ECE 278C Imaging Systems

5. Holography

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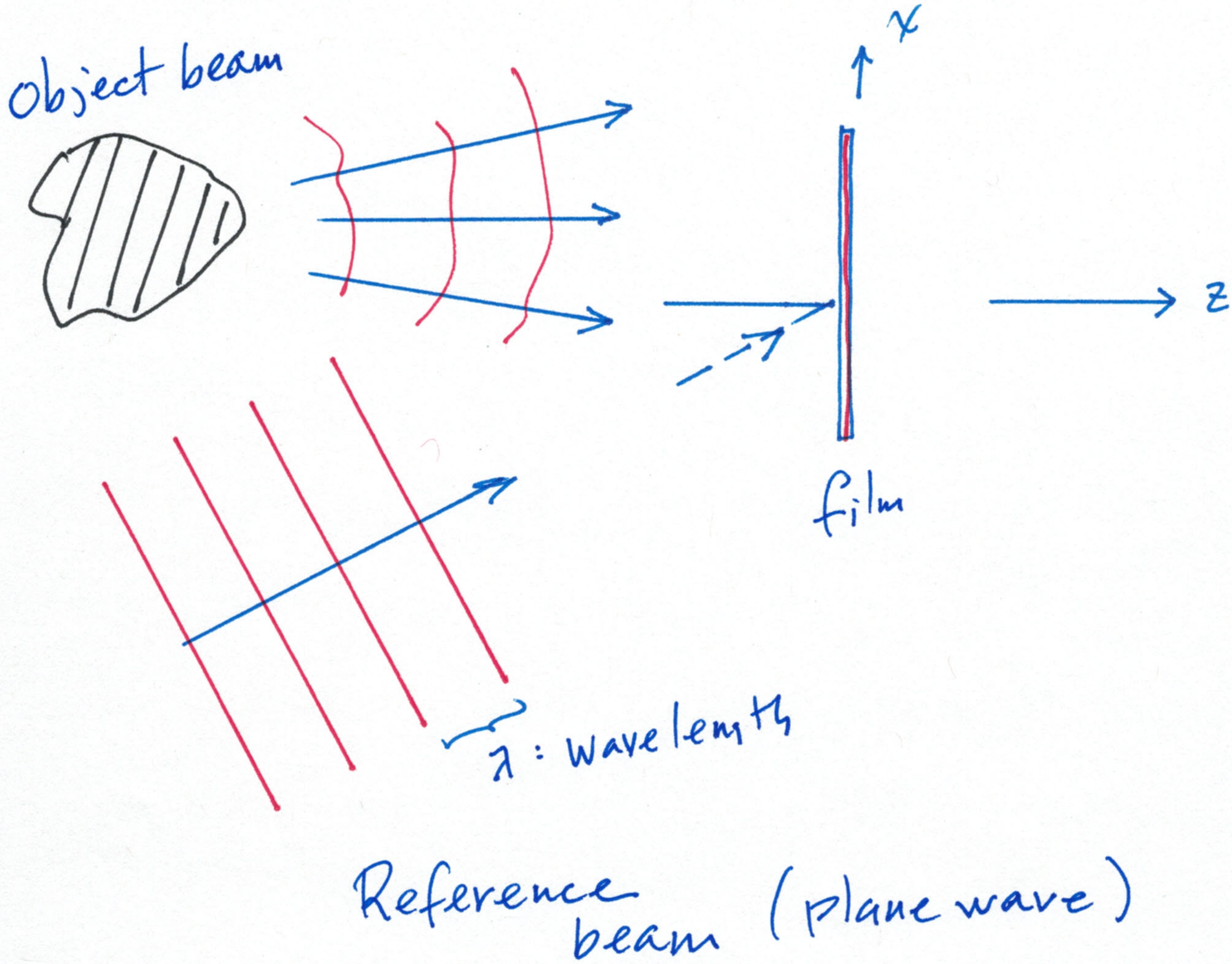
University of California, Santa Barbara

Classical Holography



① 3D object

② Reference wave

③ Film

Object beam waveform arriving at the plate

$$\left[U_o(x,y) \cdot e^{j\phi(x,y)} \right] \cdot e^{-j\omega t}$$

carrier frequency

Reference beam arriving at the plate

$$\left[U_r \cdot e^{+j2\pi f_{xr} \cdot x} \right] \cdot e^{-j\omega t}$$

$$f_{xr} = \frac{\sin\theta}{\lambda}$$

spatial frequency of plane wave

Total wavefield at plate

$U(x,y) =$ object beam + reference beam

$$= e^{-j\omega t} \cdot \left[U_o(x,y) \cdot e^{j\phi(x,y)} + U_r \cdot e^{j2\pi f_{xr} \cdot x} \right]$$

Intensity recorded by film

$$I(x,y) = |u(x,y)|^2$$

$$= u(x,y) \cdot u^*(x,y)$$

$$= \left(U_o^2(x,y) + |U_r|^2\right) \quad ①$$

$$+ \; U_r^* U_o(x,y) \cdot e^{j\phi(x,y)} \cdot e^{-j2\pi f_{xr} \cdot x} \quad ②$$

$$+ \; U_r U_o(x,y) \cdot e^{-j\phi(x,y)} \cdot e^{j2\pi f_{xr} \cdot x} \quad ③$$

After the development of the film

$$T(x,y) = k \, I(x,y)$$

transparency function

constant

1st term

$$k \{ \underbrace{U_o^2(x,y) + |U_r|^2}_{\text{mask}} \} \cdot U_r e^{j2\pi f_{xr} \cdot x} \cdot e^{-j\omega t}$$

A plane wave modulated by a real and positive mask

2nd term

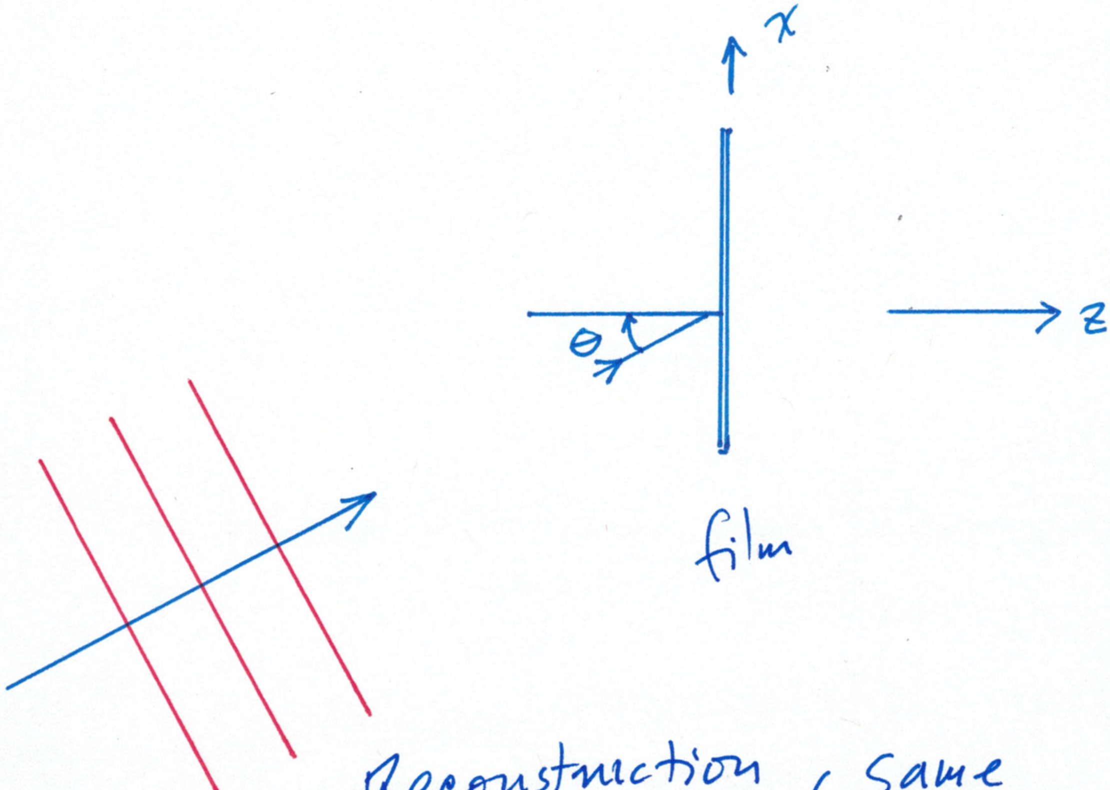
$$\underbrace{k|U_r|^2}_{\text{constant}} \cdot \underbrace{U_o(x,y) \cdot e^{j\phi(x,y)} \cdot e^{-j\omega t}}_{\text{object waveform}}$$

3D Object waveform scaled by constant

3rd term

$$\underbrace{k U_r^2}_{\substack{\text{complex}\\\text{constant}}} \cdot \underbrace{U_o(x,y) \cdot e^{-j\phi(x,y)}}_{\substack{\text{conjugated}\\\text{object}\\\text{beam}}} \cdot \underbrace{e^{j2\pi(2f_{xr}) \cdot x}}_{\substack{\text{plane}\\\text{wave}\\\text{from a}\\\text{different}\\\text{angle}}} \cdot \underbrace{e^{-j\omega t}}_{\text{carrier}}$$

Holographic reconstruction



Reconstruction beam (same plane wave)

1. film
2. Reconstruction beam

Resultant wavefield

$$\hat{u}(x,y) = T(x,y)\cdot\left(u_r e^{j2\pi f_{xr}\cdot x}\cdot e^{-j\omega t}\right)$$

reconstruction beam

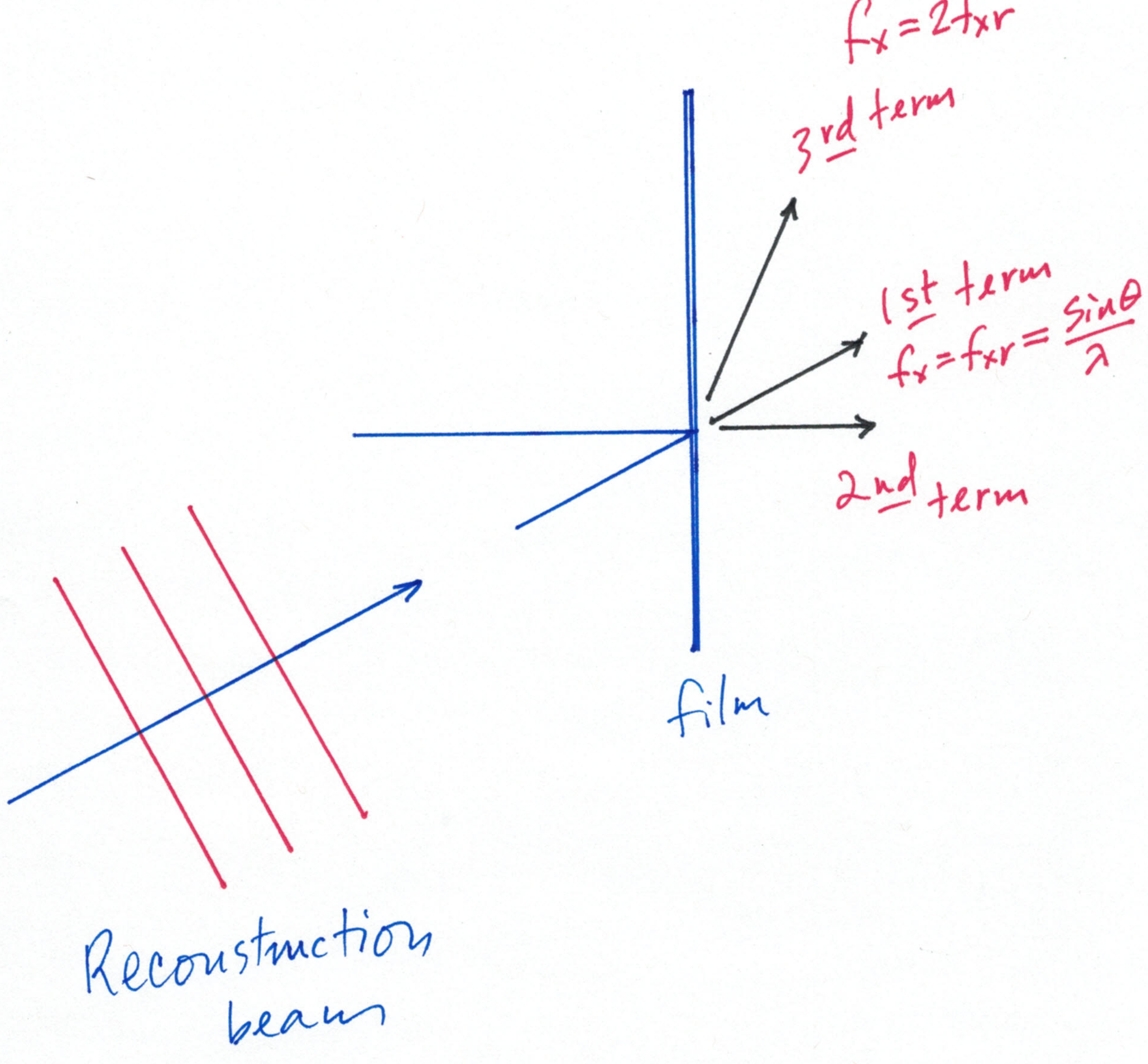
$$= k\, I(x,y)\cdot\left(u_r e^{j2\pi f_{xr}\cdot x}\cdot e^{-j\omega t}\right)$$

$$= k\left\{u_o^2(x,y) + |u_r|^2\right\}\cdot u_r e^{j2\pi f_{xr}\cdot x}\cdot e^{-j\omega t} \quad (1)$$

$$+\; k\,|u_r|^2\cdot \boxed{u_o(x,y)\cdot e^{j\phi(x,y)}\cdot e^{-j\omega t}} \quad (2)$$

$$+\; k\, u_r^2\cdot u_o(x,y)\cdot e^{-j\phi(x,y)}\cdot e^{j2\pi(2f_{xr})\cdot x}\, e^{-j\omega t} \quad (3)$$

different spatial frequency


$f_x = 2f_{xr}$
3rd term
1st term
$f_x = f_{xr} = \frac{\sin\theta}{\lambda}$
2nd term
film
Reconstruction beam

Comparison:

1. Is photography 3D?
   Of course not. It is 2D.

2. Why only 2D? Loss of phase information.
   The film can record intensity only.

3. How did holography retain phase information?
   With a reference beam.

4. Can we observe/see the phase over the holographic film?
   No, the film can record intensity only.

5. Where is the phase term then?
   Hiding in one of the 3 components.

6. How do we bring the phase term back out?
   With a reconstruction beam.

	Holography	*AM (modulation and demodulation)*
1	*In 3D space domain*	*In 1D time domain*
2	*3D object distribution*	*Time signal (information contents)*
3	*Reference beam*	*Modulation waveform*
4	*Plane wave (single spatial frequency)*	*Sinusoidal (single temporal frequency)*
5	*Modulation by absolute-square*	*Modulation by multiplication*
6	*Reconstruction beam*	*Demodulation waveform*
7	*3 resulted beams*	*3 signals after demodulation mixing*
8	*Reconstruction by view angle*	*Reconstruction by lowpassing*