1. Gyenge alak

$$\underline{\underline{\sigma}} \cdot \nabla + \vec{f} = \vec{0} \qquad / \cdot \vec{v} \qquad / \int\limits_{(v)} \dots dv$$

$$-W\left(\vec{r},\vec{v}\right) = \int\limits_{(v)} \vec{v} \cdot \overset{\downarrow}{\underline{\underline{\sigma}}} \cdot \nabla dv + \int\limits_{(v)} \vec{v} \cdot \vec{f} dv = 0$$

$$-W\left(\vec{r},\vec{v}\right) = \int\limits_{(v)} \overset{\downarrow}{\vec{v} \cdot \underline{\underline{\sigma}}} \cdot \nabla dv - \int\limits_{(v)} \overset{\downarrow}{\vec{v}} \cdot \underline{\underline{\sigma}} \cdot \nabla dv + \int\limits_{(v)} \vec{v} \cdot \vec{f} dv = 0$$

$$-W\left(\vec{r},\vec{v}\right) = \int\limits_{(a)} \vec{v} \cdot \underline{\underline{\sigma}} \cdot \vec{n} da - \int\limits_{(v)} \overset{\downarrow}{\vec{v}} \cdot \underline{\underline{\sigma}} \cdot \nabla dv + \int\limits_{(v)} \vec{v} \cdot \vec{f} dv = 0$$

$$\underline{\underline{\sigma}} \cdot \vec{n} = \vec{t} \qquad \vec{r} \in a_t$$

$$-W\left(\vec{r},\vec{v}\right) = \int\limits_{(a_t)} \vec{v} \cdot \vec{t} da - \int\limits_{(v)} \overset{\downarrow}{\vec{v}} \cdot \underline{\underline{\sigma}} \cdot \nabla dv + \int\limits_{(v)} \vec{v} \cdot \vec{f} dv = 0$$

$$W\left(\vec{r},\vec{v}\right) = \int\limits_{(v)} \overset{\downarrow}{\vec{v}} \cdot \underline{\underline{\sigma}} \cdot \nabla dv - \int\limits_{(a_t)} \vec{v} \cdot \vec{t} da - \int\limits_{(v)} \vec{v} \cdot \vec{f} dv = 0$$

$$W\left(\vec{r},\vec{v}\right) = \int\limits_{(v)} \underline{\underline{\sigma}} \cdot \cdot \vec{v} \circ \nabla dv - \int\limits_{(a_t)} \vec{v} \cdot \vec{t} da - \int\limits_{(v)} \vec{v} \cdot \vec{f} dv = 0$$

$$W\left(\vec{r},\vec{v}\right) = \int\limits_{(v)} \underline{\underline{\sigma}} \cdot \cdot \underline{\underline{F}}^{-T} \cdot \underline{\underline{F}}^{T} \cdot \vec{v} \circ \nabla \cdot \underline{\underline{F}} \cdot \underline{\underline{F}}^{-1} dv - \int\limits_{(a_t)} \vec{v} \cdot \underline{\underline{\sigma}} \cdot \vec{n} da - \int\limits_{(v)} \vec{v} \cdot \vec{f} dv = 0$$

$$W\left(\vec{r},\vec{v}\right) = \int\limits_{(v)} \underline{\underline{F}}^{-1} \cdot \underline{\underline{\sigma}} \cdot \underline{\underline{F}}^{-T} \cdot \cdot \underline{\underline{F}}^{T} \cdot \vec{v} \circ \nabla \cdot \underline{\underline{F}} dv - \int\limits_{(a_t)} \vec{v} \cdot \underline{\underline{\sigma}} \cdot \vec{n} da - \int\limits_{(v)} \vec{v} \cdot \vec{f} dv = 0$$

$$W\left(\vec{r},\vec{v}\right) = \int\limits_{(V)} \underbrace{J \underline{\underline{F}}^{-1} \cdot \underline{\underline{\sigma}} \cdot \underline{\underline{F}}^{-T}}_{\underline{\underline{S}}} \cdot \cdot \underline{\underline{F}}^{T} \cdot \vec{v} \circ \underbrace{\nabla \cdot \underline{\underline{F}}}_{\nabla_0} dV - \int\limits_{(A_t)} \vec{v} \cdot \underline{\underline{\sigma}} \cdot \underline{\underline{F}}^{-T} \cdot \vec{N} J dA - \int\limits_{(V)} \vec{v} \cdot \underbrace{\vec{f} J}_{\vec{f}_0} dV = 0$$

$$W\left(\vec{r},\vec{v}\right) = \int\limits_{(V)} \underline{\underline{S}} \cdot \cdot \underline{\underline{F}}^{T} \cdot \vec{v} \circ \nabla_0 dV - \int\limits_{(A_t)} \vec{v} \cdot \underline{\underline{F}} \cdot \underbrace{\underline{\underline{F}}^{-1} \cdot J \underline{\underline{\sigma}} \cdot \underline{\underline{F}}^{-T}}_{\underline{\underline{S}}} \cdot \vec{N} dA - \int\limits_{(V)} \vec{v} \cdot \vec{f}_0 dV = 0$$

$$W\left(\vec{r},\vec{v}\right) = \int\limits_{(V)} \underline{\underline{S}} \cdot \cdot \underline{\underline{F}}^{T} \cdot \vec{v} \circ \nabla_0 dV - \int\limits_{(A_t)} \vec{v} \cdot \underline{\underline{F}} \cdot \underbrace{\underline{\underline{S}} \cdot \vec{N}}_{\vec{t}_0} dA - \int\limits_{(V)} \vec{v} \cdot \vec{f}_0 dV = 0$$

$$W\left(\vec{r},\vec{v}\right) = \int\limits_{(V)} \underline{\underline{S}} \cdot \cdot \underline{\underline{F}}^{T} \cdot \vec{v} \circ \nabla_0 dV - \int\limits_{(A_t)} \vec{v} \cdot \underline{\underline{F}} \cdot \vec{t}_0 dA - \int\limits_{(V)} \vec{v} \cdot \vec{f}_0 dV = 0$$

2. Nemlineáris rész

$$W\left(\vec{r},\vec{v}\right)=\int\limits_{(V)}\underline{\underline{S}}\cdot\cdot\underline{\underline{F}}^{T}\cdot\vec{v}\circ\nabla_{0}dV-\int\limits_{(A_t)}\vec{v}\cdot\underline{\underline{F}}\cdot\vec{t}_{0}dA-\int\limits_{(V)}\vec{v}\cdot\vec{f}_{0}dV=0$$

$$W\left(\vec{r},\vec{v}\right)=\int\limits_{(V)}\underline{\underline{S}}\cdot\cdot\frac{1}{2}\left(\underline{\underline{F}}^{T}\cdot\vec{v}\circ\nabla_{0}+\nabla_{0}\circ\vec{v}\cdot\underline{\underline{F}}\right)dV-\int\limits_{(A_t)}\vec{v}\cdot\underline{\underline{F}}\cdot\vec{t}_{0}dA-\int\limits_{(V)}\vec{v}\cdot\vec{f}_{0}dV=0$$

$$\vec{t}_{0}\rightarrow\underline{\underline{t}}=\begin{bmatrix}t_{x}\\t_{y}\\t_{z}\end{bmatrix}$$

$$\vec{f}_{0}\rightarrow\underline{\underline{f}}=\begin{bmatrix}f_{x}\\f_{y}\\f_{z}\end{bmatrix}$$

$$\vec{v}\rightarrow\underline{\underline{v}}=\begin{bmatrix}v_{x}\\v_{y}\\v_{z}\end{bmatrix}=\begin{bmatrix}\sum_{i=1}^{N}h_{i}\left(\xi,\eta,\zeta\right)\phi_{xi}^{e}\\\sum_{i=1}^{N}h_{i}\left(\xi,\eta,\zeta\right)\phi_{yi}^{e}\\\sum_{i=1}^{N}h_{i}\left(\xi,\eta,\zeta\right)\phi_{zi}^{e}\end{bmatrix}=$$

$$=\underbrace{\begin{bmatrix}h_{1}&0&0&h_{2}&0&0&\cdots&h_{N}&0&0\\0&h_{1}&0&0&h_{2}&0&\cdots&0&h_{N}&0\\0&0&h_{1}&0&0&h_{2}&\cdots&0&0&h_{N}\end{bmatrix}}_{\underset{(3\times3N)}{\underline{\underline{H}}(\xi,\eta,\zeta)}}\underbrace{\begin{bmatrix}\phi_{x1}^{e}\\\phi_{y1}^{e}\\\phi_{z1}^{e}\\\phi_{x2}^{e}\\\phi_{y2}^{e}\\\phi_{z2}^{e}\\\vdots\\\phi_{xN}^{e}\\\phi_{yN}^{e}\\\phi_{zN}^{e}\end{bmatrix}}_{\underset{(3N\times1)}{\underline{\underline{\phi}}}}=\underline{\underline{H}}\left(\xi,\eta,\zeta\right)\underline{\underline{\phi}}$$

$$\underline{\underline{H}}\left(\xi,\eta,\zeta\right)=\underbrace{\begin{bmatrix}h_{1}&0&0&h_{2}&0&0&\cdots&h_{N}&0&0\\0&h_{1}&0&0&h_{2}&0&\cdots&0&h_{N}&0\\0&0&h_{1}&0&0&h_{2}&\cdots&0&0&h_{N}\end{bmatrix}}_{(3\times3N)}$$

$$\underline{\underline{S}}\rightarrow\underline{\underline{S}}=\begin{bmatrix}S_{x}&S_{xy}&S_{xz}\\S_{yx}&S_{y}&S_{yz}\\S_{zx}&S_{zy}&S_{z}\end{bmatrix}$$

Ha $\underline{\underline{S}}=\underline{\underline{S}}^{T}$

$$\underline{\underline{S}}\rightarrow\underline{\underline{S}}=\begin{bmatrix}S_{x}&S_{xy}&S_{zx}\\S_{xy}&S_{y}&S_{yz}\\S_{zx}&S_{yz}&S_{z}\end{bmatrix}\rightarrow\begin{bmatrix}S_{x}\\S_{y}\\S_{z}\\S_{xy}\\S_{yz}\\S_{zx}\end{bmatrix}$$

$$W^{e}\left(\vec{r},\vec{v}\right)=\int\limits_{(v)}\left(\underline{\underline{\phi}}^{e}\right)^{T}\left(\underline{\underline{B}}_{L}^{e}\right)^{T}\underline{\underline{S}}dv-\int\limits_{(a_t)}\left(\underline{\underline{\phi}}^{e}\right)^{T}\underline{\underline{H}}^{T}\underline{\underline{F}}\underline{\underline{t}}da-\int\limits_{(v)}\left(\underline{\underline{\phi}}^{e}\right)^{T}\underline{\underline{H}}^{T}\underline{\underline{f}}dv=0$$

$$W^{e}\left(\vec{r},\vec{v}\right)=\left(\underline{\underline{\phi}}^{e}\right)^{T}\left[\int\limits_{(v)}\left(\underline{\underline{B}}_{L}^{e}\right)^{T}\underline{\underline{S}}dv-\int\limits_{(a_t)}\underline{\underline{H}}^{T}\underline{\underline{F}}\underline{\underline{t}}da-\int\limits_{(v)}\underline{\underline{H}}^{T}\underline{\underline{f}}dv\right]=0$$

3. Linearizált rész

$$DW(\vec{r},\vec{v})[\vec{u}] = D\left(\int\limits_{(V)} \underline{\underline{S}} \cdot\cdot \underline{\underline{F}}^T \cdot \vec{v} \circ \nabla_0 dV - \int\limits_{(A_t)} \vec{v} \cdot \underline{\underline{F}} \cdot \vec{t}_0 dA - \int\limits_{(V)} \vec{v} \cdot \vec{f}_0 dV\right)[\vec{u}] =$$

$$= D\int\limits_{(V)} \underline{\underline{S}} \cdot\cdot \underline{\underline{F}}^T \cdot \vec{v} \circ \nabla_0 dV\,[\vec{u}] - D\int\limits_{(A_t)} \vec{v} \cdot \underline{\underline{F}} \cdot \vec{t}_0 dA\,[\vec{u}] - \underbrace{D\int\limits_{(V)} \vec{v} \cdot \vec{f}_0 dV\,[\vec{u}]}_{0} =$$

$$= \int\limits_{(V)} D\underline{\underline{S}}[\vec{u}] \cdot\cdot \underline{\underline{F}}^T \cdot \vec{v} \circ \nabla_0 dV + \int\limits_{(V)} \underline{\underline{S}} \cdot\cdot D\underline{\underline{F}}^T[\vec{u}] \cdot \vec{v} \circ \nabla_0 dV + \int\limits_{(A_t)} \vec{v} \cdot D\underline{\underline{F}}[\vec{u}] \cdot \vec{t}_0 dA$$

$$D\underline{\underline{S}}[\vec{u}] = \frac{\partial \underline{\underline{S}}}{\partial \underline{\underline{E}}} \cdot\cdot D\underline{\underline{E}}[\vec{u}]$$

$$D\underline{\underline{E}}[\vec{u}] = D\left(\frac{1}{2}\left(\underline{\underline{F}}^T \cdot \underline{\underline{F}} - \underline{\underline{I}}\right)\right)[\vec{u}] = \frac{1}{2}\left(D\underline{\underline{F}}^T[\vec{u}] \cdot \underline{\underline{F}} + \underline{\underline{F}}^T \cdot D\underline{\underline{F}}[\vec{u}]\right)$$

$$D\underline{\underline{F}}[\vec{u}] = \lim_{\varepsilon\to 0} \frac{d\left((\vec{r}+\varepsilon\vec{u}) \circ \nabla_0\right)}{d\varepsilon} = \lim_{\varepsilon\to 0} \frac{d\left(\vec{r}+\varepsilon\vec{u}\right)}{d\varepsilon} \circ \nabla_0 = \vec{u} \circ \nabla_0$$

$$D\underline{\underline{F}}^T[\vec{u}] = \lim_{\varepsilon\to 0} \frac{d\left(\nabla_0 \circ (\vec{r}+\varepsilon\vec{u})\right)}{d\varepsilon} = \nabla_0 \circ \lim_{\varepsilon\to 0} \frac{d\left(\vec{r}+\varepsilon\vec{u}\right)}{d\varepsilon} = \nabla_0 \circ \vec{u}$$

$$D\underline{\underline{E}}[\vec{u}] = D\left(\frac{1}{2}\left(\underline{\underline{F}}^T \cdot \underline{\underline{F}} - \underline{\underline{I}}\right)\right)[\vec{u}] = \frac{1}{2}\left(\nabla_0 \circ \vec{u} \cdot \underline{\underline{F}} + \underline{\underline{F}}^T \cdot \vec{u} \circ \nabla_0\right)$$

$$D\underline{\underline{S}}[\vec{u}] = \frac{\partial \underline{\underline{S}}}{\partial \underline{\underline{E}}} \cdot\cdot \frac{1}{2}\left(\nabla_0 \circ \vec{u} \cdot \underline{\underline{F}} + \underline{\underline{F}}^T \cdot \vec{u} \circ \nabla_0\right) = 2\frac{\partial \underline{\underline{S}}}{\partial \underline{\underline{C}}} \cdot\cdot \frac{1}{2}\left(\nabla_0 \circ \vec{u} \cdot \underline{\underline{F}} + \underline{\underline{F}}^T \cdot \vec{u} \circ \nabla_0\right) =$$

$$= \underbrace{\frac{\partial \underline{\underline{S}}}{\partial \underline{\underline{C}}}}_{2\underline{\underline{\mathbb{C}}}^{(4)}} \cdot\cdot \left(\nabla_0 \circ \vec{u} \cdot \underline{\underline{F}} + \underline{\underline{F}}^T \cdot \vec{u} \circ \nabla_0\right)$$

$$D\underline{\underline{S}}[\vec{u}] \cdot\cdot \underline{\underline{F}}^T \cdot \vec{v} \circ \nabla_0 = \underline{\underline{F}}^T \cdot \vec{v} \circ \nabla_0 \cdot\cdot 2\underline{\underline{\mathbb{C}}}^{(4)} \cdot\cdot \left(\nabla_0 \circ \vec{u} \cdot \underline{\underline{F}} + \underline{\underline{F}}^T \cdot \vec{u} \circ \nabla_0\right) =$$

$$= \frac{1}{2}\left(\nabla_0 \circ \vec{v} \cdot \underline{\underline{F}} + \underline{\underline{F}}^T \cdot \vec{v} \circ \nabla_0\right) \cdot\cdot 2\underline{\underline{\mathbb{C}}}^{(4)} \cdot\cdot \left(\nabla_0 \circ \vec{u} \cdot \underline{\underline{F}} + \underline{\underline{F}}^T \cdot \vec{u} \circ \nabla_0\right) =$$

$$= \left(\nabla_0 \circ \vec{v} \cdot \underline{\underline{F}} + \underline{\underline{F}}^T \cdot \vec{v} \circ \nabla_0\right) \cdot\cdot \underline{\underline{\mathbb{C}}}^{(4)} \cdot\cdot \left(\nabla_0 \circ \vec{u} \cdot \underline{\underline{F}} + \underline{\underline{F}}^T \cdot \vec{u} \circ \nabla_0\right)$$

$$\nabla_0 \circ \vec{v} \cdot \underline{\underline{F}} + \underline{\underline{F}}^T \cdot \vec{v} \circ \nabla_0 = v_{i,j}F_{ik} + F_{ji}v_{i,k} = H_{il,j}\phi_l F_{ik} + F_{ji}H_{il,k}\phi_l =$$

$$= \left(H_{il,j}F_{ik} + F_{ji}H_{il,k}\right)\phi_l$$

$$v_{i,j}F_{ik} + F_{ji}v_{i,k} \to i = x,y,z \qquad \begin{matrix} j=x \\ k=x \end{matrix}, \quad \begin{matrix} j=y \\ k=y \end{matrix}, \quad \begin{matrix} j=z \\ k=z \end{matrix}, \quad \begin{matrix} j=x \\ k=y \end{matrix}, \quad \begin{matrix} j=y \\ k=z \end{matrix}, \quad \begin{matrix} j=z \\ k=x \end{matrix}$$

$$\vec{v} \circ \nabla \cdot \underline{\underline{F}} = \underbrace{\begin{bmatrix} H_{11,1}F_{11} & H_{22,1}F_{21} & H_{33,1}F_{31} & H_{14,1}F_{11} & H_{25,1}F_{21} & H_{36,1}F_{31} & \cdots \\ H_{11,2}F_{12} & H_{22,2}F_{22} & H_{33,2}F_{32} & H_{14,2}F_{12} & H_{25,2}F_{22} & H_{36,2}F_{32} & \cdots \\ H_{11,3}F_{13} & H_{22,3}F_{23} & H_{33,3}F_{33} & H_{14,3}F_{13} & H_{25,3}F_{23} & H_{36,3}F_{33} & \cdots \\ H_{11,1}F_{12} & H_{22,1}F_{22} & H_{33,1}F_{32} & H_{14,1}F_{12} & H_{25,1}F_{22} & H_{36,1}F_{32} & \cdots \\ H_{11,2}F_{13} & H_{22,2}F_{23} & H_{33,2}F_{33} & H_{14,2}F_{13} & H_{25,2}F_{23} & H_{36,2}F_{33} & \cdots \\ H_{11,3}F_{11} & H_{22,3}F_{21} & H_{33,3}F_{31} & H_{14,3}F_{11} & H_{25,3}F_{21} & H_{36,3}F_{31} & \cdots \end{bmatrix}}_{\underline{\underline{B}}^e_{L1}(\xi,\eta,\zeta)\ (6\times 3N)} \underbrace{\begin{bmatrix} \phi^e_{x1} \\ \phi^e_{y1} \\ \phi^e_{z1} \\ \phi^e_{x2} \\ \phi^e_{y2} \\ \phi^e_{z2} \\ \vdots \\ \phi^e_{xN} \\ \phi^e_{yN} \\ \phi^e_{zN} \end{bmatrix}}_{\underline{\phi}\ (3N\times 1)} =$$

$$= \underbrace{\begin{bmatrix} \frac{\partial h_1}{\partial X}F_x & \frac{\partial h_1}{\partial X}F_{yx} & \frac{\partial h_1}{\partial X}F_{zx} & \frac{\partial h_2}{\partial X}F_x & \frac{\partial h_2}{\partial X}F_{yx} & \frac{\partial h_2}{\partial X}F_{zx} & \cdots & \frac{\partial h_N}{\partial X}F_x & \frac{\partial h_N}{\partial X}F_{yx} & \frac{\partial h_N}{\partial X}F_{zx} \\ \frac{\partial h_1}{\partial Y}F_{xy} & \frac{\partial h_1}{\partial Y}F_y & \frac{\partial h_1}{\partial Y}F_{zy} & \frac{\partial h_2}{\partial Y}F_{xy} & \frac{\partial h_2}{\partial Y}F_y & \frac{\partial h_2}{\partial Y}F_{zy} & \cdots & \frac{\partial h_N}{\partial Y}F_{xy} & \frac{\partial h_N}{\partial Y}F_y & \frac{\partial h_N}{\partial Y}F_{zy} \\ \frac{\partial h_1}{\partial Z}F_{xz} & \frac{\partial h_1}{\partial Z}F_{yz} & \frac{\partial h_1}{\partial Z}F_z & \frac{\partial h_2}{\partial Z}F_{xz} & \frac{\partial h_2}{\partial Z}F_{yz} & \frac{\partial h_2}{\partial Z}F_z & \cdots & \frac{\partial h_N}{\partial Z}F_{xz} & \frac{\partial h_N}{\partial Z}F_{yz} & \frac{\partial h_N}{\partial Z}F_z \\ \frac{\partial h_1}{\partial X}F_{xy} & \frac{\partial h_1}{\partial X}F_y & \frac{\partial h_1}{\partial X}F_{zy} & \frac{\partial h_2}{\partial X}F_{xy} & \frac{\partial h_2}{\partial X}F_y & \frac{\partial h_2}{\partial X}F_{zy} & \cdots & \frac{\partial h_N}{\partial X}F_{xy} & \frac{\partial h_N}{\partial X}F_y & \frac{\partial h_N}{\partial X}F_{zy} \\ \frac{\partial h_1}{\partial Y}F_{xz} & \frac{\partial h_1}{\partial Y}F_{yz} & \frac{\partial h_1}{\partial Y}F_z & \frac{\partial h_2}{\partial Y}F_{xz} & \frac{\partial h_2}{\partial Y}F_{yz} & \frac{\partial h_2}{\partial Y}F_z & \cdots & \frac{\partial h_N}{\partial Y}F_{xz} & \frac{\partial h_N}{\partial Y}F_{yz} & \frac{\partial h_N}{\partial Y}F_z \\ \frac{\partial h_1}{\partial Z}F_x & \frac{\partial h_1}{\partial Z}F_{yx} & \frac{\partial h_1}{\partial Z}F_{zx} & \frac{\partial h_2}{\partial Z}F_x & \frac{\partial h_2}{\partial Z}F_{yx} & \frac{\partial h_2}{\partial Z}F_{zx} & \cdots & \frac{\partial h_N}{\partial Z}F_x & \frac{\partial h_N}{\partial Z}F_{yx} & \frac{\partial h_N}{\partial Z}F_{zx} \end{bmatrix}}_{\underline{\underline{B}}^e_{L1}(\xi,\eta,\zeta) \atop (6\times 3N)} \underbrace{\begin{bmatrix} \phi^e_{x1} \\ \phi^e_{y1} \\ \phi^e_{z1} \\ \phi^e_{x2} \\ \phi^e_{y2} \\ \phi^e_{z2} \\ \vdots \\ \phi^e_{xN} \\ \phi^e_{yN} \\ \phi^e_{zN} \end{bmatrix}}_{\underline{\phi} \atop (3N\times 1)}$$

$$\vec{v}\circ\nabla\cdot\underline{\underline{F}} = \underbrace{\begin{bmatrix} H_{11,1}F_{11} & H_{22,1}F_{21} & H_{33,1}F_{31} & H_{14,1}F_{11} & H_{25,1}F_{21} & H_{36,1}F_{31} & \cdots \\ H_{11,2}F_{12} & H_{22,2}F_{22} & H_{33,2}F_{32} & H_{14,2}F_{12} & H_{25,2}F_{22} & H_{36,2}F_{32} & \cdots \\ H_{11,3}F_{13} & H_{22,3}F_{23} & H_{33,3}F_{33} & H_{14,3}F_{13} & H_{25,3}F_{23} & H_{36,3}F_{33} & \cdots \\ H_{11,2}F_{11} & H_{22,2}F_{21} & H_{33,2}F_{31} & H_{14,2}F_{11} & H_{25,2}F_{21} & H_{36,2}F_{31} & \cdots \\ H_{11,3}F_{12} & H_{22,3}F_{22} & H_{33,3}F_{32} & H_{14,3}F_{12} & H_{25,3}F_{22} & H_{36,3}F_{32} & \cdots \\ H_{11,1}F_{13} & H_{22,1}F_{23} & H_{33,1}F_{33} & H_{14,1}F_{13} & H_{25,1}F_{23} & H_{36,1}F_{33} & \cdots \end{bmatrix}}_{\underline{\underline{B}}^e_{L2}(\xi,\eta,\zeta) \atop (6\times 3N)} \underbrace{\begin{bmatrix} \phi^e_{x1} \\ \phi^e_{y1} \\ \phi^e_{z1} \\ \phi^e_{x2} \\ \phi^e_{y2} \\ \phi^e_{z2} \\ \vdots \\ \phi^e_{xN} \\ \phi^e_{yN} \\ \phi^e_{zN} \end{bmatrix}}_{\underline{\phi} \atop (3N\times 1)} =$$

$$= \underbrace{\begin{bmatrix} \frac{\partial h_1}{\partial X}F_x & \frac{\partial h_1}{\partial X}F_{yx} & \frac{\partial h_1}{\partial X}F_{zx} & \frac{\partial h_2}{\partial X}F_x & \frac{\partial h_2}{\partial X}F_{yx} & \frac{\partial h_2}{\partial X}F_{zx} & \cdots & \frac{\partial h_N}{\partial X}F_x & \frac{\partial h_N}{\partial X}F_{yx} & \frac{\partial h_N}{\partial X}F_{zx} \\ \frac{\partial h_1}{\partial Y}F_{xy} & \frac{\partial h_1}{\partial Y}F_y & \frac{\partial h_1}{\partial Y}F_{zy} & \frac{\partial h_2}{\partial Y}F_{xy} & \frac{\partial h_2}{\partial Y}F_y & \frac{\partial h_2}{\partial Y}F_{zy} & \cdots & \frac{\partial h_N}{\partial Y}F_{xy} & \frac{\partial h_N}{\partial Y}F_y & \frac{\partial h_N}{\partial Y}F_{zy} \\ \frac{\partial h_1}{\partial Z}F_{xz} & \frac{\partial h_1}{\partial Z}F_{yz} & \frac{\partial h_1}{\partial Z}F_z & \frac{\partial h_2}{\partial Z}F_{xz} & \frac{\partial h_2}{\partial Z}F_{yz} & \frac{\partial h_2}{\partial Z}F_z & \cdots & \frac{\partial h_N}{\partial Z}F_{xz} & \frac{\partial h_N}{\partial Z}F_{yz} & \frac{\partial h_N}{\partial Z}F_z \\ \frac{\partial h_1}{\partial Y}F_x & \frac{\partial h_1}{\partial Y}F_{yx} & \frac{\partial h_1}{\partial Y}F_{zx} & \frac{\partial h_2}{\partial Y}F_x & \frac{\partial h_2}{\partial Y}F_{yx} & \frac{\partial h_2}{\partial Y}F_{zx} & \cdots & \frac{\partial h_N}{\partial Y}F_x & \frac{\partial h_N}{\partial Y}F_{yx} & \frac{\partial h_N}{\partial Y}F_{zx} \\ \frac{\partial h_1}{\partial Z}F_{xy} & \frac{\partial h_1}{\partial Z}F_y & \frac{\partial h_1}{\partial Z}F_{zy} & \frac{\partial h_2}{\partial Z}F_{xy} & \frac{\partial h_2}{\partial Z}F_y & \frac{\partial h_2}{\partial Z}F_{zy} & \cdots & \frac{\partial h_N}{\partial Z}F_{xy} & \frac{\partial h_N}{\partial Z}F_y & \frac{\partial h_N}{\partial Z}F_{zy} \\ \frac{\partial h_1}{\partial X}F_{xz} & \frac{\partial h_1}{\partial X}F_{yz} & \frac{\partial h_1}{\partial X}F_z & \frac{\partial h_2}{\partial X}F_{xz} & \frac{\partial h_2}{\partial X}F_{yz} & \frac{\partial h_2}{\partial X}F_z & \cdots & \frac{\partial h_N}{\partial X}F_{xz} & \frac{\partial h_N}{\partial X}F_{yz} & \frac{\partial h_N}{\partial X}F_z \end{bmatrix}}_{\underline{\underline{B}}^e_{L2}(\xi,\eta,\zeta) \atop (6\times 3N)} \underbrace{\begin{bmatrix} \phi^e_{x1} \\ \phi^e_{y1} \\ \phi^e_{z1} \\ \phi^e_{x2} \\ \phi^e_{y2} \\ \phi^e_{z2} \\ \vdots \\ \phi^e_{xN} \\ \phi^e_{yN} \\ \phi^e_{zN} \end{bmatrix}}_{\underline{\phi} \atop (3N\times 1)}$$

$$\nabla_0\circ\vec{v}\cdot\underline{\underline{F}} + \underline{\underline{F}}^T\cdot\vec{v}\circ\nabla_0 =$$

$$= \underbrace{\begin{bmatrix} \frac{\partial h_1}{\partial X}F_x & \frac{\partial h_1}{\partial X}F_{yx} & \frac{\partial h_1}{\partial X}F_{zx} & \cdots \\ \frac{\partial h_1}{\partial Y}F_{xy} & \frac{\partial h_1}{\partial Y}F_y & \frac{\partial h_1}{\partial Y}F_{zy} & \cdots \\ \frac{\partial h_1}{\partial Z}F_{xz} & \frac{\partial h_1}{\partial Z}F_{yz} & \frac{\partial h_1}{\partial Z}F_z & \cdots \\ \frac{\partial h_1}{\partial X}F_{xy} + \frac{\partial h_1}{\partial Y}F_x & \frac{\partial h_1}{\partial X}F_y + \frac{\partial h_1}{\partial Y}F_{yx} & \frac{\partial h_1}{\partial X}F_{zy} + \frac{\partial h_1}{\partial Y}F_{zx} & \cdots \\ \frac{\partial h_1}{\partial Y}F_{xz} + \frac{\partial h_1}{\partial Z}F_{xy} & \frac{\partial h_1}{\partial Y}F_{yz} + \frac{\partial h_1}{\partial Z}F_y & \frac{\partial h_1}{\partial Y}F_z + \frac{\partial h_1}{\partial Z}F_{zy} & \cdots \\ \frac{\partial h_1}{\partial Z}F_x + \frac{\partial h_1}{\partial X}F_{xz} & \frac{\partial h_1}{\partial Z}F_{yx} + \frac{\partial h_1}{\partial X}F_{yz} & \frac{\partial h_1}{\partial Z}F_{zx} + \frac{\partial h_1}{\partial X}F_z & \cdots \end{bmatrix}}_{\underline{\underline{B}}^e_{L}(\xi,\eta,\zeta) \atop (6\times 3N)}$$