Correctness

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We define the *bounding box* $BB(t)$ of the robots as the smallest enclosing rectangle (oriented with the grid's axes) which contains all robots at step t.

Proposition 0.1. *When following the algorithm described above, the bounding box of the robots is monotonically non-inflating, i.e.,* $BB(t+1) \subseteq BB(t)$ *for all* t.

1 A single robot on the topmost row

We note $r(t)$ the single robot in the topmost row of the bounding box at step t. If there are more than one robot, $r(t)$ is not defined.

Proposition 1.1. *If* $r(t)$ *exists and is on* $(0, i)$*, then there is a robot on* $(0, i-1)$*,* $(0, i)$ *or* $(0, i+1)$ *at step* $t-1$.

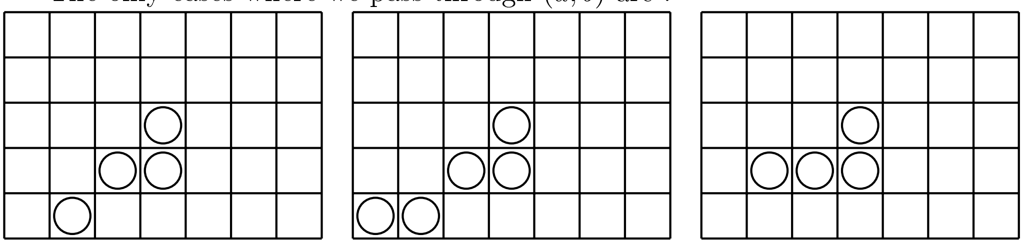
Lemma 1.2. *If* $r(t)$ *exists and there are at least three robots in the space, then after a constant number of steps, either* $BB(t+c) \subset BB(t)$ *(the topmost row moves down) or it becomes an end case*

Démonstration. The graph of figure 1 is defined as follows :

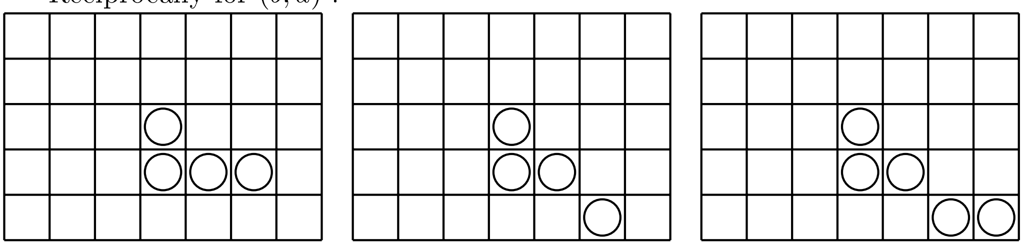
– Nodes : Possible cases for a single robot (i, j) on the topmost row.
– Edge (u, v) if u can lead to v at $t+1$ for any robot on row i.

We can see that there is a cycle from case 1.2.1 (call it a) to 1.2.3 (call it b). So theoretically the robot on the topmost row could cycle indefinitely. We show that this is not true :

The only cases where we pass through (a, b) are :



Reciprocally for (b, a) :



It is obvious that whenever the robot pass through one of the edges of the cycle at t, it can't pass through the other one at $t+1$.

□

2 More than one robot on the topmost row

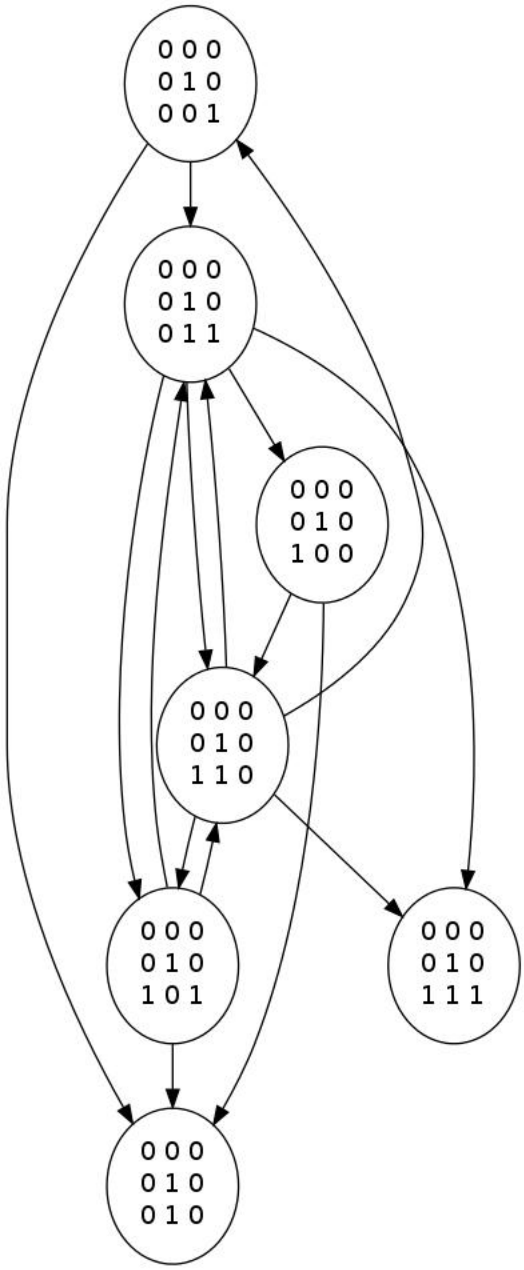


Figure 1 – Single robot

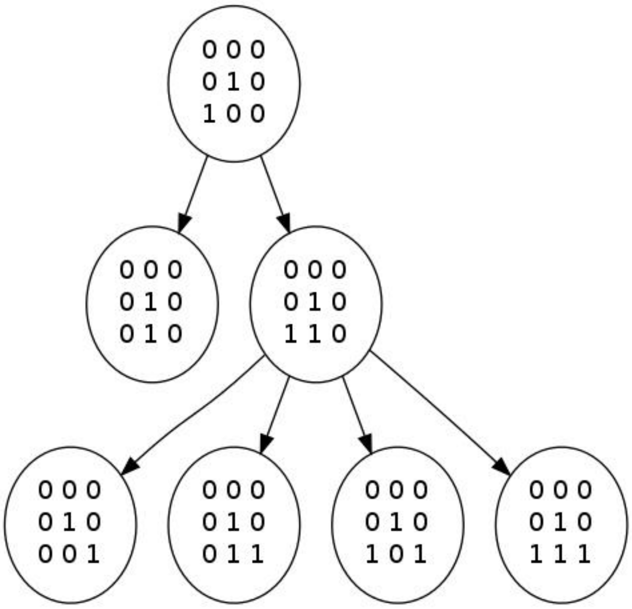


FIGURE 2 – Single robot go left

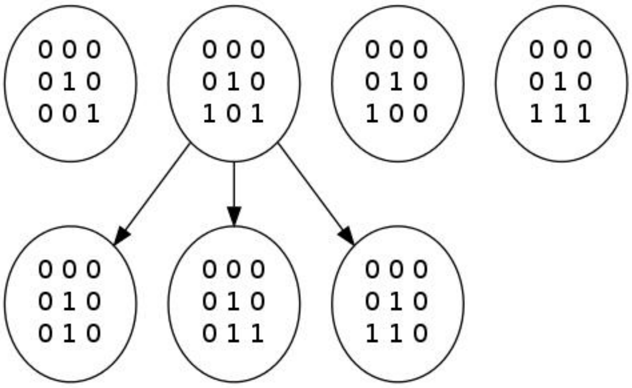


FIGURE 3 – Single robot persist

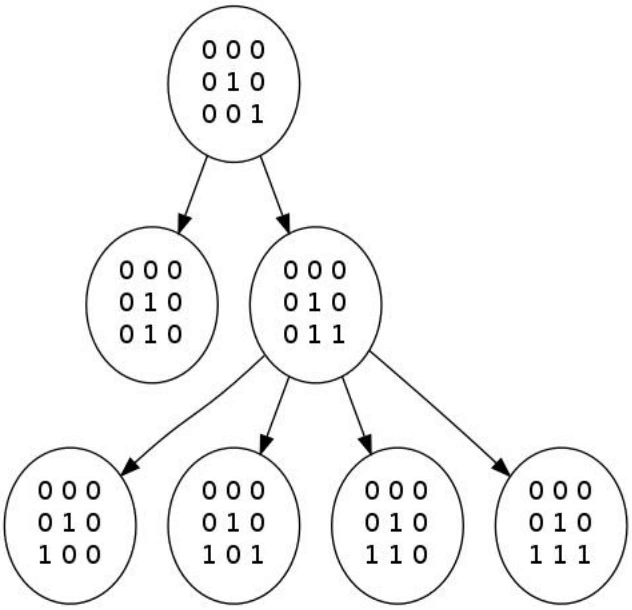


FIGURE 4 – Single robot go right

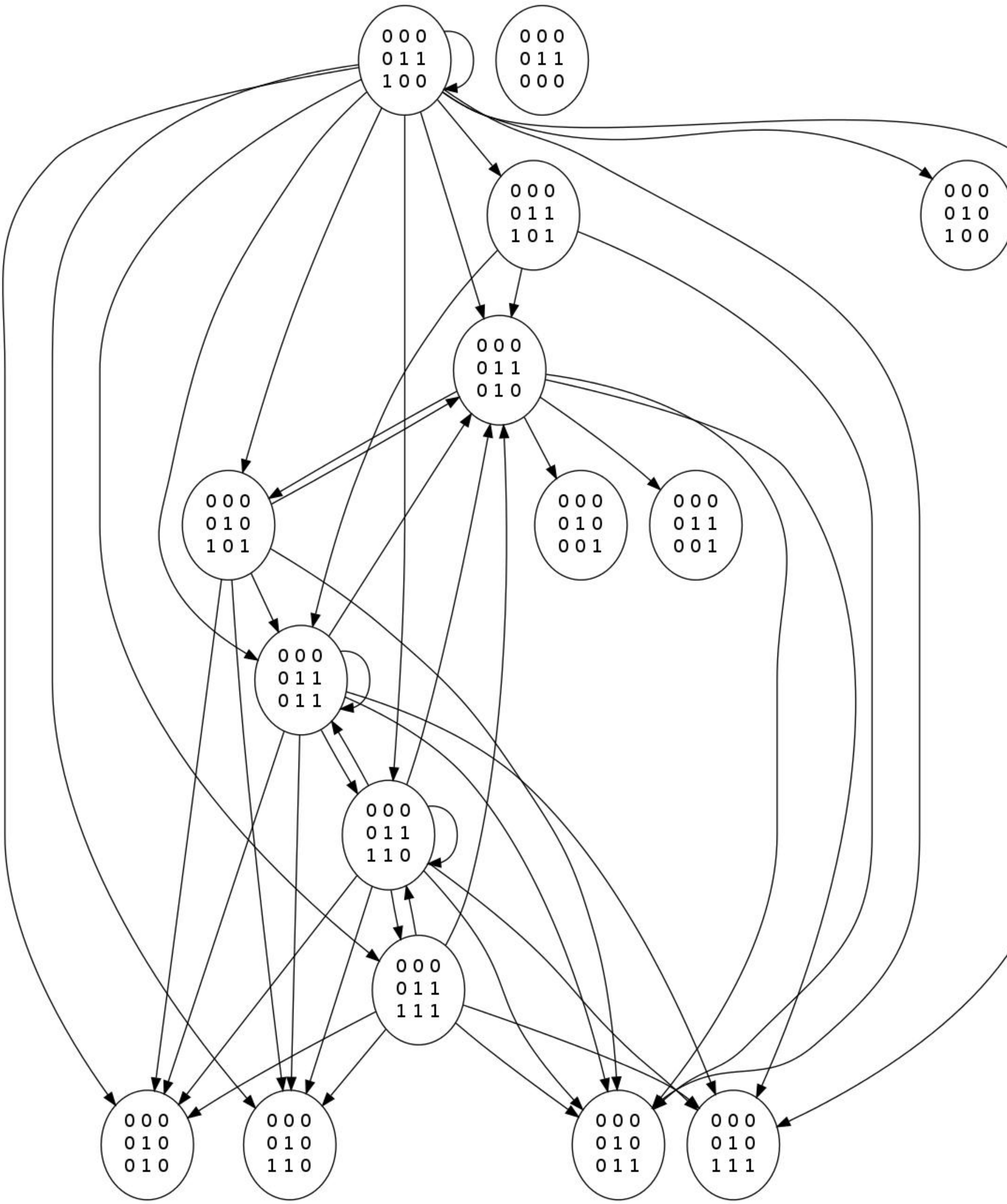


FIGURE 5 – Persistent cases

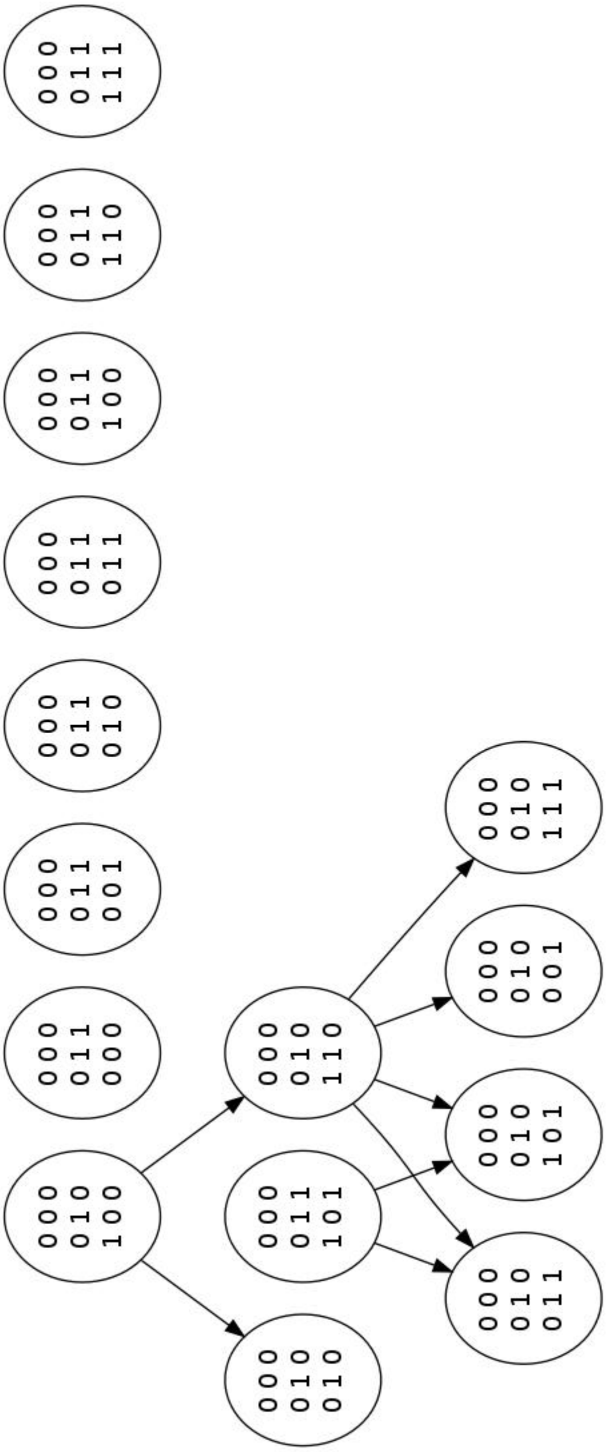


FIGURE 6 – "Go-left" cases