Measures of Relative Dispersion

These are unitless measures of dispersion and are used when one wishes to compare the dispersion of one distribution with another distribution.

1. The Coefficient of Variation

The coefficient of variation, **CV**, is the ratio of the standard deviation to the mean and is usually expressed in percentage. It is computed as:

$$CV = \frac{\sigma}{\mu} \times 100\% \quad or \quad CV = \frac{s}{\bar{x}} \times 100\%$$

depending on whether it is computed out of a population or a sample.

Example: The foreign exchange rate is one of the indicators of our country's economic performance. Given below are the means and standard deviations of the quarterly exchange rate during the years 1989 to 1991 and 1992 to 1994. On which of the two periods did a more consistent economic activity occur?

Period	Mean Exchange Rate	Standard Deviation
1989 – 1991	22.40	1.16
1992 – 1994	26.40	1.84

To answer this problem, we will make use of coefficient of variation to compare the spread of the exchange rates within the covered periods.

Computing for the CV during the period 1989 – 1991:

$$CV = \frac{1.16}{22.40} \times 100\% = \mathbf{5.18}\%$$

Computing for the CV during the period 1992 – 1994:

$$CV = \frac{1.84}{26.40} \times 100\% = \mathbf{6.97}\%$$

Based on the computed values above, we can see that the coefficient of variation of the exchange rate during the period 1989 – 1991 is lower than that of 1992 – 1994. This suggests that the economic activity of the country is **more consistent** during the period 1989 – 1991 (*5.18%)* than the economic activity during the period 1992 – 1994 (*6.97%).*

2. The Standard Score (z-score)

The standard score refers to how many standard deviations an observation is above or below the mean. This is computed as

$$Z = \frac{x - \mu}{\sigma} \qquad or \qquad Z = \frac{x - \bar{x}}{s}$$

depending on whether it is computed from a population or sample.

Example: Robert got a grade of 75% in English and a grade of 90% in Math. The mean grade in English is 65% with a standard deviation of 10% while the mean grade in Math is 80% with a standard deviation of 20%. On which subject did he perform better in comparison with his classmates?

To determine which subject Robert performed better, it is NOT ADVISABLE to simply look at the grades he obtained on each subject since these two subjects are independent of each other and there are differences in the values considered for each subject (mean and SD). A better way to gauge where Robert performed better is to transform his grades in both subjects into standard scores or z scores using the formula above.

Robert's English Score as a standard score:

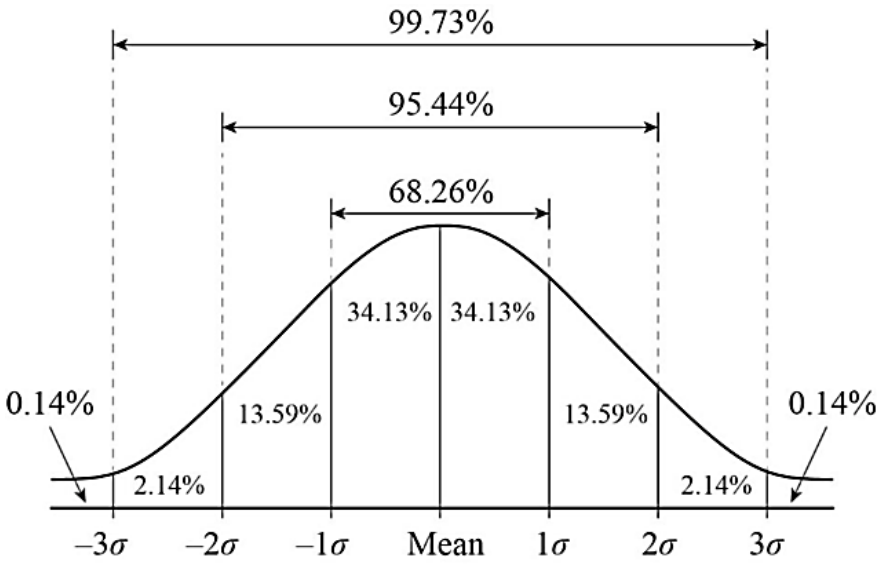
$$Z_{english} = \frac{0.75 - 0.65}{0.10} = 1.0$$

Robert's Math Score as a standard score:

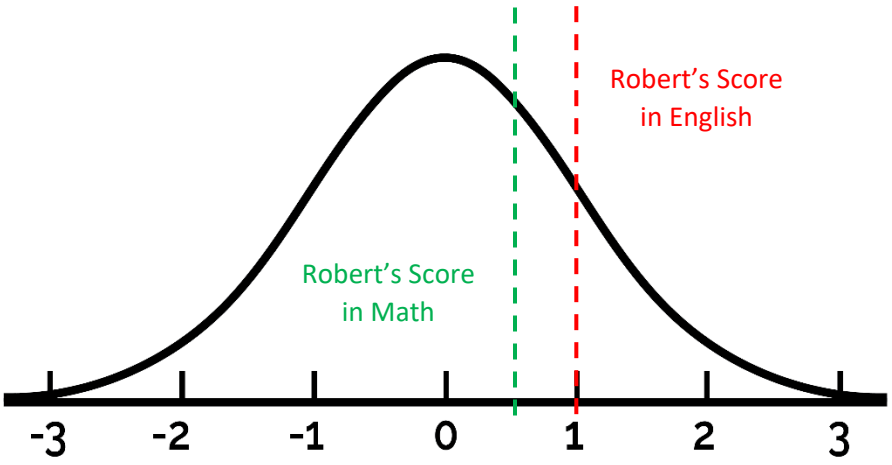
$$Z_{math} = \frac{0.90 - 0.80}{0.20} = 0.5$$

Based on the values computed above, we can see that Robert's English score is 1.0 standard deviation above the mean while his Math score is only 0.5 standard deviation above the mean. Since Robert's score in English is on a higher z-score, this indicates that Robert performed better in his English subject than on Mathematics subject.

To further understand how z-sores behave, it is best to look at them against the *Gaussian Curve* or simply known as the normal curve presented below:



Try to locate the standard score of Robert in both English and Math. Based on the locations you've identified, is it correct to say that Robert performed better in English than in Math?



The figure above further shows that Robert's score in English is on a higher standing compared to his score in Math.