ECE 278C Imaging Systems

13. FMCW imaging systems

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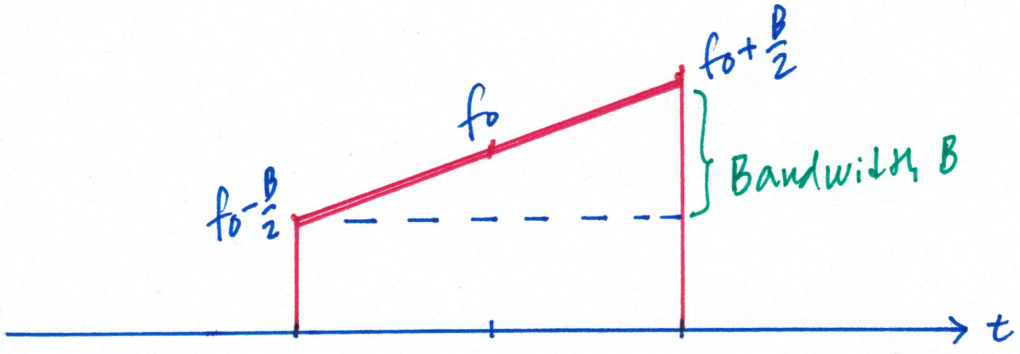
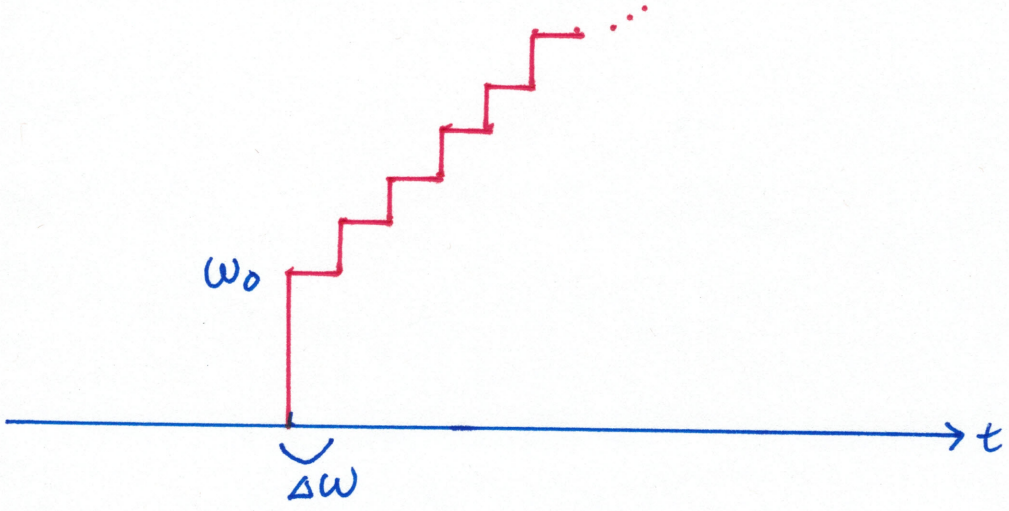
Linear Chirp waveform



Step-frequency FMCW

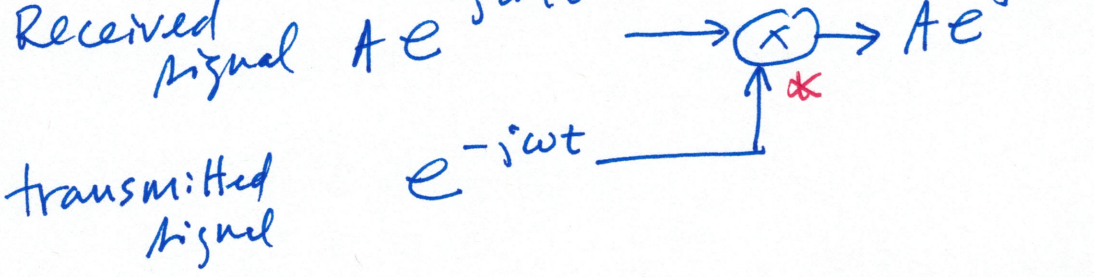


$$\omega = \omega_0 + n\Delta\omega \qquad n = 0, 1, 2, \cdots N-1$$

Time delay $\tau = \frac{2 \cdot r}{v}$ (r: range distance, v: propagation speed)

Received signal $A e^{-j\omega(t-\tau)}$

transmitted signal $e^{-j\omega t}$



$A e^{-j\omega\tau}$

$$A e^{j\omega\tau} = A \cdot e^{j(\omega_0 + n\Delta\omega)\frac{2r}{v}}$$

$$= A e^{j\omega_0 \frac{2r}{v}} \cdot e^{jn\Delta\omega\frac{2r}{v}}$$

$$= A e^{j\omega_0 \frac{2r}{v}} \cdot e^{j2\pi n \Delta f \cdot 2r/v}$$

$$= A e^{j2\pi f_0 2r/v} \cdot e^{j2\pi n(\Delta f \cdot 2r/v)}$$

DFT Kernel

$$e^{-j2\pi nk/N}$$

Single target at $r = r_0$

$$\tau_0 = \frac{2r_0}{v}$$

N-point Data sequence $= A_0 \cdot e^{j2\pi f_0 \frac{2r_0}{v}} \cdot e^{j2\pi n(\Delta f \cdot \frac{2r_0}{v})}$

$$\text{DFT}\{\text{data sequence}\} = e^{j2\pi f_0 \frac{2r_0}{v}} \cdot A_0 \cdot \delta(k - k_0)$$

$$k_0 = \frac{2B}{v} r_0$$

range profile: $A_0 \delta(k - k_0)$

Extra phase term: $e^{j2\pi f_0 \frac{2r_0}{v}} = e^{j2\pi \frac{f_0}{B} \cdot k_0}$

range profile $= \underbrace{e^{-j2\pi \frac{f_0}{B} k_0}} \cdot \{\text{DFT}\}$

$$e^{j2\pi n(\Delta f \cdot 2r/v)} \longleftrightarrow e^{j2\pi nk/N}$$

$$\Delta f \cdot 2r/v = k/N$$

$$k = (N\Delta f) \cdot \frac{2r}{v} = B \cdot \frac{2r}{v}$$

$$\boxed{k = \left(\frac{2B}{v}\right) r}$$

physical range distance

$$\boxed{r = \left(\frac{v}{2B}\right) \cdot k}$$

↑ location index of the peak

$$\text{range resolution} = \frac{v}{2B}$$

$$= \frac{\text{propagation speed}}{2 \cdot \text{Bandwidth}}$$

Multiple targets at $r = r_m$ $\quad m = 1, 2, \cdots M$

Amplitude $\quad A = A_m$

Time delay $\quad \tau_m = \frac{2r_m}{v}$

Time delay profile : $\sum_{m=1}^{M} A_m \delta(t - t_m)$

Range target dist : $\sum_{m=1}^{M} A_m \delta(r - r_m)$

Data sequence : $\sum_{m=1}^{M} A_m \cdot e^{j2\pi f_0 \frac{2r_m}{v}} \cdot e^{j2\pi u \left(\Delta f \cdot \frac{2r_m}{v}\right)}$

$$\text{DFT}\{\text{data}\} = \sum_{m=1}^{M} A_m \cdot e^{j2\pi f_0 \frac{2r_m}{v}} \delta(k - k_m)$$

$$= e^{j2\pi \frac{f_0}{B} k} \cdot \sum_{m=1}^{M} A_m \delta(k - k_m)$$

$$k_m = \frac{2B}{v} r_m$$

discrete range profile

Multi-frequency backward propagation

Operating frequency ω_n $\longrightarrow$ Complex data sample $g(n)$

$$\omega_n = \omega_0 + n\Delta\omega$$

$$f_n = f_0 + n\Delta f$$

$$\frac{1}{\lambda_n} = \frac{f_n}{v} = \frac{1}{v}[f_0 + n\Delta f]$$

Backward propagation

$$\sum_n g(n) \cdot \exp\left(-j2\pi (2r)/\lambda_n\right)$$

$$= \sum_n g(n) \cdot \exp\left(-j2\pi (2r) \cdot \frac{1}{v}(f_0 + n\Delta f)\right)$$

$$= \sum_n g(n) \cdot \exp\left(-j2\pi f_0 \frac{2r}{v}\right) \cdot \exp\left(-j2\pi (n\Delta f)\frac{2r}{v}\right)$$

$$= \exp\left(-j2\pi f_0 \frac{2r}{v}\right) \cdot \sum_n g(n) \cdot \exp\left(-j2\pi (n\Delta f)\frac{2r}{v}\right)$$

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$$\sum_n g(n) \cdot \exp\left(-j 2\pi n \left(\frac{2r}{v} \Delta f\right)\right)$$

$$\text{DTFT}\{g(n)\} = G(e^{j\Omega})$$

$$= \sum_n g(n) \cdot \exp(-jn\Omega)$$

$$\Omega = 2\pi \left(\frac{2r}{v} \Delta f\right)$$

Thus:

$$\sum_n g(n) \cdot \exp\left(-j2\pi (n\Delta f)\left(\frac{2r}{v}\right)\right)$$

$$= G(e^{j\Omega})\Big|_{\Omega = 2\pi\left(\frac{2r}{v}\Delta f\right)}$$

$$\Omega = 2\pi \left(\frac{2r}{v} \Delta f\right)$$

range distance $r = \left(\frac{v}{2\Delta f}\right) \cdot \left(\frac{\Omega}{2\pi}\right)$

DFT implementation

N samples along the unit circle

$$\Omega = k \cdot \Delta\Omega = k \cdot \frac{2\pi}{N}$$

range distance

$$r = \left(\frac{v}{2\Delta f}\right) \cdot \left(\frac{\Omega}{2\pi}\right)$$

$$= \left(\frac{v}{2\Delta f}\right) \cdot \left(\frac{2\pi k}{N \cdot 2\pi}\right)$$

$$= \frac{v}{2(N\Delta f)} \cdot k$$

$$= \left(\frac{v}{2B}\right) \cdot k$$

$$\text{range distance} = \left(\frac{\text{Propagation speed}}{2 \cdot \text{Bandwidth}}\right) \cdot \text{DFT index}$$

$$\boxed{\text{Resolution} = \frac{v}{2B}}$$

range profile (function of range distance r)

$$= \exp\left(-j2\pi f_0 \frac{2r}{v}\right) \cdot \text{N-point DFT}\{g(n)\}$$

$$= \exp\left(-j2\pi f_0 \frac{2r}{v}\right) \cdot G(k)$$

$$r = \left(\frac{v}{2B}\right) \cdot k$$