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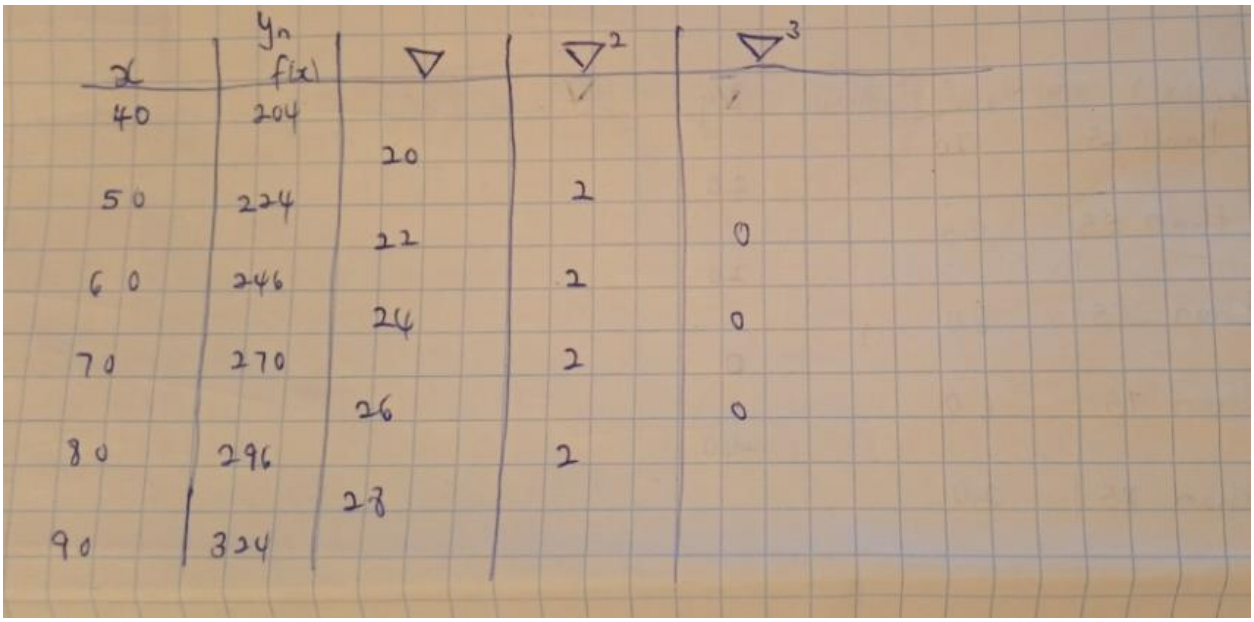
17G01ACS059

NUMERICAL METHODS ASSIGNMENT 5

1.

Backward difference table



x	y_n f(x)	∇	∇^2	∇^3
40	204			
		20		
50	224		2	
		22		0
60	246		2	
		24		0
70	270		2	
		26		0
80	296		2	
		28		
90	324			

$$f(x) = y_n + \frac{U\nabla y_n}{1!} + \frac{U(U+1)\nabla^2 y_n}{2!} + \frac{U(U+1)(U+2)\nabla^3 y_n}{3!}. + ... + \frac{U(U+1)(U+2)....(U+n-1)\nabla^n y_n}{n!}$$

$X_n = 90$, x=84, h = 10, $f(x)_n = y_n = 324$, $\nabla f(x)_n = \nabla y_n = 28$, $\nabla^2 f(x) = 2$

$$f(x) = f(x)_n + U\nabla f(x)_n + \frac{U(U+1)}{2}\nabla^2 f(x)_n$$

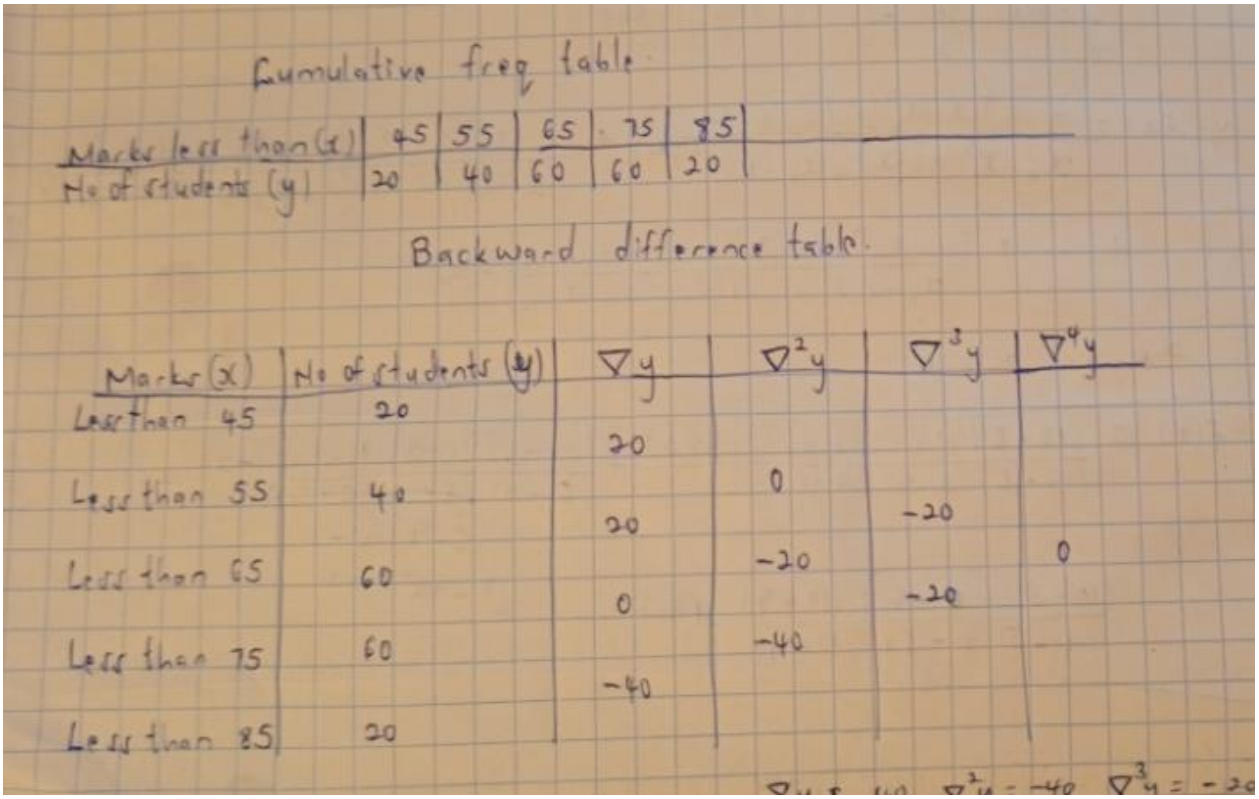
$$U = \frac{x - x_n}{h} = \frac{84-90}{10} = -0.6$$

$$f(84) = 324 + -0.6(28) + \frac{(-0.6)(-0.6+1)}{2}2$$

$$= 324 - 16.8 - 0.24$$

$$f(84) = 306.96$$

2.



Cumulative freq table

Marks less than (x)	45	55	65	75	85
No of students (y)	20	40	60	60	20

Backward difference table

Marks (x)	No of students (y)	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
Less than 45	20				
		20			
Less than 55	40		0		
		20		-20	
Less than 65	60		-20		0
		0		-20	
Less than 75	60		-40		
		-40			
Less than 85	20				

$$f(x) = y_n + \frac{U\nabla y_n}{1!} + \frac{U(U+1)\nabla^2 y_n}{2!} + \frac{U(U+1)(U+2)\nabla^3 y_n}{3!}. + ... + \frac{U(U+1)(U+2)....(U+n-1)\nabla^n y_n}{n!}$$

$X_n = 85$, $x=80$, $h = 10$, $y_n=20$, $\nabla y_n=-40$, $\nabla^2 y_n =-40$ $\nabla^3 y_{n=-20}$

$$U = \frac{x - x_n}{h} = \frac{80-85}{10} = -0.5$$

$$f(80) = 20 + -0.5(-40) + \frac{(-0.5)(-0.5+1)}{2} - 40 + \frac{(-0.5)(-0.5+1)(-0.5+2)}{6} - 20$$

$$= 20 + 20 + 5 + 1.25$$

$$= 46.25$$

Students who got between 75-80 were 46

3.

$$y = f(x|) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \times y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \times y_1 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \times y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \times y_3$$

X_0=5, x=10

$$y = f(10) = \frac{(10-6)(10-9)(10-11)}{(5-6)(5-9)(5-11)} \times 380 + \frac{(10-5)(10-9)(10-11)}{(6-5)(6-9)(6-11)} \times -2 +$$
$$\frac{(10-5)(10-6)(10-11)}{(9-5)(9-6)(9-11)} \times 196 + \frac{(10-5)(10-6)(10-9)}{(11-5)(11-6)(11-9)} \times 508$$

Y=f(10)=402.6667

4.

$X_0 = 0$, $x_1 = 1$, $x_2 = 3$, $x_3 = 4$
$y_0 = -20$, $y_1 = -12$, $y_2 = -20$, $y_3 = -24$

$$y = f(x|) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \times y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \times y_1 +$$
$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \times y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \times y_3$$

$$y = f(x|) = \frac{(x-1)(x-3)(x-4)}{(0-1)(0-3)(0-4)} \times -20 + \frac{(x-0)(x-3)(x-4)}{(1-0)(1-3)(1-4)} \times -12 +$$
$$\frac{(x-0)(x-1)(x-4)}{(3-0)(3-1)(3-4)} \times -20 + \frac{(x-0)(x-1)(x-3)}{(4-0)(4-1)(4-3)} \times -24$$

$= 3x^3 - 8x^2 + 15x - 20$

5.

$$x = \frac{(y-y_1)(y-y_2)}{(y_0-y_1)(y_0-y_2)} \times x_0 + \frac{(y-y_0)(y-y_2)}{(y_1-y_0)(y_1-y_2)} \times x_1 +$$
$$\frac{(y-y_0)(y-y_1)}{(y_2-y_0)(y_2-y_1)} \times x_1$$

$$x = \frac{(0.3-0.3332)(0.3-0.2897)}{(0.3683-0.3332)(0.3683-0.2897)} \times 0.4 + \frac{(0.3-0.3683)(0.3-0.2897)}{(0.3332-0.3683)(0.3332-0.2897)} \times 0.6 +$$
$$\frac{(0.3-0.3633)(0.3-0.3332)}{(0.2897-0.3683)(0.2897-0.3332)} \times 0.8$$

$x = 0.7186$

6.

$$y = \log_{10}(658) = \frac{(656-658)(656-659)(656-661)}{(654-658)(654-659)(654-661)} \times 2.8156 +$$
$$\frac{(656-654)(656-659)(656-661)}{(658-654)(658-659)(658-661)} \times 2.8182 + \frac{(656-654)(656-658)(656-661)}{(659-654)(659-658)(659-661)} \times 2.8189$$
$$+ \frac{(656-654)(656-658)(656-659)}{(661-654)(661-658)(661-659)} \times 2.8202$$

$$y = 2.8168$$