

Correctness

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We define the *bounding box* $BB(t)$ of the robots as the smallest enclosing rectangle (oriented with the grid's axes) which contains all robots at step t .

Proposition 0.1. *When following the algorithm described above, the bounding box of the robots is monotonically non-inflating, i.e., $BB(t+1) \subseteq BB(t)$ for all t .*

1 A single robot on the topmost row

We note $r(k)$ the single robot in the topmost row of the bounding box at step t . If there are more than one robot, r is not defined.

Proposition 1.1. *If $r(k)$ exists and is at column i , then $r(k-1)$ exist and is at column $i-1$, i or $i+1$.*

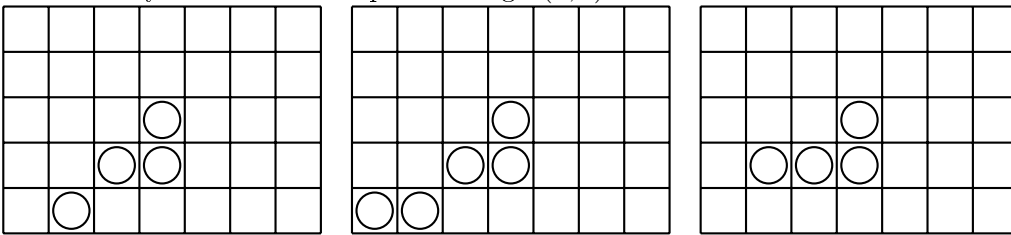
Lemma 1.2. *If $r(k)$ exists and there are at least three robots in the space (it is not an end case), then for a constant c , either $r(k+c)$ doesn't exist or there are only two cells left filled with robots in the space.*

Démonstration. The graph of figure 1 is defined as follows :

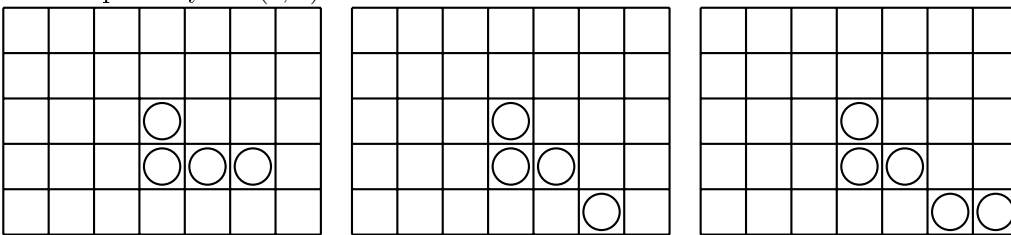
- Nodes : Possible cases for a single robot (i, j) on the topmost row.
- Edge (u, v) if u can lead to v at $t+1$ for any robot on row i .

We can see that there is a cycle from case 1.2.1 (call it a) to 1.2.3 (call it b). So theoretically the robot on the topmost row could cycle indefinitely. We show that this is not true :

The only cases where we pass through (a, b) are :



Reciprocally for (b, a) :



It is obvious that whenever the robot pass through one of the edges of the cycle at t , it can't pass through the other one at $t+1$.

□

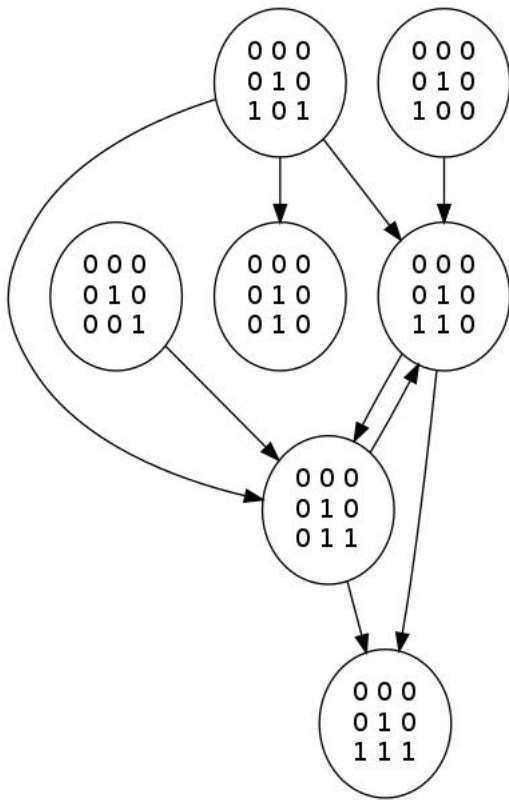


FIGURE 1 – Single robot

2 More than one robot on the topmost row

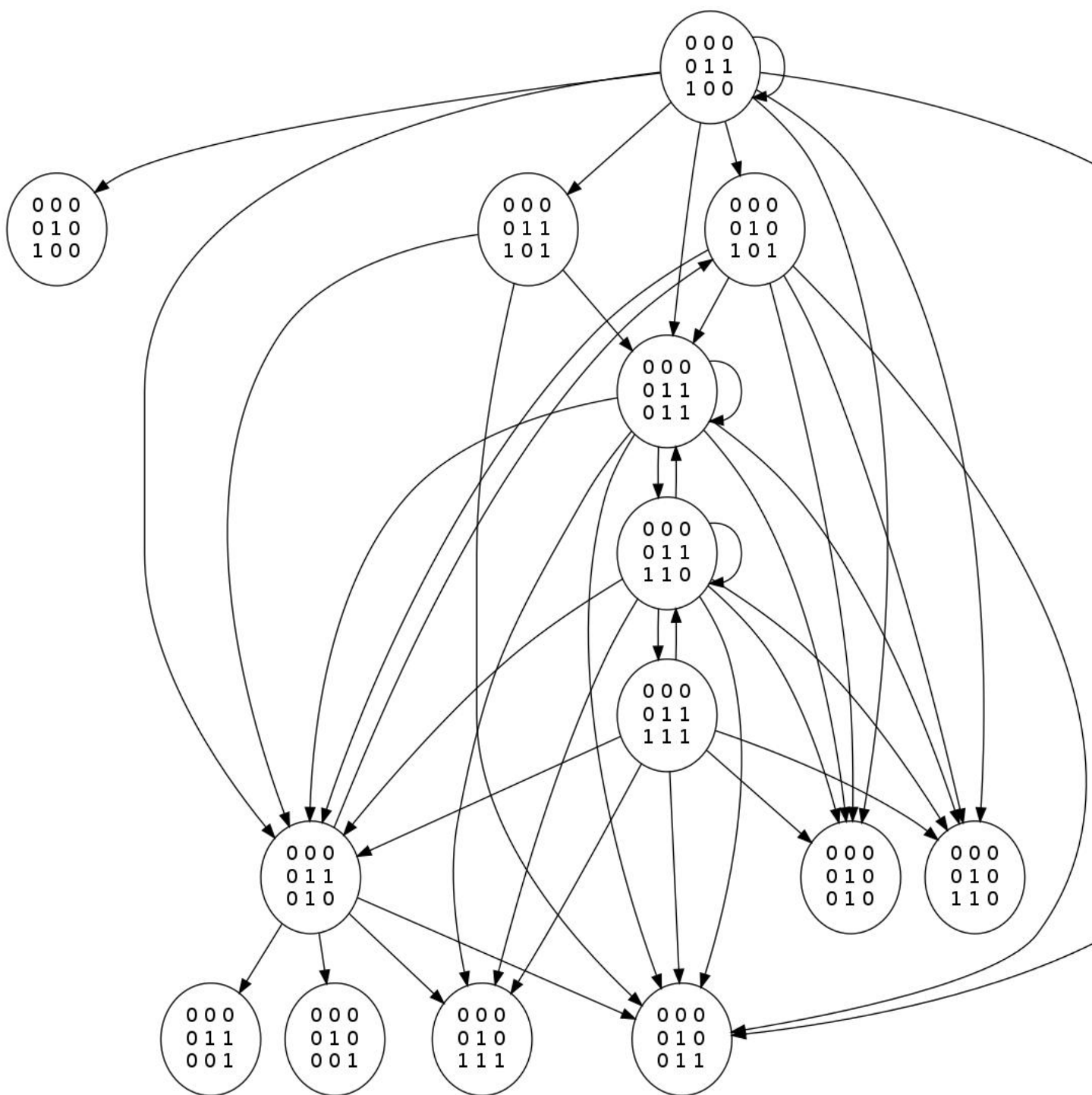


FIGURE 2 – Persistent cases

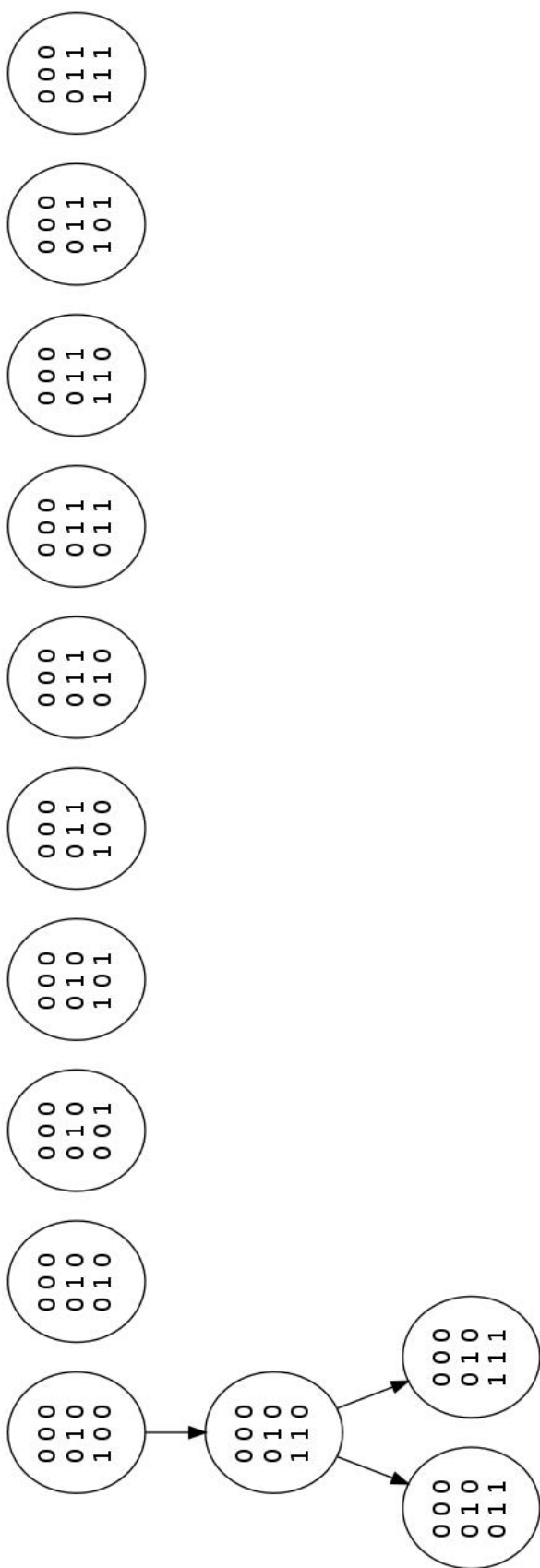


FIGURE 3 – "Go-left" cases