Out-of-OH Q&A

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1 Why

The ideal time to ask questions about the material and problem sets is during office hours. However, many of you can't make it to office hours for valid reasons, so I also generally questions sent by e-mail or other means.

Keeping this document helps you (and yours truly) for a few reasons:

- More clarifications/explanations available
- There is no 'unfair advantage' to those who ask many questions
- I don't have to answer the same thing over and over :)

2 Pset 1

2.1 About limits of sets, and Durrett and Billingsley being wrong

"Hi Gustavo,

Thank you for the class today. Can I further clarify Q4 you briefly discussed in class? I initially directly used $\lim(-\infty, x) \to \Omega$ in the steps, but you pointed out it was wrong. I'm actually still a bit confused about why it is wrong. I referred to **Durrett** (who directly used $\lim(-\infty, x) \to \Omega$) and **Billingsley** (who states "clearly"). So, unfortunately, they both would get the proof wrong. Would it be possible for you to give me some hints on which theorems would be useful in the proof? Thank you so much!

[screenshot of Durrett's book, theorem 1.1]

Best,

xx"

(I kept the bold face from the e-mail)

Dear xx,

I wouldn't dare to say that Durrett would get it wrong if he was responding to the problem set! But definitely, if he just copied and pasted from his own book, he would be discounted.

Here's the reason: earlier in the book, he states the continuity from above and below of probability measures in terms of collections of sets *indexed by natural numbers*. In order to do that, he defines what it means to say

$$A_i \uparrow A$$

for that class of collections.

Later, in the context of proving the limits of CDFs, he applies a similar statement to the collection $\{(-\infty, x]\}_{x \in \mathbf{R}}$. The problem is that it's not indexed by natural numbers. So if he wanted full credit, he'd need to clarify he meant by

$$(-\infty, x] \uparrow \mathbf{R}$$

and why continuity of $\mathbf{P}$ – defined only for limits of "increasing" sets indexed by natural numbers – also applies for these limits.

The hint is the same one I gave in the recitation. Use the fact (no need to prove it) that

$$\lim_{x \to \infty} F(x) = 1$$

if, and only if, for every *increasing* diverging sequence $x_n \uparrow \infty$,

$$F(x_n) \to 1$$

and try to apply countable additivity.

Sincerely,

Gustavo