Kinetic Physics Virtual Lab Solutions

Using one-dimensional motion equations

Questions

Exercise 1: Intro to Skee-Ball

1. Yes, the speed of the ball changed. If the ball was originally shot to the right, then it also changed direction. If it was shot to the left, then it did not change direction.

Exercise 2: Velocity

2. This exercise is for data gathering. Answers will vary.

   v_i : Possible answers range from –4.00 m/s to 4.00 m/s.

   x_1 : Possible answers range from –20 m to 20 m.

   t_1 : Possible answers range from 0 s to 5 s.

   x_2 : Possible answers range from –20 m to 20 m.

   t_2 : Possible answers range from 0 s to 5 s.

   v_{avg} : Answer should be the same as v_i.

3. The velocity stays the same.
4. Since the velocity does not change, you know that the acceleration equals zero and the average velocity is the same as the initial velocity. Use the average velocity equation.

   $$v_{avg} = \frac{\Delta x}{\Delta t}$$

   $$v_i = \frac{x}{t}$$

5. 1.50 m/s.

   Use the equation described in question 4.

   $$v_i = \frac{\Delta x}{\Delta t} = \frac{3.00 \text{ m}}{2.00 \text{ s}} = 1.50 \text{ m/s}$$

Exercise 3: Acceleration

6. The slope is the ball’s acceleration.
7. Yes, the ball is accelerating in the negative x direction.
8. A negative acceleration means that the velocity is always decreasing (that is, approaching negative infinity). This does not mean that speed is decreasing. For example, a ball traveling in the negative direction with an increasing speed has negative acceleration, but it is not slowing down. The speed of the ball when it passes the plunger on its return journey is the same as the speed of the ball when it first starts moving. The sign of the velocity at these two times is different, even though the magnitudes are the same.
9. The slope is the ball’s acceleration.

Notes on the simulations

Exercise 1 Simulation

Objective: This exercise is solely for the exploration of linear motion.

Exercise 2A Simulation

Objective: Gather data to determine the ball's average velocity and whether the velocity is changing.

Conclusion: The velocity does not change.

Exercise 2B Simulation

Objective: Use the result of Exercise 2A to sink the ball in the 50-point slot.

Answer: $v_i = 1.50$ m/s

Exercise 3 Simulation

Objective: Gather data to determine the acceleration of the ball on the ramp.

Answer: $a = -1.80$ m/s^2

Firing cannons: Calculating projectile motion

Questions

Exercise 1: Tower height

1. Drop the cannonball off the tower, giving it no initial vertical velocity. The stopwatch will tell you the time it takes for the cannonball to reach the ground. Use the equation $\Delta y = v_i t + \frac{1}{2}at^2$. The acceleration is equal to –9.80 m/s^2 and the initial velocity is zero. The only unknown will be the displacement. The displacement will be negative, and its magnitude is the height of the tower.
2. 3.19 s
3. 49.9 m

   Use a linear motion equation. The magnitude of the displacement is the height of the tower.

   $$\Delta y = v_i t + \frac{1}{2}at^2 = (0 \text{ m/s})(3.19 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(3.19 \text{ s})^2 = -49.9 \text{ m}$$

Exercise 2: Firing a cannon horizontally

4. No, the horizontal firing velocity does not affect how long it takes the cannonball to hit the ground. The horizontal firing velocity only affects how far in the x direction the cannonball travels, for shots taken from a fixed height.
5. No, the horizontal velocity does not affect the vertical velocity or the vertical acceleration of the cannonball. The component of velocity in one direction has no effect on the motion in a perpendicular direction.

Exercise 3: Blast away!

Exercise 4: Fire straight up

6. 31.3 m/s.

   Use a one-dimensional motion equation, with zero as the final velocity. The displacement is the tower's height.

$$v_f^2 = v_i^2 + 2a\Delta y$$

$$v_i = \sqrt{v_f^2 - 2a\Delta y} = \sqrt{(0\text{ m/s})^2 - 2(-9.80\text{ m/s}^2)(49.9\text{ m})} = 31.3\text{ m/s}$$

7. The cannonball takes the same amount of time to rise to its peak as it does to fall back to the y position from which it was fired. It spends 50% of its time going up, and 50% coming down. The final y velocity has the same magnitude but opposite direction as the initial y velocity.

Exercise 5: The symmetry of a projectile

8. The motion of the projectile is symmetrical: 50% of the flight time is spent on the way up, and 50% is spent on the way down.
9. The vertical velocity immediately before the cannonball hits the ground has the same magnitude as the initial vertical velocity but it points in the opposite direction. The vertical velocity component points upward initially, and just prior to the cannonball hitting the ground the vertical velocity component points downward.
10. The initial horizontal velocity of the cannonball is the same as the horizontal velocity immediately before the cannonball hits the ground. Since there is no x acceleration, the horizontal velocity never changes while the ball is in flight.

Exercise 6: Artillery fire!

Exercise 7: Use trigonometry to destroy the castle!

11. Castle door

    Distance from cannon: 200 m

    Firing speed: Possible values range from 15.0 m/s to 60 m/s

    Firing angle: Answers may vary, but are dependent upon firing speed and distance.

    Battering ram

    Distance from cannon: Answers may vary.

    Firing speed: Possible values range from 15.0 m/s to 60 m/s

    Firing angle: Answers may vary, but are dependent upon firing speed and distance.

    To determine how the firing angle, speed and distance are related, begin by writing an expression for the time that the ball will be in the air. The total time that the cannonball is in flight is twice the time it takes to reach its peak. The cannonball's vertical velocity is 0 m/s at its peak.

$$v_{yf} = v_{yi} + at$$

$$t_{peak} = \frac{v_{yf} - v_{yi}}{a} = \frac{(0\text{ m/s}) - (v_{yi})}{-9.80\text{ m/s}^2} = \frac{v_{yi}}{9.80\text{ m/s}^2}$$

$$t_{total} = 2t_{peak} = 2\left(\frac{v_{yi}}{9.80\text{ m/s}^2}\right)$$

Use the definition of velocity to calculate the initial horizontal velocity in terms of the total flight time, then substitute the above expression for the time. The horizontal velocity does not change throughout the cannonball's flight because there is no acceleration in the horizontal direction.

$$v_x = \frac{\Delta x}{t_{total}} = \frac{\Delta x\left(9.80 \text{ m/s}^2\right)}{2v_{yi}}$$

This exercise requires the firing speed and angle, rather than the horizontal and vertical velocities. Change to polar coordinates.

$$v_x = v\cos\theta,\ v_y = v\sin\theta$$

Insert these two equations into the equation above for v_x.

$$v\cos\theta = \frac{\Delta x\left(9.80 \text{ m/s}^2\right)}{2v\sin\theta}$$

$$v = \sqrt{\frac{\Delta x\left(9.80 \text{ m/s}^2\right)}{2\sin\theta\cos\theta}}$$

Exercise 8 (optional): Establishing a range equation

12. Begin with a horizontal motion equation using only v and θ.

$$\Delta x = v_x t$$

$$\Delta x = v\cos\theta t$$

Next use a vertical motion equation using only v and θ that is relevant for the period when the cannonball is rising to its peak. At its peak the cannonball has zero vertical velocity and the time elapsed is half the total time of flight.

$$v_{yf} = v_{yi} + a_y\left(\frac{t}{2}\right)$$

$$0 = v\sin\theta + a_y\left(\frac{t}{2}\right)$$

Solve the second equation for t and substitute it into the first equation.

$$t = \frac{-2v\sin\theta}{a_y}$$

$$\Delta x = v\cos\theta\frac{-2v\sin\theta}{a_y}$$

Finally, use a double angle identity to simplify the final equation.

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\Delta x = \frac{-v^2\sin 2\theta}{a_y}$$

13. Any two complementary angles will give the same range. The equation developed in question 12 confirms this. The $\sin 2\theta$ function is symmetrical about 45° in the first quadrant.

Exercise 9 (optional): Establishing a range equation for different elevations

14. The range equals the negative of the firing speed squared times the sine of theta, times cosine theta, minus the speed times the cosine of theta times the square root of, the square of, the speed times the sine of theta, plus two times the acceleration times the change in height, all divided by the acceleration.

$$\Delta x = \frac{-v^2 \sin\theta \cos\theta - v\cos\theta\sqrt{(v\sin\theta)^2 + 2a_{\mathrm{y}}\Delta y}}{a_{\mathrm{y}}}$$

15. Begin with the following equation.

$$\Delta y = v_{\mathrm{y}}t + \frac{1}{2}a_{\mathrm{y}}t^2$$

$$\frac{1}{2}a_{\mathrm{y}}t^2 + v_{\mathrm{y}}t - \Delta y = 0$$

Solve this equation using the quadratic formula. The later time corresponds to when the projectile reaches that displacement on its way down. The denominator (the acceleration) is negative, so choose the minus sign in the numerator in order to obtain the later time.

$$t = \frac{-v_{\mathrm{y}} - \sqrt{v_{\mathrm{y}}^{\,2} + 2a_{\mathrm{y}}\Delta y}}{a_{\mathrm{y}}}$$

Now substitute this into the linear motion equation that applies to the ball's motion in the x direction.

$$\Delta x = v_{\mathrm{x}}t$$

$$\Delta x = v_{\mathrm{x}}\left(\frac{-v_{\mathrm{y}} - \sqrt{v_{\mathrm{y}}^{\,2} + 2a_{\mathrm{y}}\Delta y}}{a_{\mathrm{y}}}\right)$$

Finally replace v_{x} and v_{y} with expressions using only v and θ.

$$\Delta x = v\cos\theta\left(\frac{-v\sin\theta - \sqrt{(v\sin\theta)^2 + 2a_{\mathrm{y}}\Delta y}}{a_{\mathrm{y}}}\right)$$

$$\Delta x = \frac{-v^2 \sin\theta \cos\theta - v\cos\theta\sqrt{(v\sin\theta)^2 + 2a_{\mathrm{y}}\Delta y}}{a_{\mathrm{y}}}$$

16. The assumption is that the ball is being launched from the same location, no matter what the angle. However, if the cannon is tilted up to a higher starting angle, it starts at a slightly greater height than if it is aimed, for example, horizontally (at zero degrees). Therefore, it lands with a slightly greater speed than you might expect, and with a slightly greater range.

Notes on the simulations

Exercise 1 Simulation

Objective: Devise a method of determining the height of the tower using a cannonball and a stopwatch, then test your method.

Answer: $\Delta y = -49.9$ m

Exercise 2 Simulation

Objective: Hit the haystack with the cannonball by giving it an initial horizontal velocity.

Answer: $v_x = 15.7$ m/s

Solution: The cannonball takes 3.19 seconds to reach the ground. Calculate the initial horizontal velocity that will propel the cannonball a distance of 50.0 meters in 3.19 seconds.

$$v_x = \frac{\Delta x}{t} = \frac{50.0 \text{ m}}{3.19 \text{ s}} = 15.7 \text{ m/s}$$

Exercise 3 Simulation

Objective: Play against a classmate or the computer and try to hit your opponent's targets before your opponent hits yours. You can adjust the horizontal velocity only.

Solution: The towers are the same height as in exercise 2, so the time it takes a ball to fall will again be 3.19 s. If a target is located Δx meters from your tower, calculate the initial horizontal velocity that will propel the cannonball a distance of Δx meters in 3.19 seconds.

$$v_x = \frac{\Delta x}{t} = \frac{\Delta x}{3.19 \text{ s}}$$

Exercise 4 Simulation

Objective: Shoot a cannonball straight up so that it has zero velocity at the top of the tower.

Answer: $v_i = 31.3$ m/s

Exercise 5 Simulation

Objective: This exercise is used to experiment with projectile motion and to answer questions 8-10.

Exercise 6 Simulation

Objective: Play against a classmate or the computer and try to hit your opponent's targets before your opponent hits yours. You can adjust the horizontal velocity and vertical firing velocities.

Solution: Begin by writing an expression for the time that a ball will be in the air. The total time that the cannonball is in flight is twice the time it takes to reach its peak. The cannonball's vertical velocity is 0 m/s at its peak.

$$v_{yf} = v_{yi} + at$$

$$t_{peak} = \frac{v_{yf} - v_{yi}}{a} = \frac{(0 \text{ m/s}) - (v_{yi})}{-9.80 \text{ m/s}^2} = \frac{v_{yi}}{9.80 \text{ m/s}^2}$$

$$t_{total} = 2t_{peak} = 2\left(\frac{v_{yi}}{9.80 \text{ m/s}^2}\right)$$

Use a linear motion equation to calculate the initial horizontal velocity in terms of the total flight time, and substitute the expression above for the time. The horizontal velocity does not change throughout the cannonball's flight because there is no acceleration in the horizontal direction.

$$\Delta x = v_x t_{\text{total}}$$

$$v_x = \frac{\Delta x}{t_{\text{total}}} = \frac{\Delta x\left(9.80\text{ m/s}^2\right)}{2v_{yi}}$$

To hit a target at a distance Δx, choose a reasonable value for the initial vertical velocity and input this value and Δx into the equation above to determine the corresponding horizontal velocity.

For example, the opposing castle is 200 meters away. To hit it, choose a reasonable value for the initial vertical velocity, say 35.0 m/s. Use this value and the distance to the castle to calculate the correct horizontal firing velocity.

$$v_x = \frac{\Delta x\left(9.80\text{ m/s}^2\right)}{2v_{yi}} = \frac{(200\text{ m})\left(9.80\text{ m/s}^2\right)}{2(35.0\text{ m/s})} = 28.0\text{ m/s}$$

This is only one possible solution; there are many other horizontal and vertical firing velocity combinations that will hit the castle.

Exercise 7 Simulation

Objective: Play against a classmate or the computer and try to hit your opponent's targets before your opponent hits yours. You can adjust the firing speed and angle.

Notes: For some combinations of angle and target distance, the required firing speed may be more than the maximum possible firing speed of the cannon (60 m/s). In those cases, try a smaller angle, or input a speed into the equation and solve for the angle.

Exercise 8 Simulation

Objective: Develop an equation that will tell you the range of a cannonball for a given firing speed and angle, and then test this equation in the simulation.

Exercise 9 Simulation

Objective: Develop an equation for calculating the horizontal range of a cannonball shot from a different elevation than the target. Then test the equation in the simulation.

Juggling objects: The physics behind juggling

Questions

Exercise 1: Juggling one ball

1. 6.9 m/s

   Solve a motion equation for the initial vertical velocity. The ball makes a round trip in 1.4 seconds, and the vertical displacement is zero.

$$\Delta y = v_i t + \frac{1}{2}at^2$$

$$v_i = \frac{\Delta y - \frac{1}{2}at^2}{t} = \frac{\left[(0\text{ m/s}) - \frac{1}{2}\left(-9.80\text{ m/s}^2\right)(1.4\text{ s})^2\right]}{1.4\text{ s}}$$

$$v_i = 6.9\text{ m/s}$$

2. Yes, the times are the same.
3. Since there is no x acceleration, if the x velocity is anything other than zero, there will be some x displacement which means it will not land in the same hand when it comes back down.

Exercise 2: Two balls with one hand

4. 0.70 s

   Since the ball is in the air for 1.4 s and it takes the same amount of time to rise to the top as to fall back down, the time to the peak is half of 1.4 seconds, or 0.70 seconds.
5. 2.4 m

   For the total hang time to be 1.4 seconds, the initial y velocity must be 6.9 m/s as was determined in question Use the time needed to reach the peak, 0.70 s, to find the vertical displacement of the ball at that time.

$$\Delta y = v_i t + \frac{1}{2}at^2$$

$$\Delta y = (6.9\text{ m/s})(0.70\text{ s}) + \frac{1}{2}\left(-9.80\text{ m/s}^2\right)(0.70\text{ s})^2$$

$$\Delta y = 2.4\text{ m}$$

Exercise 3: Two balls with two hands

6. 0.50 m/s

   Since there is zero acceleration in the x direction, the velocity will be constant. Use the definition of velocity.

$$v_x = \frac{\Delta x}{t} = \frac{0.70\text{ m}}{1.4\text{ s}} = 0.50\text{ m/s}$$

7. Yes, the prediction was correct.
8. No, the horizontal velocity has no effect on the vertical velocity. The only thing that affects the time the ball is in the air is the initial y velocity, and the y acceleration (which is constant in this lab).

Exercise 4: The amazing juggler

9. The initial x velocity will decrease. If you increase the initial y velocity of the ball, the hang time will increase. Since the hang time has increased and the distance between the hands remains the same, the initial x velocity must decrease.

Exercise 5: Juggling to a specific height

10. 10.8 m/s

    Use a motion equation to find the initial y velocity.

$$v_{\mathrm{f}}{}^2 = v_{\mathrm{i}}{}^2 + 2a\Delta y$$

$$v_{\mathrm{i}} = \sqrt{v_{\mathrm{f}}{}^2 - 2a\Delta y} = \sqrt{(0\ \mathrm{m/s})^2 - 2\left(-9.80\ \mathrm{m/s^2}\right)(6.0\ \mathrm{m})}$$

$$v_{\mathrm{i}} = 10.8\ \mathrm{m/s}$$

11. 2.2 s

First calculate the time it takes a ball to rise 6.0 m.

$$v_{\mathrm{f}} = v_{\mathrm{i}} + at$$

$$at = v_{\mathrm{f}} - v_{\mathrm{i}}$$

$$t = \frac{v_{\mathrm{f}} - v_{\mathrm{i}}}{a} = \frac{(0\ \mathrm{m/s}) - (10.8\ \mathrm{m/s})}{-9.80\ \mathrm{m/s^2}} = 1.1\ \mathrm{s}$$

Since the time to rise 6.0 m is 1.1 seconds, the time to fall is also 1.1 seconds. So the hang time must be 2.2 seconds.

12. 0.50 m/s

Since there is no acceleration in the x direction, the x velocity is constant throughout the path of the ball.

$$v_{\mathrm{x}} = \frac{\Delta x}{t} = \frac{1.1\ \mathrm{m}}{2.2\ \mathrm{s}} = 0.50\ \mathrm{m/s}$$

Exercise 6: Throwing angle

13. $v = 7.07$ m/s, $\theta = 82.0°$

First use a motion equation to calculate the initial y velocity.

$$v_{\mathrm{fy}}{}^2 = v_{\mathrm{iy}}{}^2 + 2a\Delta y$$

$$v_{\mathrm{iy}} = \sqrt{v_{\mathrm{fy}}{}^2 - 2a\Delta y} = \sqrt{(0\ \mathrm{m/s})^2 - 2\left(-9.80\ \mathrm{m/s^2}\right)(2.50\ \mathrm{m})}$$

$$v_{\mathrm{iy}} = 7.00\ \mathrm{m/s}$$

Now calculate the hang time.

$$v_{\mathrm{fy}} = v_{\mathrm{iy}} + at$$

$$t = \frac{v_{\mathrm{fy}} - v_{\mathrm{iy}}}{a} = \frac{(0\ \mathrm{m/s}) - (7.00\ \mathrm{m/s})}{-9.80\ \mathrm{m/s^2}}$$

$$t = 0.714\ \mathrm{s}$$

Since the time to rise 2.50 m is 0.714 seconds, the hang time must be 1.43 seconds. Next calculate the initial x velocity. Since there is no acceleration in the x direction, the x velocity is constant throughout the path of ball.

$$v_{\mathrm{x}} = \frac{\Delta x}{t} = \frac{1.40\ \mathrm{m}}{1.43\ \mathrm{s}} = 0.979\ \mathrm{m/s}$$

Finally use the individual components of the velocity to find the angle and magnitude of the velocity.

$$v^2 = v_x^{\ 2} + v_y^{\ 2}$$

$$v = \sqrt{v_x^{\ 2} + v_y^{\ 2}} = \sqrt{(0.979\ \text{m/s})^2 + (7.00\ \text{m/s})^2}$$

$$v = 7.07\ \text{m/s}$$

Use the x component of v to calculate θ.

$$v_x = v\cos\theta$$

$$\frac{v_x}{v} = \cos\theta$$

$$\theta = \arccos\left(\frac{v_x}{v}\right) = \arccos\left(\frac{0.979\ \text{m/s}}{7.07\ \text{m/s}}\right)$$

$$\theta = 82.0°$$

Notes on the simulations

Exercise 1 Simulation

Objective: Determine the initial vertical velocity required for the ball to have a flight time of 1.4 s. Then test your answer in the simulation.

Answer: $v_{iy} = 6.9$ m/s

Exercise 2 Simulation

Objective: Specify a time interval between throws that will cause one ball to be at its peak when the other is in the hand of the juggler.

Answer:

$v_{iy} = 6.9$ m/s

$t = 0.7$ s

Exercise 3 Simulation

Objective: Specify a horizontal velocity that will allow you to pass the ball from one hand to the other while the ball is in the air.

Answer:

$v_{iy} = 6.9$ m/s

$v_{ix} = 0.5$ m/s

$t = 0.7$ s

Exercise 4 Simulation

Objective: Specify a time interval that will allow you to juggle three balls.

Answer:

$v_{iy} = 6.9$ m/s

$v_{ix} = 0.5$ m/s

$t = 0.9$ s

Solution: The round trip time is 2.8 seconds, twice the hang time of 1.4 seconds. Divide the round trip time by the number of balls in the simulation, 3 in this case. The correct time interval is 0.9 seconds.

Exercise 5 Simulation

Objective: Determine the initial x and y velocities and the hang time required to juggle the balls through the hoop.

Answer:

$v_{iy} = 10.8$ m/s

$v_{ix} = 0.5$ m/s

$t = 2.2$ s

Exercise 6 Simulation

Objective: Determine the speed, throwing angle, and time interval between throws to juggle three balls through the hoop.

Answer:

$v = 7.07$ m/s

$\theta = 82.0°$

$t = 0.93$ s

Exercise 7 Simulation

Objective: Calculate a combination of throwing speed, angle, and time interval between throws to juggle five chainsaws.

Answer: Answers will vary.

Solution: Choose a height to throw the chainsaws to. In this solution, 5.00 m is used. Use a motion equation to calculate the initial y velocity.

$$v_{fy}^2 = v_{iy}^2 + 2a\Delta y$$

$$v_{iy} = \sqrt{v_{fy}^2 - 2a\Delta y} = \sqrt{(0 \text{ m/s})^2 - 2\left(-9.80 \text{ m/s}^2\right)(5.00 \text{ m})}$$

$$v_{iy} = 9.90 \text{ m/s}$$

Now calculate the hang time.

$$v_f = v_i + at$$

$$t = \frac{v_f - v_i}{a} = \frac{(0 \text{ m/s}) - (9.90 \text{ m/s})}{-9.80 \text{ m/s}^2}$$

$$t = 1.01 \text{ s}$$

Since the time to rise 5.00 m is 1.01 seconds, the hang time must be 2.02 seconds. The round trip time is double the hang time, or 4.04 seconds. The correct time interval between throws is one fifth that number, or 0.808 seconds. (This is a very short time to catch and throw a chainsaw before the next one arrives!) Next calculate the initial x velocity. Since there is no acceleration in the x direction, the x velocity of the chainsaw is constant throughout its path.

$$v_x = \frac{\Delta x}{t} = \frac{1.40 \text{ m}}{2.02 \text{ s}} = 0.693 \text{ m/s}$$

Finally use the individual components of the initial velocity to find its angle and magnitude.

$$v^2 = {v_x}^2 + {v_y}^2$$

$$v = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{(0.693 \text{ m/s})^2 + (9.90 \text{ m/s})^2}$$

$$v = 9.92 \text{ m/s}$$

Use the x component of v to calculate θ.

$$v_x = v\cos\theta$$

$$\frac{v_x}{v} = \cos\theta$$

$$\theta = \arccos\left(\frac{v_x}{v}\right) = \arccos\left(\frac{0.693 \text{ m/s}}{9.92 \text{ m/s}}\right)$$

$$\theta = 86.0°$$

Circular motion

Questions

Exercise 1: Speed, velocity and acceleration

1. The x velocity increases but the y velocity never changes in the Speed change zone.
2. Yes.
3. Neither the x nor the y velocity changes in the Constant speed zone.
4. No.

Exercise 2: Racing around a curve

5. Yes, both the x and y velocities change on the curve.
6. Yes.
7. Velocity is a vector composed of speed and direction. Although the speed does not change as the car goes around the curve, the direction does and thus the velocity vector changes. A change in the velocity vector means there is acceleration.

Exercise 3: The velocity vector

8. The car accelerates at points B and C. Although the speed does not necessarily need to change at these points, the direction does, and therefore there is acceleration.
9. The speed equals the square root of the sum of the x velocity squared and the y velocity squared. The angle of the velocity is the arctangent of the ratio of the y velocity to the x velocity.
10. There are many different answers to this question. Begin with any value for the initial x velocity. This

example uses 15.0 m/s. The initial y velocity is 0 m/s so the speed for the first segment is 15.0 m/s. Use the fact that the x and y velocities need to have the same magnitudes in the last two segments (because these segments are at angles of $45°$ and $225°$) to find their values.

$$v = \sqrt{v_x{}^2 + v_y{}^2} = \sqrt{2v_x{}^2}$$

$$v_x = \sqrt{\frac{v^2}{2}} = \pm\sqrt{\frac{(15.0 \text{ m/s})^2}{2}} = \pm 10.6 \text{ m/s}$$

For the second segment, the x velocity is 10.6 m/s and the y velocity is –10.6 m/s. For the third segment the x velocity is –10.6 m/s and the y velocity is –10.6 m/s.

Exercise 4: Centripetal acceleration

11. Answers will vary, but for each turn the acceleration should equal the speed squared, divided by the radius of the turn.
12. The centripetal acceleration decreases with an increase in radius. More precisely, it varies inversely with the curve radius.
13. Acceleration is the change in velocity over time. With a smaller radius, and for a car moving at the same speed, the velocity vector will be changing direction more rapidly, so it will have a larger acceleration.

Exercise 5: Speed and centripetal acceleration

14. Answers will vary.
15. Centripetal acceleration increases as speed increases. More precisely, centripetal acceleration is proportional to the square of the speed.

Exercise 6: Grand Prix of physics

16. $v_1 = 15.3$ m/s, $v_2 = 22.2$ m/s, $v_3 = 16.0$ m/s

    Use the definition of centripetal acceleration to calculate the maximum speed you can take each turn without spinning off the track.

$$a_c = \frac{v^2}{r}$$

$$v = \sqrt{a_c r} = \sqrt{\left(29.4 \text{ m/s}^2\right) r}$$

$$v_1 = \sqrt{\left(29.4 \text{ m/s}^2\right)(8.00 \text{ m})} = 15.3 \text{ m/s}$$

$$v_2 = \sqrt{\left(29.4 \text{ m/s}^2\right)(16.75 \text{ m})} = 22.2 \text{ m/s}$$

$$v_3 = \sqrt{\left(29.4 \text{ m/s}^2\right)(8.75 \text{ m})} = 16.0 \text{ m/s}$$

Exercise 7: Control the car with keys

Exercise 8 (optional): Use the force

17. 12670 N

    Multiply the normal force by the coefficient of friction to find the maximum force of friction. Since the track is level, the normal force equals the weight of the car.

$$F = \mu_k mg = (0.75)(1724 \text{ kg})\left(9.80 \text{ m/s}^2\right) = 12670 \text{ N}$$

18. The car will experience more centripetal acceleration around the smaller turn, which is the first one.
19. 8.4 m/s

    Using Newton's second law, set the force due to centripetal acceleration equal to the force of friction calculated in question 17.

$$F = m\frac{v^2}{r}$$

$$v = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{(12670 \text{ N})(9.6 \text{ m})}{1724 \text{ kg}}} = 8.4 \text{ m/s}$$

Notes on the simulations

Exercise 1 Simulation

Objective: Review speed, velocity and acceleration on a straight track. Try to beat the blue car to the finish line.

Exercise 2 Simulation

Objective: Experiment with and observe the speed, velocity and acceleration of a car going around a curved track. Try to beat the blue car to the finish line without spinning off the track.

Exercise 3 Simulation

Objective: Determine the x and y velocities for each segment of the track that will allow your car to win the race.

Solution: To keep the car on the track, on the first segment the x velocity must be positive and the y velocity must be zero. On the second segment, the x velocity must be positive and the y velocity must have the same magnitude as the x velocity, but be negative. On the third segment, the x and y velocities must have the same magnitude and both be negative. Once the car is made to stay on the track, the magnitudes of the velocity components can be adjusted until the red car wins.

Exercise 4 Simulation

Objective: Experiment with the simulation to determine the relationship between the radius of a turn and centripetal acceleration. Try to beat the blue car to the finish line without spinning off the track.

Exercise 5 Simulation

Objective: Drive the car around the track at varying speeds and note the accelerations. Determine how the speed around a circular track is related to centripetal acceleration.

Exercise 6 Simulation

Objective: Use the definition of centripetal acceleration to calculate the maximum speed you can take each turn without spinning off the track. Use the calculated speeds to beat the blue car to the finish line.

Answer:

Turn 1: $v_1 = 15.3$ m/s

Turn 2: $v_2 = 22.2$ m/s

Turn 3: $v_3 = 16.0$ m/s

Exercise 7 Simulation

Objective: Use the arrow keys on your keyboard to drive the car in uniform circular motion.

Exercise 8 Simulation

Objective: Use Newton's second law and your knowledge of frictional forces to determine the maximum speed at which you can run the race without spinning off the track.

Answer: $v = 8.4$ m/s

Exercise 9 Simulation

Objective: Use the arrow keys on your keyboard to drive the car in uniform circular motion.

Helicopters in flight: Applying Newton's force laws

Questions

Exercise 1: Going airborne

1. No. If the helicopter is moving at a constant velocity, then it is not accelerating. Newton's second law states that if there is no acceleration, then there is no net force.
2. Yes. If the helicopter was first moving upward, then eventually hovered (was at rest), then it changed from positive to zero velocity. When the helicopter changed from a positive velocity to zero velocity, there must have been a negative acceleration. A negative acceleration can only be caused by a negative net force.

Exercise 2: What is the mass of the helicopter?

3. Yes, the acceleration does change with net force. By Newton's second law, the acceleration is proportional to the net force. The mathematical expression for the relationship between acceleration and net force is $\boldsymbol{F} = m\boldsymbol{a}$.
4. 2041 kg

   Use Newton's second law to find the mass of the helicopter. Values for the force and acceleration will vary. Forces of 1000 N or more must be used to obtain a mass that is accurate to four significant digits. In this example we use F = 2000 N, a = 0.98m/s^2.

   $$F = ma$$

   $$m = \frac{F}{a} = \frac{2000 \text{ N}}{0.98 \text{ m/s}^2}$$

   $$m = 2041 \text{ kg}$$

Exercise 3: Net force and acceleration

5. 32,000 N

   L is the lift force, which acts in the upward direction on the helicopter. The weight of the helicopter is a force that acts downward. Write an expression for the net force, which, by Newton's second law, equals the mass multiplied by the acceleration.

$$\sum \mathbf{F} = m\mathbf{a}$$

$$L - mg = ma$$

$$L = m(a+g) = (2041\ \text{kg})\left(5.88\ \text{m/s}^2 + 9.80\ \text{m/s}^2\right)$$

$$L = 32000\ \text{N}$$

Exercise 4: Air resistance

6. The force of air resistance increases as the helicopter speed increases.
7. The magnitude of the force of air resistance is proportional to the square of the speed of the helicopter.

Exercise 5: Flying your helicopter

8. A horizontal thrust force of 0 N is required for the helicopter to have either zero horizontal velocity or a constant horizontal velocity. (Recall that air resistance has been "turned off" in this exercise, so when the horizontal thrust is zero, there is no net horizontal force.)

Exercise 6 (optional): Save the day!

9. $L = 25{,}500$ N, $\Sigma F_{\text{x}} = 4429$ N , $\Sigma F_{\text{y}} = 5123$.

   First find the net horizontal force. From Newton's second law, this is the mass multiplied by the horizontal acceleration. L is the lift force.

$$\Sigma F_{\text{x}} = ma_{\text{x}}$$

$$\Sigma F_{\text{x}} = (2041\ \text{kg})\left(2.17\ \text{m/s}^2\right)$$

$$\Sigma F_{\text{x}} = 4429\ \text{N}$$

   Next find the magnitude of the lift force using trigonometry. The following equation is true because the only force in the horizontal direction is the horizontal component of the lift force.

$$F_x = L\cos\theta$$

$$L = \frac{F_x}{\cos\theta} = \frac{4429\ \text{N}}{\cos 80^\circ}$$

$$L = 25500\ \text{N}$$

   The net vertical force is the mass times the vertical acceleration.

$$\Sigma F_{\text{y}} = ma_{\text{y}}$$

$$\Sigma F_{\text{y}} = (2041\ \text{kg})\left(2.51\ \text{m/s}^2\right)$$

$$\Sigma F_{\text{y}} = 5123\ \text{N}$$

Notes on the simulations

Exercise 1 Simulation

Objective: Experiment with changing the net force on the helicopter. Try to get it to lift off the ground, and then hover.

Exercise 2 Simulation

Objective: Apply a net force to the helicopter and observe the resulting acceleration. Use Newton's second law to determine the mass of the helicopter.

Exercise 3 Simulation

Objective: Calculate the lift force (not the net force) that will cause the helicopter to accelerate at 5.88 m/s^2.

Exercise 4 Simulation

Objective: Gather data to determine how speed is related to the force of air resistance.

Exercise 5 Simulation

Objective: Use a horizontal thrust force and vertical lift force to pilot the helicopter from one landing pad to another.

Exercise 6 Simulation

Objective: Determine the lift force required to cause the helicopter to have a vertical acceleration of 2.51 m/s^2 and a horizontal acceleration of 2.17 m/s^2. The lift force is directed at $80°$ to the horizontal.

Gravity

Questions

Exercise 1: Gravitational attraction and mass

1. (Data table should show a linear relationship between mass and force.)
2. The force of gravity increases as the mass of the satellite increases.
3. (Graph) The relationship between the mass of the satellite and the force of gravity in linear.

Exercise 2: Force and distance

4. (Data table should show that the force is inversely proportional to the square of the distance.)
5. The force of gravity decreases as the distance between Earth and the satellite increases.
6. (Graph) The relationship between distance and force is not linear.

Exercise 3 (optional): Newton's Law of Gravitation

7. If the distance is doubled, Newton's Law of Gravitation states that the force will be quartered.
8. Doubling one of the masses will double the force between them.
9. The resulting force on each of the masses is the same as their weight, $F = mg$, the mass multiplied by the acceleration due to gravity at the surface of the Earth.
10. $G = 6.67 \times 10^{-11}\ N{\cdot}m^2/kg^2$

    To calculate *G*, first solve Newton's law of gravitation for *G*.

$$F = \frac{GMm}{r^2}$$

$$G = \frac{Fr^2}{Mm}$$

Now substitute one of your data points for the variables. Data points used may vary, but the resulting value of G should be near its accepted value.

$$G = \frac{(9.78\ \text{N})\left(6.38\times10^{6}\ \text{m}\right)^2}{\left(5.97\times10^{24}\ \text{kg}\right)(1.0\ \text{kg})} = 6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$$

Exercise 4 (optional): Gravity and motion

11. They do not move toward each other at the same speed. While they exert the same force on each other, they have different masses. By Newton's second law, the lighter mass accelerates more for a given amount of force.
12. Drag the two masses out so that they are 10 squares apart. Then let them fall toward each other until they touch. By counting how far they moved before touching, you can determine how fast they moved relative to each other. For instance, if the *m* mass moved 8 squares and the 4*m* mass moved 2 squares, then the *m* mass was traveling 4 times faster. You can try other combinations of masses to see if this relationship holds in general. If you place *m* and 2*m* masses 10 squares apart, and the point they meet is 6.6 squares from the *m* starting point and 3.3 squares from the 2*m* starting point, then the *m* mass traveled twice as fast as the 2*m* mass.
13. The relationship holds true in general. The ratio of the masses is the inverse of the ratio of their accelerations.

Exercise 5 (optional): The Solar System

14. When the Sun's mass decreases, the force of gravity on the planets also decreases.
15. When the Sun's mass increases, the force of gravity on the planets also increases.

Notes on the simulations

Exercise 1 Simulation

Objective: Gather data to determine the relationship between mass and the force of gravity.

Conclusion: The relationship between mass and gravitational force is linear.

Exercise 2 Simulation

Objective: Gather data to determine the relationship between distance and the force of gravity.

Conclusion: The force of gravity is inversely proportional to the square of the distance.

Exercise 3 Simulation

Objective: Verify Newton's Law of Gravitation and determine the value of the gravitational constant *G*.

Answer: $G = 6.67 \times 10^{-11}$ N·m^2/kg^2

Exercise 4 Simulation

Objective: Explore the relationship between gravitational force and motion.

Exercise 5 Simulation

Objective: Explore the relationship between the Sun's mass and the force of gravity exerted by it on the planets, as well as their resulting motion.

Collisions

Questions

Exercise 1: Conservation of momentum

1. (Data table)
2. (Data table)
3. Yes, it is possible to have negative momentum.
4. No, velocity is not conserved.
5. Yes, momentum is conserved.
6. (Data table)
7. No. Since the velocity is squared in the kinetic energy formula, kinetic energy is positive regardless of the sign of the velocity.
8. Yes, kinetic energy is conserved.

Exercise 2: Conservation of momentum and inelastic collisions

9. Yes. Momentum is conserved in an inelastic collision. To show this, you could sum the momenta of two disks to show that the sums are equal before and after an inelastic collision.
10. No. An inelastic collision means kinetic energy is not conserved. To show this, you could sum the kinetic energies of two disks to show that the sums are equal before and after an inelastic collision.
11. (Data table)
12. (Data table)
13. (Graph)
14. (Graph)
15. (Graph)
16. (Graph)
17. Yes, momentum is conserved in an inelastic collision. The sum of the two disks' momenta before the collision is the same as the sum after. In the stacked bar chart, the total height of the bar remains the same.
18. No, kinetic energy is not conserved in an inelastic collision. The sum of the two disks' kinetic energies before the collision is not the same as the sum after. In the stacked bar chart, the total height of the bar

changes.

19. The kinetic energy lost is transformed to other types of energy.

Exercise 3: Extra credit: Collisions

20. Collision A is impossible because momentum is not conserved.
21. Collision B is elastic because momentum and kinetic energy are both conserved.
22. Collision C is inelastic because momentum is conserved but kinetic energy is not.

Notes on the simulations

Exercise 1 Simulation

Objective: Observe and record data before and after collisions. Determine whether velocity, momentum, or kinetic energy is conserved during an elastic collision.

Conclusion: Velocity is not conserved. Momentum and kinetic energy are conserved during an elastic collision.

Exercise 2 Simulation

Objective: Observe and record data before and after an inelastic collision. Determine whether momentum and kinetic energy are conserved.

Conclusion: In an inelastic collision, momentum is conserved but kinetic energy is not.

Exercise 3 Simulation

Objective: Observe three collisions and determine whether each one is elastic, inelastic, or impossible.

Conclusion: Collision A is impossible, collision B is elastic, and collision C is inelastic.

Force between charged particles

Questions

Exercise 1: Sign and direction of force

1. The arrows point away from each other.
2. The arrows point away from each other.
3. The arrows point toward each other.
4. Like charges repel.
5. Opposite charges attract.
6. Set one sphere’s charge equal to zero, and try various positive and negative values for the charge of the other sphere. The results show no electrostatic force between a charged sphere and an uncharged sphere.

Exercise 2: How electric forces change with distance

7. The amount of force decreases as the distance increases.
8. The relationship between force and distance is not linear. The force is inversely proportional to the square of the distance between the spheres.

9. The spheres exert equal amounts of force on each other, in accordance with Newton's third law.

Exercise 3: How the charge values affect the force

10. Increasing the amount of charge also increases the force.
11. (Graph) The relationship between force and charge is linear.

Exercise 4: Extra credit – Determining Coulomb's law

12. $k = 8.99\times10^{9}$ N·m^{2}/C^{2}. The force between the spheres is proportional to the amounts of charge on each sphere and inversely proportional to the square of the distance between them.
13. The electric force is repulsive when the charges have the same sign and it is attractive when the charges have opposite signs.

Exercise 5: Electric golf: the practice green

Exercise 6: Electric golf: Q-school

14. Charge on putter = 1.60×10^{-19} C, Ball to putter distance = 0.053 m. (More solutions are possible.)
15. Initial acceleration = 49.1 m/s^{2}.

Exercise 7: Electric golf – the world tour

16. Yes, the initial heading of the ball was in the approximate direction of the force exerted on it by the putter.
17. No, the ball did not always continue to move in the same direction. As the ball moves through a non-uniform electric field, the net force on the ball changes in both magnitude and direction, and therefore the direction of its path can also change.

Notes on the simulations

Exercise 1 Simulation

Objective: Experiment with charges to determine the direction of force that like and opposite charges exert of each other.

Conclusion: Like charges repel each other, while opposite charges attract.

Exercise 2 Simulation

Objective: Vary the distance between the spheres to determine how the electric force changes with distance.

Conclusion: The electric force between the two spheres is inversely proportional to the square of the distance between them.

Exercise 3 Simulation

Objective: Vary the charges on the spheres to determine how the electric force changes with charge.

Conclusion: The electric force is linearly proportional to the charge of each sphere.

Exercise 4 Simulation

Objective: Write out what is known so far about the relationships between force, charge and distance. Use the simulation to determine the value of the constant *k* in Coulomb's Law.

Answer: The force between the spheres is proportional to the amounts of charge on each sphere and inversely proportional to the square of the distance between them. $k = 8.99\times10^{9}$ N·m^{2}/C^{2}.

Exercise 5 Simulation

Objective: Practice using the principles of electric force and charge to play a game of electric golf.

Exercise 6 Simulation

Objective: Use Coulomb's law to sink the ball in the hole.

Answer: Charge on putter = 1.60×10^{-19} C, Ball to putter distance = 0.053 m. (More solutions are possible.)

Exercise 7 Simulation

Objective: Use Coulomb's law to sink the ball in each hole.

Electromagnetic induction

Questions

Exercise 1: A generator

1. The emf in the coil changes as the wire loop rotates in the magnetic field. The emf varies over time as a sinusoidal function. This is related to the fact that the magnetic force is a function of the sine of the angle between the velocity and magnetic field vectors.
2. The induced emf is at a minimum when the plane of the loop is perpendicular to the magnetic field, that is, when the angle between the velocity vector and the magnetic field is $\theta = 0°$ or $180°$. The induced emf is a maximum when $\theta = 90°$ or $270°$. The equation for the magnetic force, $F = |q|vB \sin\theta$, corroborates this conclusion. When $\theta = 0°$ or $180°$ the sine function will equal zero, so the magnetic force will be zero. When the magnetic force is zero the buildup of charge or current will dissipate and the emf will go to zero. When $\theta = 90°$ or $270°$ the magnetic field lies in the plane of the loop and the sine function's magnitude will be at a maximum. This corresponds to the strongest magnetic force, and thus the strongest buildup of charge or current.

Exercise 2: Using mutual induction to transform emf

3. As you increase the number of secondary loops, the maximum induced emf increases. This is because the area through which magnetic field is passing has increased, increasing the maximum magnetic flux. The rate at which the flux changes is also greater, and so is the emf.
4. $30\ \text{V}$
5. If the secondary solenoid has twice as many loops, the induced emf will be twice as large as the primary emf. The equation for the ratio of emf magnitudes is:

$$\frac{N_2}{N_1} = \frac{\Delta V_2}{\Delta V_1}$$

6. The secondary coil should have half as many coils as the primary coil.

Exercise 3 (optional): Configuring a transformer

7. The primary solenoid must have 40 times as many coils as the secondary solenoid.

$$\frac{N_\mathrm{p}}{N_\mathrm{s}} = \frac{\Delta V_\mathrm{p}}{\Delta V_\mathrm{s}} = \frac{6800\ \mathrm{V}}{170\ \mathrm{V}} = 40$$

Notes on the simulations

Exercise 1 Simulation

Objective: Experiment with a wire loop rotating in a magnetic field, the basis of a generator.

Exercise 2 Simulation

Objective: Experiment with mutual induction in two solenoids with a different number of loops.

Exercise 3 Simulation

Objective: Configure the transformer to deliver a peak emf of 170 V to the homes.

Answer: There are two possible answers given the choices available in the simulation:

Primary solenoid: 800 loops, Secondary solenoid: 20 loops

Primary solenoid: 400 loops, Secondary solenoid: 10 loops

Thermodynamics and states of matter

Questions

Exercise 1: Simulating an ideal gas

1. After 15 seconds the molecules will have varying speeds. It is very likely that some of the molecules will have collided with each other and therefore changed speeds.
2. The speeds of the molecules varied from around 20 m/s to 300 m/s.
3. Enter data.
4. Since the potential energy of the molecules is not changing, the kinetic energy is the only part of the overall energy of the system that is changing. Conservation of energy states that the overall energy must remain constant, and therefore the average KE should stay the same.

Exercise 2: Speed distribution

5. The graph has a peak near the middle, and tapers off to each side. The peak is near the initial speed, so a molecule is more likely to be moving near the initial speed than at speeds far from the initial speed.
6. The graph of molecules' speeds is almost, but not quite symmetrical. The graph extends farther to the right, at the higher speeds. It makes sense that the graph cannot be symmetrical, since the speed cannot be less than zero, so there is a cutoff at zero speed, but it is possible for some of the gas molecules to randomly get hit in their direction of motion by other fast-moving particles, and thus move at a very high speed.

Exercise 3: Temperature and average molecular speed

7. The average speed of the gas molecules increased as the temperature increased.
8. The temperature is proportional to the square of the average speed. We use C to represent a proportionality constant.

   $T = C \cdot v_{\text{avg}}{}^{2}$

Exercise 4: Pressure and temperature

9. The average pressure increased as the temperature increased.
10. The relationship is linear.

    $P = C_1 T$

11. As the temperature rises, the molecules move faster, which in turn increases the rate at which they hit the walls of the container. A faster hit also applies more force to the wall. These two effects raise the pressure.

Exercise 5: Pressure and volume

12. The average pressure decreased as the volume of the gas increased.
13. There is an inverse relationship.

    $P = \dfrac{C_2}{V}$

14. As the volume increases, each gas molecule has more volume to travel in before hitting a wall. Thus the molecules will hit the wall less frequently, and exert less force on the walls over time, which causes the pressure to drop.

Exercise 6: Pressure and number of molecules, N

15. The average pressure increased as the number of molecules increased.
16. There is a linear relationship.

    $P = C_3 N$

Exercise 7: The ideal gas law

17. Begin by combining the first two equations. The combined equation shows that pressure is directly proportional to temperature, and inversely proportional to the volume.

    $P = C_4 \dfrac{T}{V}$

    Next, take the last equation into account. The pressure is also directly proportional to the number of gas molecules.

    $P = k \dfrac{NT}{V}$

18. This solution uses data from exercise 5. The value of k and the error will vary somewhat due to the very small number of molecules in our experiment. As the number of molecules increases, the error in the value of k becomes smaller.

$$P = k\frac{NT}{V}$$

$$k = \frac{PV}{NT} = \frac{(145 \text{ Pa})(1.6\times10^{-22} \text{ m}^3)}{(16)(100 \text{ K})} = 1.45\times10^{-23} \text{ J/K}$$

19. Use 1.38×10^{-23} J/K as the actual value to calculate the percent error.

$$\% \text{ error} = \frac{|measured - actual|}{actual} * 100\%$$

$$\% \text{ error} = \frac{|(1.45\times10^{-23} \text{ J/K}) - (1.38\times10^{-23} \text{ J/K})|}{1.38\times10^{-23} \text{ J/K}} * 100\%$$

$$\% \text{ error} = 5.07\%$$

Notes on the simulations

Exercise 1 Simulation

Objective: Observe the speeds of eight molecules in a container over time.

Exercise 2 Simulation

Objective: Observe the speed distribution of 50 molecules over time.

Exercise 3 Simulation

Objective: Change the temperature of the gas to determine how it is related to average molecular speed.

Conclusion: The temperature is proportional to the square of the average molecular speed.

Note: Students are asked to graph their data on paper.

Exercise 4 Simulation

Objective: Determine how the pressure of a gas is related to its temperature. The amount (number of molecules) and volume of the gas are held constant.

Conclusion: The pressure is linearly proportional to the temperature.

Note: Students are asked to graph their data on paper.

Exercise 5 Simulation

Objective: Determine how the pressure of a gas is related to its volume. The amount (number of molecules) and temperature of the gas are held constant.

Conclusion: The pressure is inversely proportional to the volume.

Note: Students are asked to graph their data on paper.

Exercise 6 Simulation

Objective: Determine how the pressure of a gas is related to the number of molecules. The volume and temperature of the gas are held constant.

Conclusion: The pressure is linearly proportional to the number of molecules.

Note: Students are asked to graph their data on paper.

Birds on a wire: The nature of waves

Questions

Exercise 1: A wave in a wire

1. The amplitude is equal to the height of the peak above equilibrium. It is also equal to the depth of a trough below equilibrium (where the depth is a positive number).
2. The wave pulses traveled to the right. The direction of the red parrot's motion was perpendicular to the direction of travel of the wave pulse.
3. The wire as a whole does not move over time. Individual particles in the wire move vertically up and down as the wave pulse moves horizontally through the wire.

Exercise 2: Continuous waves

4. The amplitude of the wave does not change when the frequency changes.
5. The frequency does not change when the amplitude is changed. They are independent of each other.

Exercise 3: Wavelength

6. As the frequency of a wave increases, the wavelength decreases.
7. If you double the frequency, the wavelength will be halved. If you triple the frequency, the wavelength will be reduced by a factor of three.

Exercise 4: Wave speed

8. (Data table) Speed in each case should be the same. $v = 0.8\ \text{m/s}$

   Calculate the wave speed using a linear motion equation. Values for x and t may vary. In this example we use x = 2.5 m, and t = 3.12 s.

   $$v = \frac{x}{t} = \frac{2.5\ \text{m}}{3.12\ \text{s}}$$
   $$v = 0.8\ \text{m/s}$$

9. No, amplitude does not have an effect on the speed of the wave.
10. No, frequency does not have an effect on the speed of the wave.

Exercise 5: Frequency and wavelength

11. (Data table) Speed in each case should be the same. $v = 0.8\ \text{m/s}$

    Use the equation relating wavelength, frequency and wave speed. Values for f and λ will vary, but the wave speed should always be 0.8 m/s. In this example we take f = 0.4 Hz and λ = 2.0 m.

    $$v = f\lambda = (0.4\ \text{Hz})(2.0\ \text{m})$$
    $$v = 0.8\ \text{m/s}$$

12. Yes, the wave speeds obtained in this Exercise and Exercise 4 are the same.

Exercise 6: It is the wire

13. $v = 0.55$ m/s

    Use the equation relating wavelength, frequency and wave speed.

    $$v = f\lambda = (1.1 \text{ Hz})(0.5 \text{ m})$$

    $$v = 0.55 \text{ m/s}$$

14. The wave will travel slower in the thicker wire.

    $$\frac{v_{thick}}{v_{thin}} = \frac{\sqrt{\dfrac{F}{2(m/L)}}}{\sqrt{\dfrac{F}{m/L}}} = \frac{\sqrt{\dfrac{F}{2}}}{\sqrt{\dfrac{F}{1}}}$$

    $$\frac{v_{thick}}{v_{thin}} = \sqrt{\frac{1}{2}} \approx 0.707$$

15. Yes, our experimental findings from Exercises 4, 5, and 6 support our answer to Question 13. Waves traveled faster in the thinner wire in Exercises 4 and 5 than in the thicker wire of Exercise 6.

Exercise 7 (optional): An equation for a particle in a traveling wave

16. $y = 0.35 \sin(7.3x - 8.8t)$

    The amplitude is 0.35 m as stated in the problem. The coefficient of t is $2\pi f$, which is equal to 8.8 rad/s.

17. $\lambda = 0.86$ m

    Use the definition for the coefficient of x to calculate the wavelength.

    $$\frac{2\pi}{\lambda} = 7.3 \text{ rad/m}$$

    $$\lambda = \frac{2\pi}{7.3 \text{ rad/m}}$$

    $$\lambda = 0.86 \text{ m}$$

Exercise 8 (optional): Timing your wave

18. There are many correct answers to this question. This example uses 1.0 m as the amplitude to simplify the problem. (The amplitude of the wave must be at least this value.) Since the amplitude is the same as the desired vertical displacement at 5 seconds, the baby bird must be at a peak then. In order for the baby bird to be at a peak of the wave, the sine function must equal 1. This example uses $\pi/2$ radians as the argument of the sine function.

$$\frac{2\pi}{\lambda}x + 2\pi ft = \frac{\pi}{2}$$
$$\frac{2\pi f}{v}x + 2\pi ft = \frac{\pi}{2}$$
$$\frac{f}{v}x + ft = \frac{1}{4}$$
$$f\left(\frac{x}{v} + t\right) = \frac{1}{4}$$
$$f = \frac{1}{4\left(\frac{x}{v} + t\right)} = \frac{1}{4\left(\frac{-4.0\text{ m}}{2.0\text{ m/s}} + 5.0\text{ s}\right)}$$
$$f = 0.083\text{ Hz}$$

The coefficient of t should be $2\pi f$, or 0.52 rad/s. Thus, an equation for the wave that will vertically displace the baby bird correctly is $y = (1.0\text{ m})\sin(0.3x + 0.52t)$. This answer corresponds to the mother bird reaching the baby at the first peak of the wave.

Notes on the simulations

Exercise 1 Simulation

Objective: Experiment with different wave amplitudes to create a wave pulse in the wire that will dislodge the other birds.

Exercise 2 Simulation

Objective: Experiment with different wave frequencies and amplitudes to dislodge the other birds from the wire.

Exercise 3 Simulation

Objective: Find the frequency that will cause the wavelength to be one meter, thereby dislodging the hummingbirds.

Exercise 4 Simulation

Objective: Measure the speed of the wave and determine whether it depends on frequency or amplitude.

Conclusion: Wave speed in a wire does not depend on frequency or amplitude.

Exercise 5 Simulation

Objective: Create waves and use their frequency and measured wavelength to calculate their speed.

Exercise 6 Simulation

Objective: Gather data in order to calculate the speed of a wave in a thicker wire and compare it to that in a thinner wire with the same tension.

Conclusion: Waves move slower in thicker wires of the same tension.

Exercise 7 Simulation

Objective: Use the equation for the vertical displacement of a particle in a wire to create a wave of specific amplitude and frequency.

Answer: $y = 0.35 \sin(7.3x - 8.8t)$

Exercise 8 Simulation

Objective: Create a wave that has a vertical displacement of 1.0 m at $x = -4.0$ m and $t = 5.0$ s.

Answer: Many possible answers. One is $y = (1.0 \text{ m}) \sin(0.3x + 0.52t)$

Resonance

Questions

Exercise 1: Music and waves

1. As the string is shortened, the pitch goes up. The notes sound higher.
2. The frequency goes up when the string is shortened.
3. As frequency goes up, the perceived pitch goes up. The note sounds higher on the musical scale.

Exercise 2: Wave fundamentals: a review

4. $A = 1.0$ m, $\lambda = 2.0$ m, $f = 150$ Hz, $v = 300$ m/s

   The amplitude and wavelength can be measured using the scale on the graph. To calculate the frequency, advance the time step by a particular amount of time and observe how many cycles pass by (or alternately, note how much time it takes to advance the wave by one cycle).

   $$f = \frac{\text{cycles}}{\text{second}} = \frac{1.5 \text{ cycles}}{10 \text{ ms}} \times \frac{1000 \text{ s}}{1 \text{ s}}$$
   $$f = 150 \text{ Hz}$$

   To calculate the wave speed, note how much time is required to advance the wave a certain distance.

   $$v = \frac{\Delta x}{t} = \frac{3.0 \text{ m}}{10.0 \text{ ms}}$$
   $$v = 300 \text{ m/s}$$

5. Enter the values from the last question into the wave speed equation.

   $$v = \lambda f = (2.0 \text{ m})(150 \text{ Hz})$$
   $$v = 300 \text{ m/s}$$

   The value matches.

Exercise 3: Superposition

6. A wave pulse with twice the amplitude, 3.0 m, is momentarily created when the two pulses are in the same place.
7. The two waves momentarily cancel out when the two pulses are in the same place.

8. A wave pulse is momentarily created when the two pulses are in the same place; its amplitude is the sum of the amplitudes of the individual pulses.

Exercise 4: Reflection

9. The wave pulse is inverted.
10. The wave pulse's amplitude has the same magnitude, and its speed is unchanged.

Exercise 5: Reflection and standing waves

11. After about 100 ms, when the situation reaches a steady state, the wave stands in the same place. The horizontal location of the peaks stays the same over time.
12. Most of the particles are moving up and down, but there are points on the string that remain stationary. These stationary points are exactly between the particles that move with the most amplitude, and occur every 3.0 meters on the scale in the simulation.
13. The final waveform is the combination of a right-traveling wave and its equal-amplitude reflection. The final waveform has an amplitude that is greater than the amplitude of the original wave; it is twice the original.
14. The wavelength of the resulting wave is 6.0 m, the same as that of the original wave. The frequency of the resulting wave is 50 Hz, the same as that of the original wave.

Exercise 6: Standing waves and frequency

15. $f = 110$ Hz, $\lambda = 1.82$ m

    Use the equation for wave speed to calculate the wavelength.

    $$v = \lambda f$$

    $$\lambda = \frac{v}{f} = \frac{200\text{ m/s}}{110\text{ Hz}}$$

    $$\lambda = 1.82\text{ m}$$

16. 2/3

    Use the equation given in this exercise to express the ratio of string lengths.

    $$f = \frac{v}{2L}$$

    $$\frac{L_2}{L_1} = \frac{\frac{v}{2f_2}}{\frac{v}{2f_1}}$$

    $$\frac{L_2}{L_1} = \frac{f_1}{f_2}$$

    $$\frac{L_2}{L_1} = \frac{110\text{ Hz}}{165\text{ Hz}}$$

    $$\frac{L_2}{L_1} = \frac{2}{3}$$

Exercise 7: Harmonics

17. Place the finger at ¼ of the length. This increases the frequency over the fundamental frequency by a factor of 4.
18. To increase the frequency by a factor of 5, to 330 Hz, the finger should be placed 1/5 of the way along the string. This makes the wavelength 2/5 of the string length, which is a factor of 5 lower than the fundamental wavelength of 2 string lengths. Therefore the frequency is 5 times as great.

Exercise 8: Playing Beethoven's Fifth Symphony

Exercise 9: Playing the Physics Blues

Notes on the simulations

Exercise 1 Simulation

Objective: Experiment with the string, varying its length and listening to the sounds produced.

Exercise 2 Simulation

Objective: Measure the wave's amplitude, wavelength, frequency and speed.

Answer:

$A = 1.0$ m

$\lambda = 2.0$ m

$f = 150$ Hz

$v = 300$ m/s

Exercise 3 Simulation

Objective: Experiment with the superposition of two wave pulses.

Exercise 4 Simulation

Objective: Observe the reflection of a wave pulse and determine what properties (if any) of the pulse change as it reflects.

Conclusion: The wave pulse is inverted upon reflection, but nothing else about it changes.

Exercise 5 Simulation

Objective: Observe how standing waves are created by reflection.

Exercise 6 Simulation

Objective: Vary the length of the string and predict what frequency of standing wave will be produced.

Exercise 7 Simulation

Objective: Experiment with harmonics by placing the finger at different points along the string.

Exercise 8 Simulation

Objective: Play the opening to Beethoven's Fifth Symphony by setting each string to play the correct frequency.

Answer:

String 1: harmonic at 1/4

String 2: length of 5/6

String 3: length of 3/4

String 4: harmonic at 1/3

Solution:

String 1: To increase the frequency by a factor of 4, to 198 Hz, put the finger 1/4 the way along the string.

String 2: Use the equation obtained in question 15.

$$\frac{L_2}{L_1} = \frac{f_1}{f_2} = \frac{132 \text{ Hz}}{158.4 \text{ Hz}}$$

$$\frac{L_2}{L_1} = \frac{5}{6}$$

String 3:

$$\frac{L_2}{L_1} = \frac{f_1}{f_2} = \frac{132 \text{ Hz}}{176 \text{ Hz}}$$

$$\frac{L_2}{L_1} = \frac{3}{4}$$

String 4: To increase the frequency by a factor of 3, to 148.5 Hz, put the finger 1/3 of the way along the string.

Exercise 9 Simulation

Objective: Play the Physics Blues by setting the strings of a piano, guitar and bass to play the correct frequencies.

Answer: For most of the strings, there are two ways to set the correct frequency, by either changing the length of the string or by creating a harmonic. The tables below show one way to correctly tune each string.

Bass:

String #	Tuning method	Fundamental frequency	Target frequency	Musical note	Length fraction
1	Harmonic	16.5	66	C2	1/4
2	Change length	66	74.25	D2	8/9
3	Harmonic	16.5	82.5	E2	1/5
4	Change length	66	88	F2	3/4
5	Harmonic	16.5	99	G2	1/6
6	Change length	66	110	A2	3/5
7	Change length	66	118.8	B^b2	5/9
8	Change length	66	123.75	B2	8/15
9	Harmonic	16.5	132	C3	1/8
10	Harmonic	16.5	148.5	D3	1/9
11	Change length	66	158.4	E^b3	5/12

Guitar:

String #	Tuning method	Fundamental frequency	Target frequency	Musical note	Length fraction
1	Change length	66	88	F2	3/4
2	Harmonic	16.5	99	G2	1/6
3	Change length	66	110	A2	3/5
4	Change length	66	118.8	B^b2	5/9
5	Harmonic	16.5	132	C3	1/8
6	Harmonic	16.5	148.5	D3	1/9
7	Change length	66	158.4	E^b3	5/12
8	Change length	66	176	F3	3/8

Piano:

String #	Tuning method	Fundamental frequency	Target frequency	Musical note	Length fraction
1	Change length	33	123.75	B2	4/15
2	Harmonic	44	132	C3	1/3
3	Change length	132	148.5	D3	8/9
4	Change length	132	158.4	E^b3	5/6
5	Harmonic	33	165	E3	1/5
6	Harmonic	44	176	F3	1/4
7	Harmonic	66	198	G3	1/3
8	Change length	132	220	A3	3/5
9	Change length	132	237.6	B^b3	5/9

Refraction

Questions

Exercise 1: Refraction

1. The beam bends the most when the angle between the beam and a line perpendicular (normal) to the surface is the largest. The beam bends the least when the angle between the beam and the normal to the surface is smallest. It does not bend at all when fired straight down.

Exercise 2: Speed of light

2. From slowest to fastest: 3, 2, 1.
3. From most to least bending: 3, 2, 1.
4. The ray bends more as the speed of light decreases. There is a direct relationship between the speed of light in a material and how much the light ray changes direction when it enters the medium.

Exercise 3: Index of refraction

5. From lowest to highest index of refraction: 1, 2, 3. Since the light had the highest speed in medium 1, that medium must have the lowest index of refraction. The light had the lowest speed in medium 3, so that medium has the highest index of refraction.

Exercise 4: Angle of refraction

6. Line 1 is the normal line and Line 2 is the interface.
7. The light bent toward the normal.
8. Angle 1 is the angle of incidence and Angle 2 is the angle of refraction.
9. The medium of the submarine has a higher index of refraction. Since the angle in the sub's medium is less than the angle in the helicopter's medium, this means that the speed of the laser in the submarine's medium must have been less than the speed of the laser in the helicopter's medium. Since the index of refraction is inversely related to the speed of light in that medium, the index of refraction must be greater for the submarine's medium.

Exercise 5: Snell's law

10. The correct answers for the angle of incidence to hit each sub are below.

Sub	n of air, n_1	angle of incidence, θ_1	n of water, n_2	angle of refraction, θ_2
1	1.00	65.6°	1.33	43.2°
2	1.00	52.3°	1.33	36.5°
3	1.00	32.6°	1.33	23.9°

Use Snell's law to calculate the angle of incidence for Sub 1.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\sin\theta_1 = \frac{n_2 \sin\theta_2}{n_1}$$

$$\theta_1 = \arcsin\left(\frac{n_2 \sin\theta_2}{n_1}\right) = \arcsin\left(\frac{(1.33)\sin(43.2°)}{1.00}\right)$$

$$\theta_1 = 65.6°$$

Do the same for Sub 2.

$$\theta_1 = \arcsin\left(\frac{n_2 \sin\theta_2}{n_1}\right) = \arcsin\left(\frac{(1.33)\sin(36.5°)}{1.00}\right)$$

$$\theta_1 = 52.3°$$

Do the same for Sub 3.

$$\theta_1 = \arcsin\left(\frac{n_2 \sin\theta_2}{n_1}\right) = \arcsin\left(\frac{(1.33)\sin(23.9°)}{1.00}\right)$$

$$\theta_1 = 32.6°$$

Exercise 6: Double refraction

11. Once a sub is hit, the information on the screen can be used to calculate the index of refraction for each of the mediums. Begin by using Snell's law to find the index of refraction of the brown medium. Then use the index of refraction of the brown medium and Snell's law to calculate the index of refraction for the light blue medium. Finally use the index of refraction equation to find the speed of light in each material.

12. $1.50{\times}10^8$ m/s

Find the index of refraction of the brown goop using Snell's law and then use the index of refraction equation to find the speed of light in the brown goop. The angles used to calculate the speed will depend on how the student fires the laser. In this solution we use $\theta_{air} = 28.9°$ and $\theta_{bg} = 14.0°$.

$$n_{air}\sin\theta_{air} = n_{bg}\sin\theta_{bg}$$

$$n_{bg} = \frac{n_{air}\sin\theta_{air}}{\sin\theta_{bg}}$$

$$n_{bg} = \frac{1.00\sin(28.9°)}{\sin(14.0°)}$$

$$n_{bg} = 2.00$$

$$n_{bg} = \frac{\text{speed of light in vacuum}}{\text{speed of light in brown goop}}$$

$$v_{bg} = \frac{c}{n_{bg}}$$

$$v_{bg} = \frac{299{,}792 \text{ km/s}}{2.00}$$

$$v_{bg} = 1.50\times10^8 \text{ m/s}$$

13. $2.50{\times}10^8$ m/s

Use the calculated index of refraction of the brown goop in Snell's law to determine the index of refraction of the light blue medium. Then use the definition of the index of refraction to calculate the speed of light in the light blue medium. The angles used to calculate the answer may differ.

$$n_{lb} = \frac{n_{bg}\sin\theta_{bg}}{\sin\theta_{lb}}$$

$$n_{lb} = \frac{(2.00)\sin(14.0°)}{\sin(23.8°)}$$

$$n_{lb} = 1.20$$

$$n_{lb} = \frac{\text{speed of light in vacuum}}{\text{speed of light in light blue medium}}$$

$$v_{lb} = \frac{c}{n_{lb}}$$

$$v_{lb} = \frac{299{,}792 \text{ km/s}}{1.20}$$

$$v_{lb} = 2.50\times10^8 \text{ m/s}$$

Exercise 7: Total internal reflection

14. $48.8°$

Use the definition of the critical angle.

$$\theta_{\text{critical}} = \arcsin\left(\frac{n_{\text{fast}}}{n_{\text{slow}}}\right)$$

$$\theta_{\text{critical}} = \arcsin\left(\frac{1.00}{1.33}\right)$$

$$\theta_{\text{critical}} = 48.8°$$

Notes on the simulations

Exercise 1 Simulation

Objective: Experiment with refraction. Try to hit each of the submarines with a laser beam before they hit your helicopter.

Exercise 2 Simulation

Objective: Observe the different speeds of the laser beams in different media and how much they are bent at the interface.

Exercise 4 Simulation

Objective: Try to hit the submarine to practice the terminology of refraction.

Exercise 5 Simulation

Objective: Use Snell's law to calculate the correct angle of incidence to hit each submarine with a laser.

Answer:

Sub 1: $65.6°$

Sub 2: $52.3°$

Sub 3: $32.6°$

Exercise 6 Simulation

Objective: Fire your laser through two unknown media to hit the submarines. Use the information given after a hit to determine the speed of light in each medium.

Exercise 7 Simulation

Objective: Hit the other submarine by shooting at the critical angle or at a slightly greater angle.

Answer: The critical angle is $48.8°$.