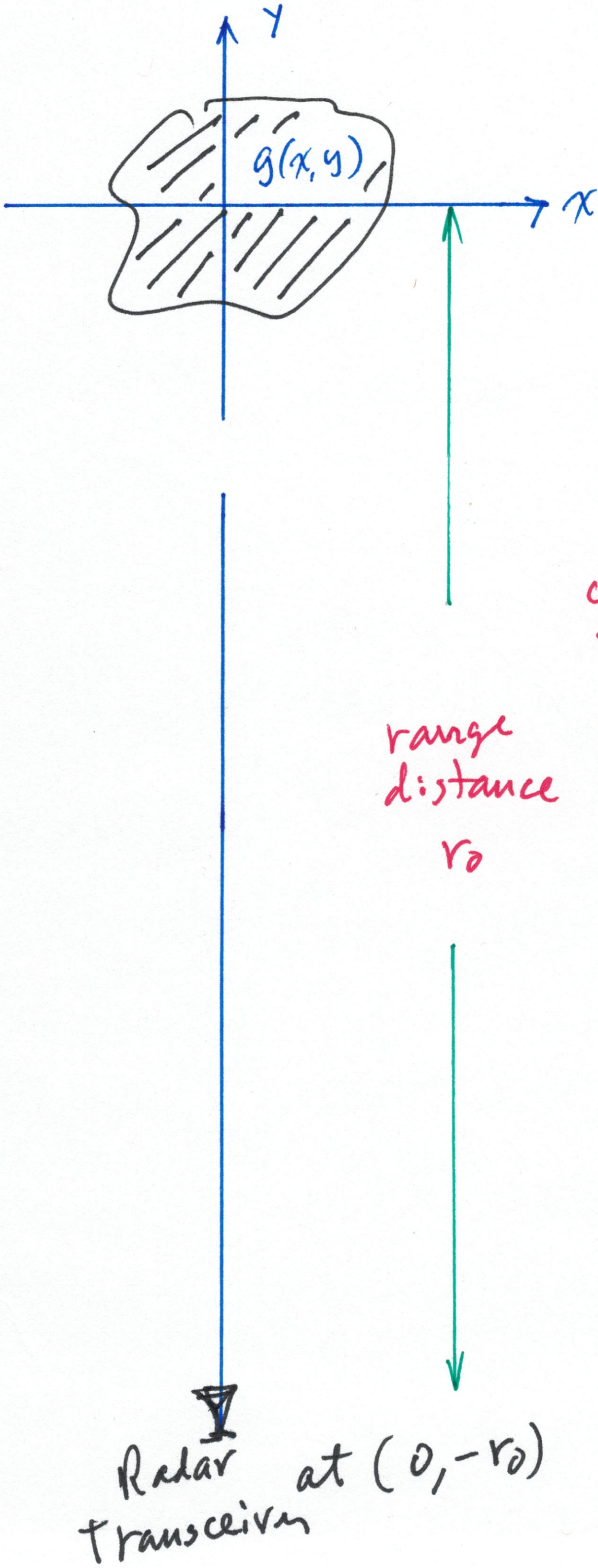
Synthetic aperture radar (SAR) imaging

Level #1: monostatic CW systems

↳ single wavelength λ



Detected wavefield sample at $(x,y)=(0,-r_0)$

$$p(0,-r_0) = \iint_{-\infty}^{\infty} g(x,y)\, \frac{\exp(j2\pi \, 2r/\lambda)}{j\lambda(2r)}\, dx\,dy$$

complex scalar

$$\frac{\exp(j2\pi(2r)/\lambda)}{j\lambda(2r)} \approx \frac{\exp(j2\pi \cdot 2(r_0+y)/\lambda)}{j\lambda(2r_0)}$$

$$= \frac{\exp(j4\pi r_0/\lambda)}{j2\lambda r_0} \cdot \exp(j2\pi(2y)/\lambda)$$

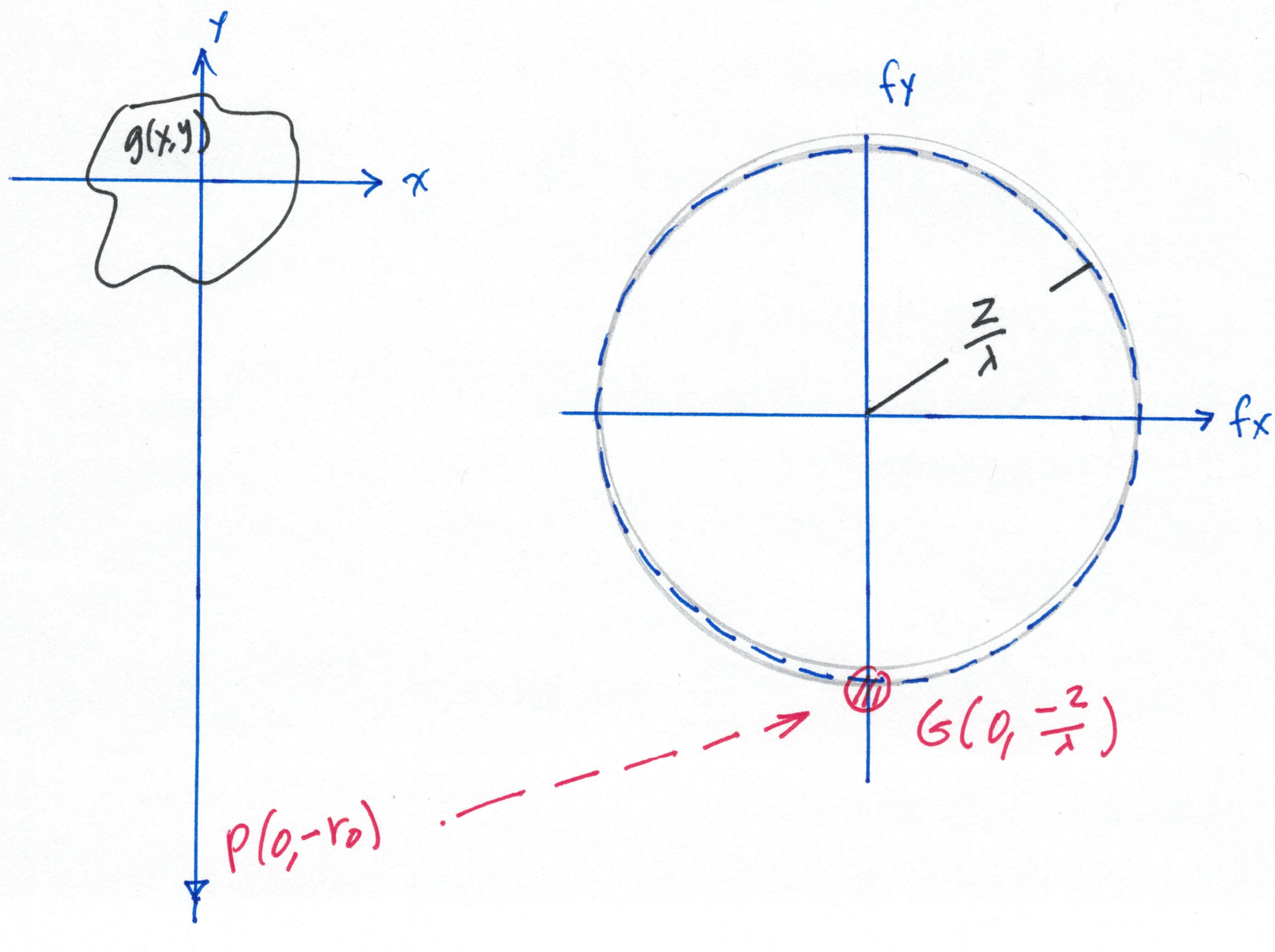
$$= \text{complex constant} \cdot \exp\left(j2\pi\left(\frac{2}{\lambda}\right)\cdot y\right)$$

Received signal

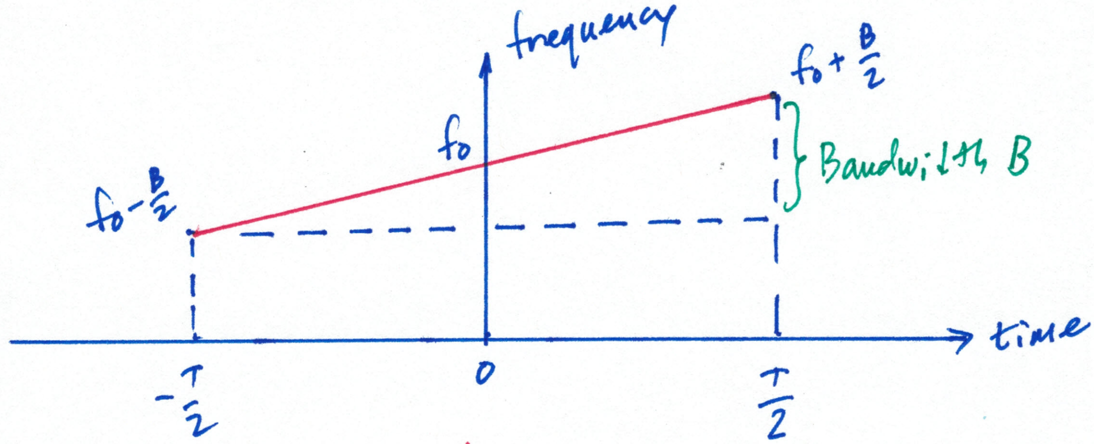
$$P(0,-r_0) = \text{constant} \iint_{-\infty}^{\infty} g(x,y) \cdot \exp(j2\pi\, 2y/\lambda)\, dx\, dy$$

$$= \text{Constant} \iint_{-\infty}^{\infty} g(x,y) \cdot \exp\left(-j2\pi \left(\frac{-2}{\lambda}\right) y\right) \cdot dx\, dy$$

$$= \text{constant} \cdot G(f_x, f_y) \Big|_{\substack{f_x = 0 \\ f_y = -\frac{2}{\lambda}}}$$



Linear Chirp waveform



Chirp period = T

$$e(t) = E \cdot \exp\left(-j\left(2\pi f_0 t + \pi \frac{B}{T} t^2\right)\right)$$

$$= E \cdot \exp(-j\, 2\pi f_0 t) \cdot \exp\left(-j\, \pi \frac{B}{T} t^2\right)$$

$$\text{Frequency} \triangleq \frac{\partial}{\partial t}\,\text{phase}$$

$$= \frac{\partial}{\partial t}\left[2\pi f_0 t + \pi \frac{B}{T} t^2\right]$$

$$= 2\pi\left(f_0 + \frac{B}{T} t\right)$$

Frequency as a function of time

$t = -\frac{T}{2}$ $\quad$ $\text{min freq} = 2\pi\left(f_0 + \frac{B}{T}\left(-\frac{T}{2}\right)\right) = 2\pi\left(f_0 - \frac{B}{2}\right)$

$t = \frac{T}{2}$ $\quad$ $\text{max freq} = 2\pi\left(f_0 + \frac{B}{T}\left(\frac{T}{2}\right)\right) = 2\pi\left(f_0 + \frac{B}{2}\right)$

$$\text{Bandwidth} = \text{max freq} - \text{min freq}$$

$$= 2\pi(B)$$

A simple case:

one single target : range distance $= r = r_0$

Time delay $= \dfrac{2r_0}{v} = t_0$

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transmit

$$e_T(t) = E_T \cdot \exp\left(-j\left(2\pi f_0 t + \pi \frac{B}{T} t^2\right)\right)$$

Received

$$e_r(t) = E_r \cdot \exp\left(-j\, 2\pi f_0 (t - t_0)\right)$$

$$\cdot \exp\left(-j\, \pi \frac{B}{T} (t - t_0)^2\right)$$

$$= E_r \cdot \exp(-j\, 2\pi f_0 t) \cdot \exp(+j\, 2\pi f_0 t_0)$$

$$\cdot \exp\left(-j\, \pi \frac{B}{T} t^2\right)$$

$$\cdot \exp\left(-j\, \pi \frac{B}{T} t_0^2\right)$$

$$\cdot \exp\left(+j\, 2\pi \frac{B}{T} t_0\, t\right)$$

$$e_r(t)\cdot e_T^*(t) = \overset{\text{not a time function}}{\boxed{E_r\cdot E_T^*\cdot \exp(j2\pi f_0 t_0)\cdot \exp\left(-j\pi\frac{B}{T}t_0^2\right)}}\cdot \underbrace{\exp\left(+j2\pi\frac{B}{T}t_0\, t\right)}_{\text{function of } t}$$

Define

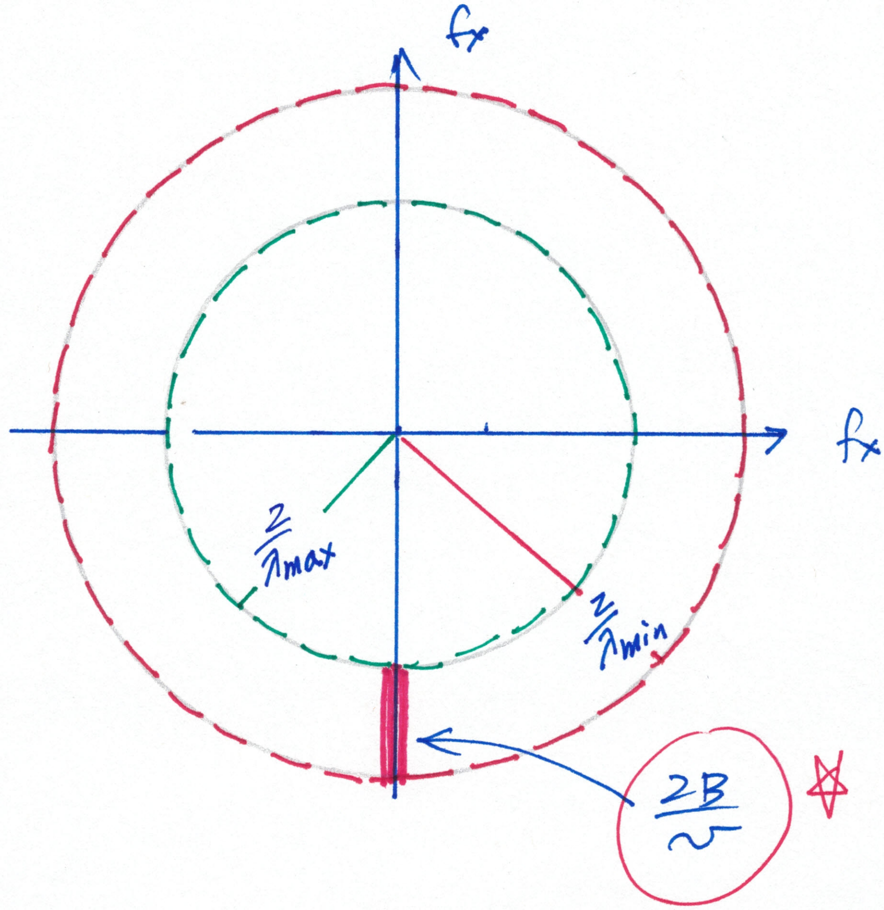
$$c = E_r\cdot E_T^*\cdot \exp(j2\pi f_0 t_0)\cdot \exp\left(-j\pi\frac{B}{T}t_0^2\right)$$

Received signal (after mixing)

$$e_r(t)\cdot e_T^*(t) = c\cdot \exp\left(+j2\pi\left(\frac{B}{T}t_0\right)t\right)$$

$$\begin{tikzcd} e_r(t) \arrow[r] & \otimes \arrow[r] & c\cdot \exp\left(+j2\pi\left(\frac{B}{T}t_0\right)t\right) \\ & e_T(t) \arrow[u, "*"'] & \end{tikzcd}$$

Spectrum peaks at $\boxed{f=\frac{B}{T}t_0=\frac{2B}{Tv}\cdot r_0}$



$$\frac{f}{v} = \frac{1}{\lambda}$$

$$f_{min} = f_0 - \frac{B}{2}$$

$$f_{max} = f_0 + \frac{B}{2}$$

$$\frac{2}{\lambda_{min}} - \frac{2}{\lambda_{max}} = \frac{2}{v}\left(f_{max} - f_{min}\right)$$

spatial frequency bandwidth $\boxed{= \frac{2B}{v}}$

$$\boxed{\text{Resolution (in range)} = \frac{v}{2B}}$$