Correctness

Éloi Perdereau

4 mai 2014

We define the *bounding box* $BB(t)$ of the robots as the smallest enclosing rectangle (oriented with the grid's axes) which contains all robots at step t.

Proposition 0.1. *When following the algorithm described above, the bounding box of the robots is monotonically non-inflating, i.e.,* $BB(t+1) \subseteq BB(t)$ *for all* t.

1 A single robot on the topmost row

We note $r(t)$ the single robot in the topmost row of the bounding box at step t. If there are more than one robot, $r(t)$ is not defined.

Proposition 1.1. *If* $r(t)$ *exists and is on* $(0, i)$*, then there was a robot on* $(0, i-1)$*,* $(0, i)$ *or* $(0, i+1)$ *at step* $t-1$.

This shows that studying the neighborhood of $r(t)$ for new robots on the topmost row is enough for the whole row.

Lemma 1.2. *If* $r(t)$ *exists and there are at least three robots in the space, then after a constant number of steps, either* $BB(t+c) \subset BB(t)$ *(the topmost row moves down) or it becomes an end case*

Démonstration. We define the graph $G_{single}(V_{single}, E_{single})$ as follows :

- V_{single} : neighborhood cases of $r(t)$
- $(u, v) \in E_{single}$ if u can lead to v at $t+1$ for any robot on row i, and $t+1$ is not an end case.

The graph have been generated and is shown on figure 1. We notice multiple cycles, i.e. row 0 might never move down. However, edges are only representative for one step. In other words, there are paths that $r(t)$ cannot go through. We will study 4 of them and it will be enough to prove that $BB(t+c) \subset BB(t)$.

Note A to G the 7 nodes of G_{single} from top to bottom and left to right.
Left moves are when $r(t) > r(t+1)$, mid moves and right moves are defined analogously.
The middle robot in the start node is at $(0,0)$ at t.

- path($D \to A \to B$) : $D \to A$ and $A \to B$ are respectively a left and a right move.
  a. In order to have $(0,-1)$ filled at $t+1$, $(0,1)$ must be on case 1.2.6 ($-90°$) at t. So $(1,-1)$, $(1,0)$ and $(1,1)$ are empty at t.
  b. Giving a., in order to have $(1,-1)$ empty at $t+1$, $(1,-1)$ must be on case 1.3.5 at t. So at $t+1$, $(2,-1)$ will be filled.
  c1. In order to have $(0,0)$ filled at $t+2$, $(0,1)$ must be on case 1.2.5 or 1.3.6 (both $+180°$). Which is impossible because of b.
  c2. Giving b., there is no way $(0,0)$ can be filled at $t+2$.

  Therefore if $r(t)$ pass through $D \to A$, $r(t+1)$ cannot pass through $A \to B$. This invalidates all cycles involving both D and A.

□

2 More than one robot on the topmost row

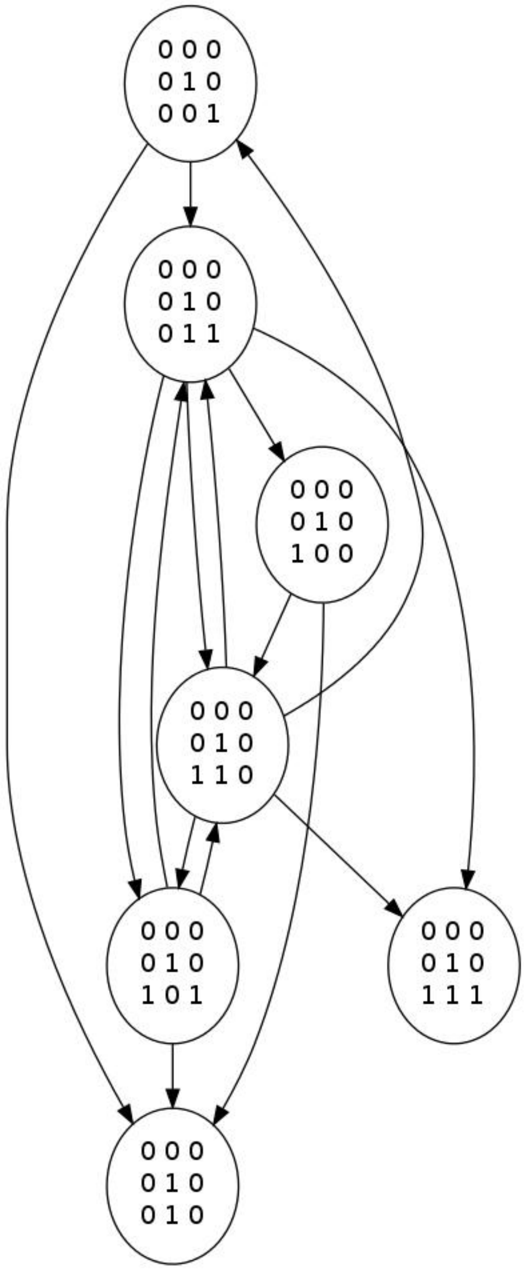

FIGURE 1 – Single robot

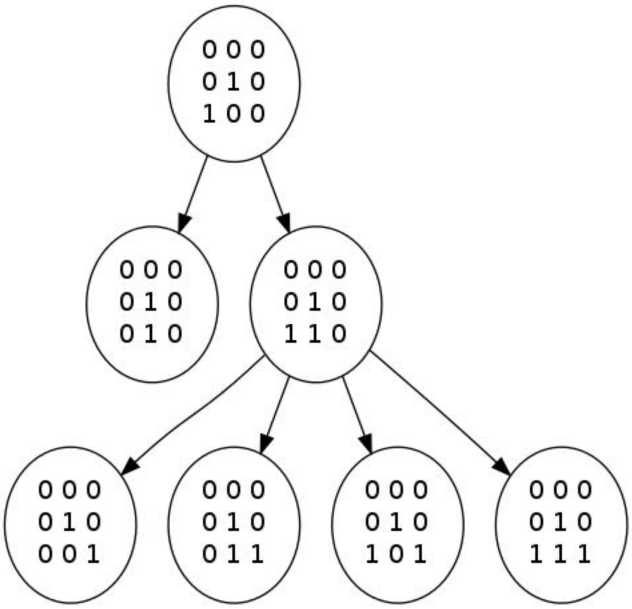

FIGURE 2 – Single robot : left moves

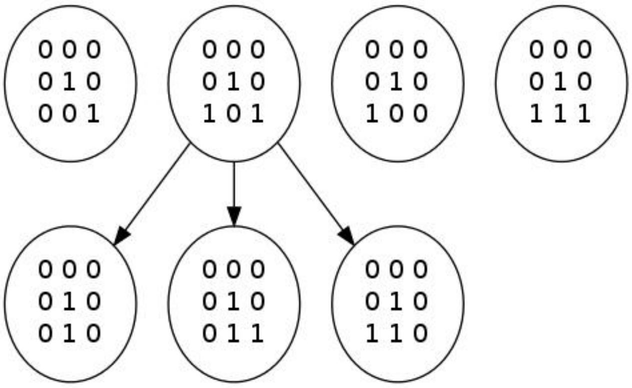

FIGURE 3 – Single robot : mid moves

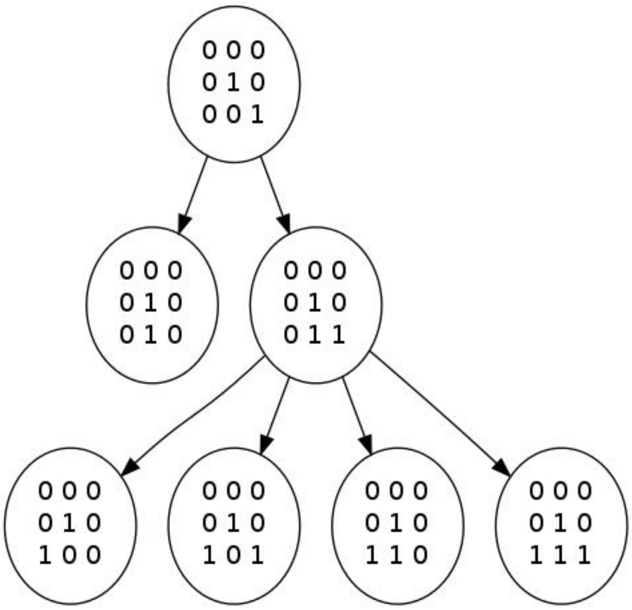

FIGURE 4 – Single robot : right moves

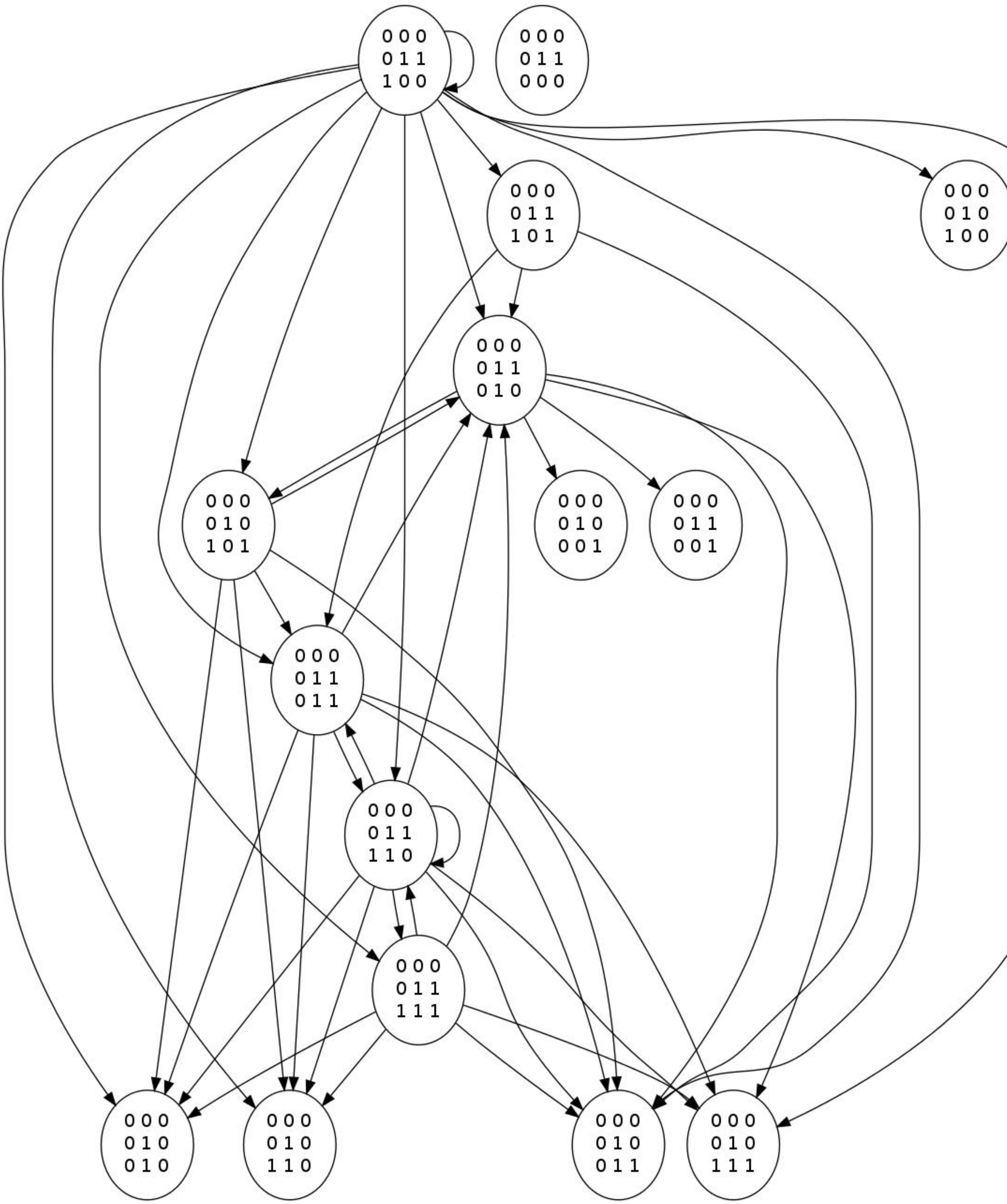

FIGURE 5 – Mid moves

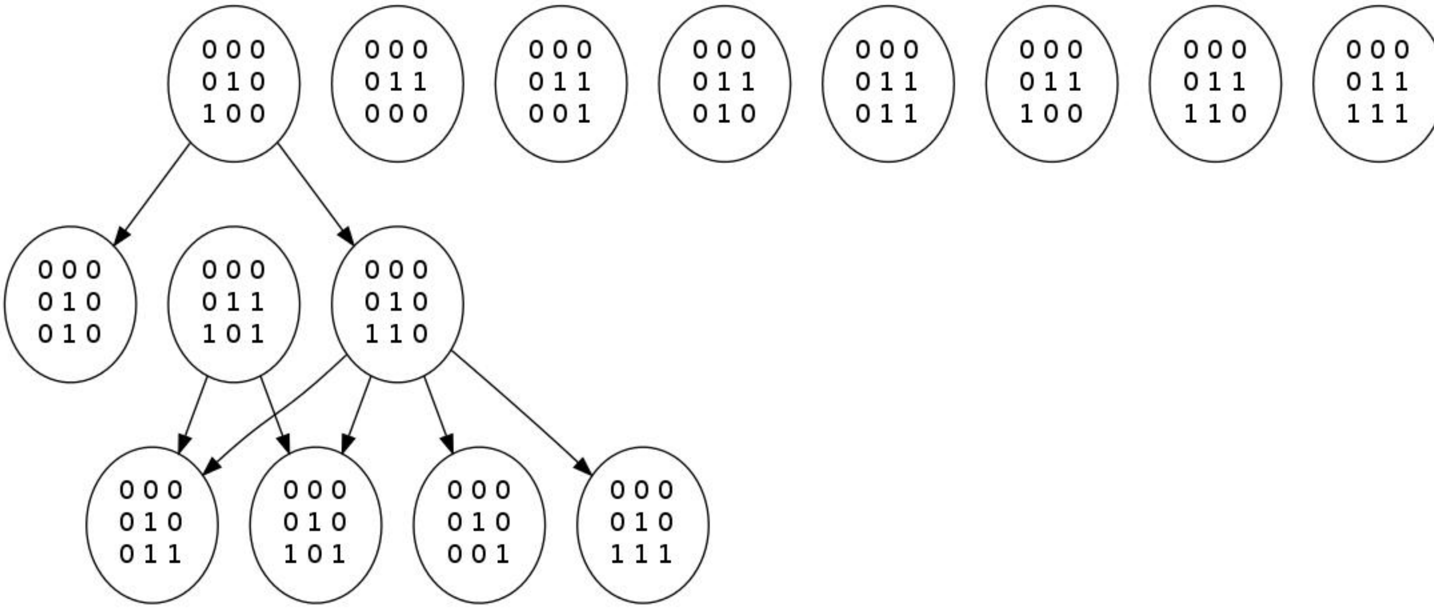

FIGURE 6 – Left moves