Recitation 10

Gustavo Pereira, Felipe Netto

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1 The Diamond-Dybvig model

[intro about bank runs, liquidity mismatch, non-performing loans or other shocks to assets causing people to freak out vs apparently irrational "purely demand driven bank runs"]

The structure of the Diamond-Dybvig economy seen in class is the following. Technology is very simple: whatever amount a is saved on period 0 turns to $R_1 a$ goods on period 1; similarly, an amount a saved on period 1 turns to $R_2 a$ units of the good in period 2. The production possibilities are thus described by the triples

$$(-a, \lambda R_1 a, (1-\lambda) R_1 R_2 a)$$

where $\lambda \in [0, 1]$ denotes the fraction of resources made available for consumption – thus not saved – in period 1.[1]

Now let's turn to the consumer side. A total of N agents live for three periods, $t \in \{0, 1, 2\}$. The agents' types are *patient* and *impatient*, and they are labeled by the superscripts $h \in \{1, 2\}$, where 1 means impatient and 2 means patient.[2] The fraction of agents of each type is denoted by α^h.

There is a single consumption good in this economy, which can only be consumed in periods 1 and 2. Each agent starts their life with y units of the consumption good. Type h has preferences for consumption over time described by the following utility function:[3]

[1]Note that one could equivalently describe the production set by

$$(-a, R_1 a - b, R_2 b)$$

where $b \leq R_1 a$ is the total amount of resources saved in period 1.

[2]Mnemonically you can think of the agent's label as the period when they like to consume the most; those who prefer to postpone consumption to period 2 (patient) are labeled as 2, and similarly for the impatient ones, who like to consume on period 1.

[3]The fact that there is a function $g(\cdot)$ is related to risk aversion from the period 0 perspective; hopefully it will become clear once the timing and informational structure of the model are discussed.

$$u^h(c_1, c_2) = g(c_1 + \delta^h c_2) \tag{1}$$

where we add the assumption that $0 < \delta^1 < \frac{1}{R_2} < \delta^2$. This assumption justifies the labels given to the agents, for the following reason. If agents had access to the technology at time $t = 1$, consumption plans for type $h = 2$ would necessarily involve choosing the corner $c_1^2 = 0$, and for type $h = 1$ the corner $c_2^1 = 0$.

[draw indifference curves to explain the paragraph above]

We make however another crucial assumption about the structure of this economy: **in period $t = 0$, the agents do not know their own types.** Their type is only revealed *to them* during period $t = 1$, so that no one else but each agent knows their type. Everyone does, however, know the true proportion of types in the economy.

1.1 Autarkic allocation

The meaning of "autarky" here is that there is no transfer between agents at any point; in other words, neither do agents contract with each other, nor is there an entity in charge of transferring resources between them. Given the preferences specified in 1, this means that impatient agents consume $R_1 y$ in period $t = 1$ and patient ones consume $R_2 R_1 y$ in period $t = 2$.

How well-off are agents in this allocation? In period 1, the impatient agents consume $R_1 y$. Hence their autarky utility – denoted $\bar{u}^1$ – is

$$\bar{u}^1 = g(R_1 y) \tag{2}$$

In period 2, the patient agents consume $R_2 R_1 y$, hence

$$\bar{u}^2 = g(\delta^2 R_2 R_1 y)$$

Before agents know their types, they assign a probability α^1 of being impatient and α^2 of being patient. Thus, their *ex-ante*[4] *utility* is:

$$E[\bar{u}^h] = \alpha^1 \bar{u}^1 + \alpha^2 \bar{u}^2 = \alpha^1 g(R_1 y) + \alpha^2 g(\delta^2 R_2 R_1 y)$$

(We will proceed under the assumption that expected utility is the correct way of measuring welfare ex-ante.)

[4]"Ex-ante" in this context means *before they know their types.*

1.2 Can people become better off relative to autarky?

If the world started in period $t = 1$, autarky would be Pareto efficient.[5] However, because the world starts in $t = 0$, people would like to insure against uncertainty about their own types. This insurance motive is governed by the function $g(\cdot)$, and of course depends on how likely it is to be of each type. Let's elaborate a bit on this.

In order to improve people's welfare, we need to understand what are the available allocations in this economy. Pooling all resources at time 0, we can get Ny units of the good to be invested. Thus, the available amount for consumption at time 1 will be $\lambda R_1 Ny$, and at time 2 it will be $(1-\lambda)R_2R_1Ny$.

$$\left\{ (c_1^1, c_1^2, c_2^1, c_2^2) : \begin{array}{c} (\alpha^1 N)c_1^1 + (\alpha^2 N)c_1^2 = \lambda R_1 Ny \\ (\alpha^1 N)c_2^1 + (\alpha^2 N)c_2^2 = (1-\lambda)R_2R_1Ny \end{array} \right\}$$

Given agents' preferences, we know that any (ex-ante) Pareto efficient allocation will feature $c_2^1 = c_1^2 = 0$. So may restrict our search for an improvement relative to autarky to

$$\begin{aligned} \alpha^1 c_1^1 &= \lambda R_1 y \\ \alpha^2 c_2^2 &= (1-\lambda)R_2R_1y \end{aligned}$$

which, solving for λ, is equivalent to

$$\alpha^2 c_2^2 = R_2 \left[R_1 y - \alpha^1 c_1^1 \right] \tag{3}$$

(Exercise: show that autarky is feasible)

(Optional from this point on; otherwise move to next section)

Hence the question becomes: are there feasible allocations – that is, allocations that satisfy 1.2 – that give the consumer a higher ex-ante utility than

$$\alpha^1 g(R_1 y) + \alpha^2 g(\delta^2 R_2 R_1 y)?$$

We can proceed by a variational argument. Infinitesimal changes in c_2^2 and c_1^1 will be feasible, given 1.2 if

$$\alpha^2 dc_2^2 = -\alpha^1 R_2 dc_1^1$$

The variation of ex-ante utility given dc_1^1 and dc_2^2 is thus given by

$$dU = \alpha^1 g'(R_1 y)dc_1^1 + \alpha^2 \delta^2 g'(\delta^2 R_2 R_1 y)dc_2 = \left[\alpha^1 g'(R_1 y) - \delta^2 \alpha^2 R_2 g'(\delta^2 R_2 R_1 y) \right] dc_1^1 \tag{4}$$

[5] A direct proof of this is easy and a good exercise. The claim also follows from the first welfare theorem, since (for time 1 onward) autarky is an equilibrium if price of c_2 is given by $1/R_2$.

If we assume that g, R_2, y and δ^2 are such that

$$\frac{g'(R_1 y)}{g'(\delta^2 R_1 R_2 y)} > \delta^2 R^2$$

then we get

$$\begin{aligned} dU &= \left[\alpha^1 g'(R_1 y) - \delta^2 \alpha^2 R_2 g'(\delta^2 R_2 R_1 y)\right] dc_1^1 \\ &> [g'(R_1 y) - \delta^2 R_2 g'(\delta^2 R_2 R_1 y)] dc_1^1 > 0 \end{aligned}$$

if $dc_1^1 > 0$. What the above derivation shows is that, if inequality 4 holds, autarky isn't optimal; in particular, in order to get closer to the optimal solution, one needs to *increase* c_1^1 (and thus *decrease* c_2^2).

1.3 Optimal allocation

From the discussion above, finding the first best allocation amounts to solving the following problem

$$\begin{aligned} \max_{c_1^1, c_2^2} \quad & \alpha^1 g(c_1^1) + \alpha^2 g(\delta^2 c_2^2) \\ \text{s.t.} \quad & \alpha^2 c_2^2 = R_2 \left[R_1 y - \alpha^1 c_1^1\right] \end{aligned}$$

which can be solved by substituting the constraint in the objective and taking the first order condition. The optimal allocation is thus the feasible allocation that satisfies

$$\frac{g'(c_1^1)}{\delta^2 g'(\delta^2 c_2^2)} = \delta^2 R_2$$

We denote this optimal allocation by $(\tilde{c}_1, \tilde{c}_2)$.

1.4 Implementation

If agents types were observable, and if they could write binding contracts at period 0, they would choose the optimal allocation found above. In period 1, patient agents would then be required to withdraw some of their $R_1 y$ units of consumption to transfer to impatient agents, thus fulfilling the contracts defined the period before.

These extra goods showing up in period 1 (relative to the Ny before autarky) are interpreted as *liquidity*. The autarky allocation is relatively illiquid. There aren't enough goods around to insure agents against the unlucky outcome of being impatient.

If types aren't observable, liquidity provision becomes a problem. In that scenario, it's unlikely that agents will write contracts in time zero to begin with. How, then, can we implement optimal allocations?

That is when we turn to banks. We'll study the case when a bank is able to collect the good at time 0 as *deposits*. Then those deposits are used in production, generating assets for the bank on periods 1 and 2. The bank's balance sheet here will comprise:

- on the asset side, the output of production using this economy's technology
- on the liabilities side, claims of depositors to consumption goods;

Diamond and Dybvig (1983) describe two arrangements for the bank, among others:

- Demand deposit contracts
- Suspension of convertibility

The demand deposits work as follows. Every agent has in period 1 a claim to $\tilde{c}_1$ units of consumption. Agents are put in a line randomly, where, once their serviced, they can either claim their $\tilde{c}_1$ units of consumption or wait until period 2. The bank services withdrawals until it (potentially) runs out of assets. The remaining assets of the bank in period 2 are then distributed equally among those who didn't withdraw in period 1. One feature of this type of contract is that, if the bank offers the *optimal* period 1 amount found above, then one Nash equilibrium is that patient agents wait until period 2.

However, in addition to this "optimal equilibrium", there is another important equilibrium where patient agents choose to withdraw in period 1 instead of postponing consumption. This is interpreted as a **bank run**. The idea is that if patient agents believe that other patient agents will withdraw $\tilde{c}_1$ at time 1, they will (correctly) believe the bank won't have enough assets to provide them with optimal consumption in period 2. The bank can avoid the possibility of running out of assets by reducing the claims offered in period 1. The highest claim to period 1 consumption that avoids bank runs is exactly $\bar{c}_1 = R_1 y$: the autarky value! (See the paper for a proof of this.) Therefore, by avoiding bank runs in this arrangement, banks provide no liquidity.

The *suspension of convertibility* arrangement is one way of avoiding bank runs. Instead of agents to deposit until it runs out of assets, the bank allows them to withdraw only up until total withdrawals equal $\alpha_1 N \tilde{c}_1$. At that point, the bank suspends conversion of deposits into goods. Because now there is no reason to believe the bank will have insufficient assets next period, patient agents optimally choose to wait instead of withdrawing at period 1. Thus, in this setting, suspension of convertibility is incentive compatible.