Correctness

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We define the bounding box BB(t) of the robots as the smallest enclosing rectangle (oriented with the grid's axes) which contains all robots at step t.

Proposition 0.1. When following the algorithm described above, the bounding box of the robots is monotonically non-inflating, i.e., $BB(t+1) \subseteq BB(t)$ for all t.

1 A single robot on the topmost row

We note r(t) the single robot in the topmost row of the bounding box at step t. If there are more than one robot, r(t) is not defined.

Proposition 1.1. If r(t) exists and is on (0,i), then there was a robot on (0,i-1), (0,i) or (0,i+1) at step t-1.

This shows that studying the neighborhood of r(t) for new robots on the topmost row is enough for the whole row.

Lemma 1.2. If r(t) exists and there are at least three robots in the space, then after a constant number of steps, either $BB(t+c) \subset BB(t)$ (the topmost row moves down) or it becomes an end case

Démonstration. We define the graph $G_{single}(V_{single}, E_{single})$ as follows:

- V_{single} : neighborhood cases of r(t)
- $-(u,v) \in E_{single}$ if u can lead to v at t+1 for any robot on row i, and t+1 is not an end case.

The graph have been generated and is shown on figure 1. We notice multiple cycles, i.e. row 0 might never move down. However, edges are only representative for one step. In other words, there are paths that r(t) cannot go through. We will study 4 of them and it will be enough to prove that $BB(t+c) \subset BB(t)$.

Note A to G the 7 nodes of G_{single} from top to bottom and left to right.

Left moves are when r(t) > r(t+1), mid moves and right moves are defined analogously.

The middle robot in the start node is at (0,0) at t.

- path $(D \to A \to B): D \to A$ and $A \to B$ are respectively a left and a right move.
 - a. In order to have (0, -1) filled at t + 1, (0, 1) must be on case 1.2.6 (-90°) at t. So (1, -1), (1, 0) and (1, 1) are empty at t.
 - b. Giving a., in order to have (1, -1) empty at t + 1, (1, -1) must be on case 1.3.5 at t. So at t + 1, (2, -1) will be filled.
 - c1. In order to have (0,0) filled at t+2, (0,1) must be on case 1.2.5 or 1.3.6 (both $+180^{\circ}$). Which is impossible because of b.
 - c2. Giving b., there is no way (0,0) can be filled at t+2.

Therefore if r(t) pass through $D \to A$, r(t+1) cannot pass through $A \to B$. This invalidates all cycles involving both D and A.

2 More than one robot on the topmost row

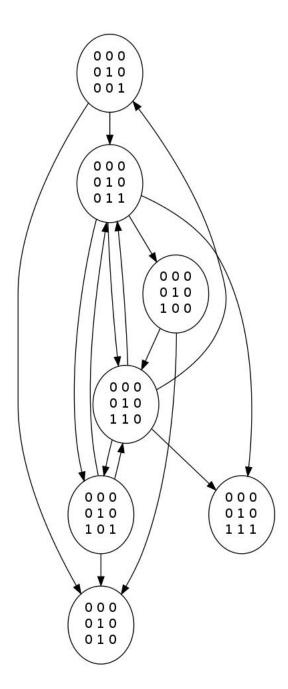
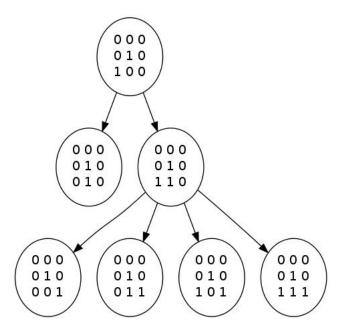
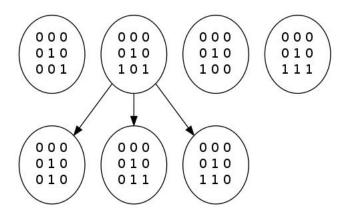


Figure 1 – Single robot



 $Figure\ 2-Single\ robot: left\ moves$



 $Figure \ 3-Single \ robot: mid \ moves$

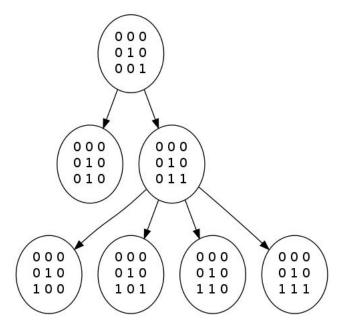


Figure 4 – Single robot : right moves

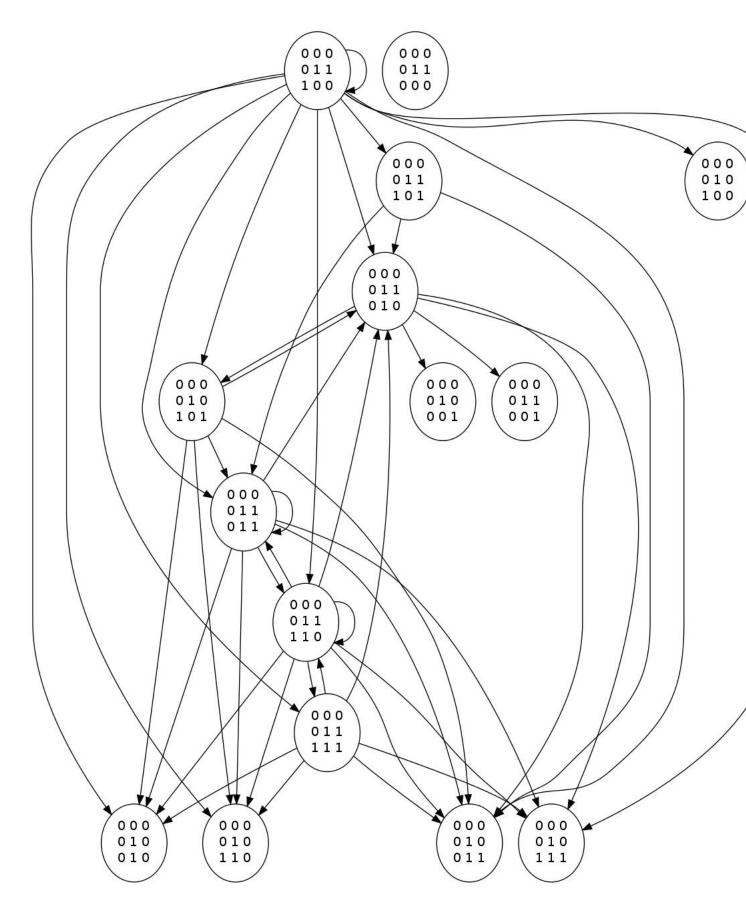


Figure 5 – Mid moves

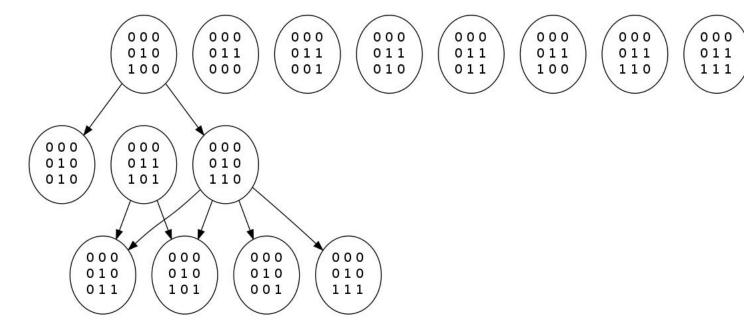


Figure 6 – Left moves