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**1.Determine the complete partial fractions decomposition of the function:**

$$\frac{2x+1}{(x-3)(x-1)^2(x^2+4)^2}$$

*the partial fraction decomposition will be*

$$\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{Cx+D}{(x^2+4)}+\frac{Ex+F}{(x^2+4)^2}+\frac{G}{(x-3)}$$

$$=\frac{(x-3)(x-1)^2(x^2+4)(Cx+D)+(x-3)(x-1)^2(Ex+F)+(x-3)(x-1)(x^2+4)^2A+(x-3)(x^2+4)^2B+(x-1)^2(x^2+4)^2G}{(x-3)(x-1)^2(x^2+4)^2}$$

**since denominators are equal so we equate the numerator;**

$$2x+1=(x-3)(x-1)^2(x^2+4)(Cx+D)+(x-3)(x-1)^2(Ex+F)+(x-3)(x-1)(x^2+4)^2A+(x-3)(x^2+4)^2B+(x-1)^2(x^2+4)^2G$$

$$\begin{aligned}2x+1={}&x^6A+x^6C+x^6G-4x^5A+x^5B-5x^5C+x^5D-2x^5G+11x^4A-3x^4B+11x^4B-11x^4C\\&-5x^4D+x^4E+9x^4G-32x^3A+8x^3B-23x^3C+11x^3D-5x^3E+x^3F-16x^3G\\&+40x^2A-24x^2B+28x^2C-23x^2D+7x^2E-5x^2F+24x^2G-64xA+16xB-12xC\\&+28xD-3xE+7xF-32xG+48A-48B-12D-3F+16G\end{aligned}$$

**coefficient to be equated**

$A+C+G=0$

$-4A+B-5C+D-2G=0$

$11A-3B+11C-5D+E+9G=0$

$-32A+8B-23C+11D-5E+F-16G=0$

$40A-24B+28C-23D+7E-5F+24G=0$

$-64A+16B-12C+28D-3E+7F-32G=2$

$48A-48B-12D-3F+16G=1$

$A=\frac{11}{500}$ , $B=\frac{-3}{50}$, $C=\frac{246}{21125}$, $D=\frac{1076}{21125}$, $E=\frac{31}{325}$, $F=\frac{41}{325}$, $G=\frac{7}{676}$

$$\frac{2x+1}{(x-3)(x-1)^2(x^2+4)^2}=\frac{\frac{-11}{500}}{(x-1)}+\frac{\frac{-3}{50}}{(x-1)^2}+\frac{\frac{246x}{21125}+\frac{1076}{21125}}{x^2+4}+\frac{\frac{31x}{325}+\frac{41}{325}}{(x^2+4)^2}+\frac{\frac{7}{676}}{(x-3)}$$

$$\textbf{answer}=\frac{\frac{-11}{500}}{(x-1)}+\frac{\frac{-3}{50}}{(x-1)^2}+\frac{\frac{246x}{21125}+\frac{1076}{21125}}{x^2+4}+\frac{\frac{31x}{325}+\frac{41}{325}}{(x^2+4)^2}+\frac{\frac{7}{676}}{(x-3)}$$

**2.Determine the arc length of the curve represented by the parametric curve $x(t) = t^3, y(t) = t^4 [0, 1]$**

the arc length for a parametric curve $\mathbf{l} = \int_a^b \sqrt{(x'(t)^2 + (y'(t)^2} \, dt$

der of$(\mathbf{t^3})' = 3t^2$

$(\mathbf{t^4})' = 4t^3$

$l = \int_0^1 \sqrt{(3t^2)^2 + (4t^3)^2 \, dt} = \int_0^1 t^2\sqrt{16t^2 + 9 \, dt}$

first calculate the corresponding indefinite integral

$$\int t^2\sqrt{16t^2 + 9 \, dt} = \frac{81\sinh(\operatorname{asinh}\left(\frac{4t}{3}\right))}{2048} - \frac{81\operatorname{asinh}\left(\frac{4t}{3}\right)}{512}$$

fundamental theorem $\int_a^b F(x)dx = f(b) - f(a)$

$$\frac{81\sinh(\operatorname{asinh}\left(\frac{4t}{3}\right))}{2048} - \frac{81\operatorname{asinh}\left(\frac{4t}{3}\right)}{512}\Big|_{t=1} = -\frac{81\operatorname{asinh}\left(\frac{4}{3}\right)}{512} + \frac{81\sinh\left(4\operatorname{asinh}\left(\frac{4}{3}\right)\right)}{2048}$$

$$\frac{81\sinh(\operatorname{asinh}\left(\frac{4t}{3}\right))}{2048} - \frac{81\operatorname{asinh}\left(\frac{4t}{3}\right)}{512}\Big|_{t=0} = 0$$

$$\int_0^1 \left(t^2\sqrt{16t^2 + 9}\right) dt = -\frac{81\operatorname{asinh}\left(\frac{4}{3}\right)}{512} + \frac{81\sinh\left(4\operatorname{asinh}\left(\frac{4}{3}\right)\right)}{2048}$$

$= \mathbf{1.427758 \text{ square units}}$

**3. Given that $\int_2^6 f(x) = -5, \int_2^6 g(x) = 7$**

**i. $\int_2^6 [g(x) - f(x)]dx$**

$$\int_2^6 [7 + 5]dx$$

$$\int_2^6 12x$$

$$12 \times 6 - 12 \times 2$$

$$= \mathbf{48 \text{ sq. units}}$$

**ii. $\int_2^6 [4f(x) - g(x)]dx$**

$$\int_2^6 [4 \times -5 + 7]dx$$

$$\int_2^6 [-20 + 7]dx$$

$$-13 \times 6 - -13 \times 2$$

$$= \mathbf{-52 \text{ sq. units}}$$

**4. Integrate $\int \mathbf{x^3 f'(x) dx}$**

$$\text{defferientiate } \mathbf{f'(x)} = 1$$

$$= \int x^3 dx$$

$$= \frac{x^4}{4} + c$$

$$= \mathbf{\frac{1}{4}x^4 + c}$$

**5. Find the $\int_1^0 \mathbf{5x^2 - 1\ dx}$ and $\int_0^1 \mathbf{5x^2 - 1dx}$**

$$\int_1^0 5x^2 - 1\ dx$$

$$\left(\frac{5x^3}{3} - x\right)|_1^0$$

$$0 - \left(\frac{5}{3} - 1\right)$$

$$= 0 - \frac{2}{3}$$

$$= -\frac{\mathbf{2}}{\mathbf{3}}\ \mathbf{sq.units}\ \text{i. e lies under the x} - \text{axis}$$

$$\int_0^1 5x^2 - 1\ dx$$

$$\left(\frac{5x^3}{3} - x\right)|_0^1$$

$$\left(\frac{5}{3} - 1\right) - 0$$

$$= \frac{2}{3} - 0$$

$$= \frac{\mathbf{2}}{\mathbf{3}}\ \mathbf{sq.units\ i.e}\ \text{lies above x} - \text{axis}$$