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**17G01ACS059**

## NUMERICAL METHODS THIRD ASSIGNMENT

1. **Prove that $e^x = \frac{e^2}{E} e^x \cdot \frac{Ee^x}{\Delta^2 e^x}$ the interval of differencing being h**

We know that $Ef(x) = f(x+h)$

Hence $Ee^x = e^{x+h}$

$$\Delta e^x = e^{x+h} - e^x = e^x(e^h - 1)$$
$$= \Delta^2 e^x = e^x \bullet (e^h - 1)^2$$

Hence

$$\frac{\Delta^2}{E} e^x = (\Delta^2 E^{-1}) e^x = \Delta^2 e^{x-h} = e^{-h}(\Delta^2 e^x) = e^{-h} e^x (e^h - 1)^2$$

The right hand-side

$$= e^{-h} e^x (e^h - 1) \bullet \frac{e^{x+h}}{e^x (e^h - 1)} = e^x$$

2.

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $y = f(x)$ | 0.975 | -0.6083 | -3.5250 | -5.5250 | -6.3583 | 4.2250 | 36.475 |

| $x$ | $y$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ | $\Delta^4 y$ | $\Delta^5 y$ |
|---|---|---|---|---|---|---|
| 1 | 0.975 | | | | | |
| | | -15833 | | | | |
| 2 | -0.6083 | | -1.3334 | | | |
| | | $-2.9167+E$ | | $2.2501+E$ | | |
| 3 | -3.5250 | | $0.9167+E$ | | $-2.0001-4E$ | |
| | | $-2+E$ | | $0.25-3E$ | | $12+10E$ |
| 4 | $-5.5250+E$ | | $1.1667-2E$ | | $9.9999+6E$ | |
| | | $-0.8333-E$ | | $10.2499+3E$ | | $-9.9997-10E$ |
| 5 | -6.3583 | | $11.4166+E$ | | $0.0002-4E$ | |
| | | 10.5833 | | $10.2501-E$ | | |
| 6 | 4.2250 | | 21.6667 | | | |
| | | 32.25 | | | | |
| 7 | 36.475 | | | | | |

$$12+10E = -9.9997-10E$$

$$12+9.9997 = -10E-10E$$

$$\frac{\cancel{-20}E}{\cancel{-20}} = \frac{21.9997}{-20}$$

$$E = -1.099985$$

$$y_4 = -5.5250 - 1.099985$$

$$y_4 = -6.624985$$

3.

i. 4 values given, let y be a polynomial of degree 3

$$\Delta^4 y_0 = 0$$
$$(E-1)^4 y_0 = 0$$
$$(E^4 - 4E^3 + 6E^2 - 4E + 1)y_0 = 0$$
$$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$$
$$97 - 4(y_3) + 6(17) - 4(4) + 1 = 0$$
$$97 - 4y_3 + 6(17) - 4(4) + 1 = 0$$
$$97 - 4y_3 + 102 - 16 + 1 = 0$$
$$\frac{184}{4} = \frac{4y_3}{4}$$
$$y_3 = 46$$

ii. 5 values given, let y be a polynomial of degree 4

$$\Delta^5 y_0 = 0$$
$$(E-1)^5 y_0 = 0$$
$$(E^5 - 4E^4 + 10E^3 - 10E^2 + 5E - 1)y_0 = 0$$
$$y_5 - 5y_4 + 10y_3 - 10y_2 + 5y_1 - y_0 = 0$$
$$491 - 5(189) + 10(y_3) - 10(11) + 5(3) - 1 = 0$$
$$491 - 945 + 10y_3 - 110 + 15 - 1 = 0$$
$$10y_3 - 550 = 0$$
$$\frac{10y_3}{10} = \frac{550}{10}$$
$$y_3 = 55$$

iii. 4 values given, assume it's a polynomial of degree 3

$$\Delta^4 y_0 = 0$$
$$(E-1)^4 y_0 = 0$$
$$(E^4 - 4E^3 + 6E^2 - 4E + 1)y_0 = 0$$
$$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$$
$$y_4 - 4y_1 = 45 \text{.........................................................} 1$$

$$\Delta^5 y_0 = 0$$
$$(E-1)^5 y_0 = 0$$
$$(E^5 - 4E^4 + 10E^3 - 10E^2 + 5E - 1)y_0 = 0$$
$$y_5 - 5y_4 + 10y_3 - 10y_2 + 5y_1 - y_0 = 0$$
$$116 - 5(y_4) + 10(28) - 10(11) + 5(y_1) - 1 = 0$$
$$116 - 5(y_4) + 10(28) - 10(11) + 5y_1 - 1 = 0$$
$$-5y_4 + 5y = -285 \text{....................................................} 2$$

$$(y_4 - 4y_1 = 45) \times 1$$
$$(-5y_4 + 5y_1 = -285) \times 1$$

$$-5y_4 + 20y_1 = -225$$
$$-$$
$$-5y_4 + 5y_1 = -285$$

$$y_1 = 4$$
$$y_4 = 61$$