Dynamic Programming

- A paradigm to design algorithms.

- We want to compute the *optimal solution* of a problem.

- We divide the problem into sub-problems of smaller size.

- We calculate *optimal solutions* of sub-problems, which *will be pieces* of the *optimal solution* of the original problem

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- Sub-problems may overlap! They may appear again and again… Think about the Fibonacci problem F(n) = F(n-1) + F(n-2)

- Let’s keep a table of solutions to store *optimal solutions* of sub-problems. (In this way, we calculate the solution of a sub-problem just once!)

Knapsack Problem

Item:					
Weight:	10	2	4	3	11
Value:	20	8	14	13	35

...and we have a knapsack with (max. weight) capacity 10

Which items should we insert in the knapsack to get
The maximum possible value without exceeding the capacity?

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Unbounded knapsack → assume infinite copies of items

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weight=10
value=42

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Two variants of this problem!

Unbounded knapsack → assume infinite copies of items

weight=10
value=42

0/1 Knapsack → assume only one copy of each item

weight=9
value=35

(Unbounded) Knapsack Problem

Let us assume n classes of items, and a knapsack of capacity x.

Item of class i has weight w_i and value v_i

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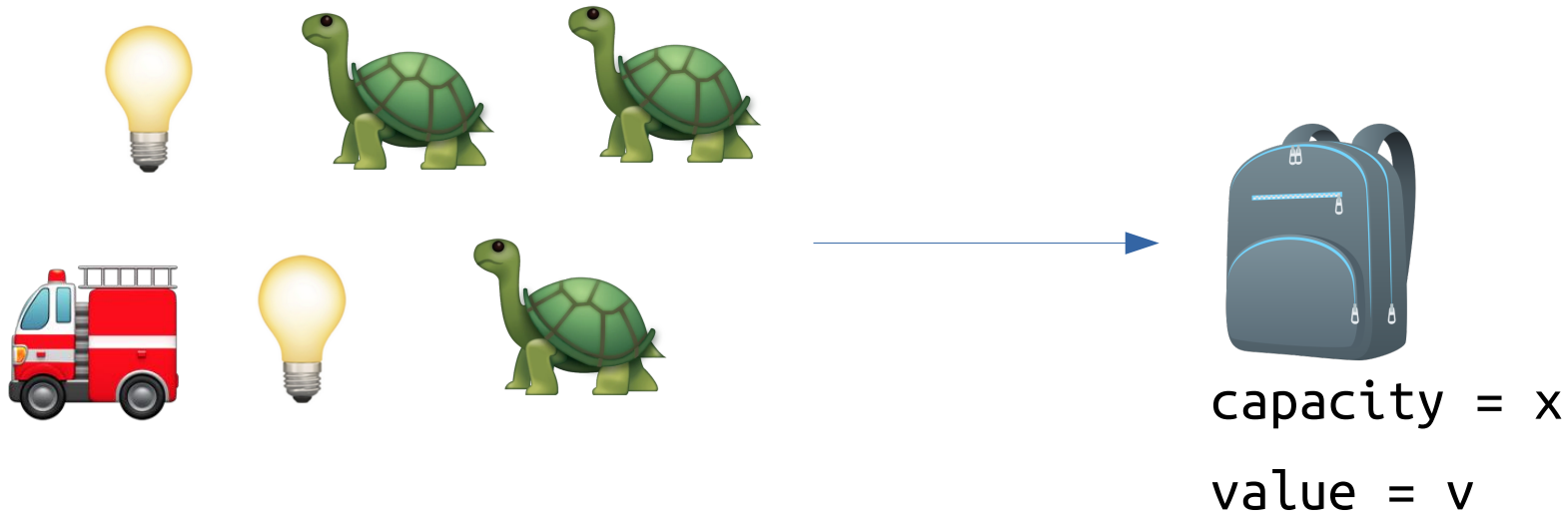
Suppose this is the optimal solution for capacity x which has at least one copy of item i



(Unbounded) Knapsack Problem

Let us assume n classes of items, and a knapsack of capacity x.

Item of class i has weight w_i and value v_i

Then, this is the optimal solution for capacity x - w_i which has at least one copy of item i

capacity = x - w_i

value = v - v_i

(Unbounded) Knapsack Problem

Let K(x) be the *optimal value* for a knapsack with capacity x

$K(x) = \max_i($  +  $)$

Optimal value for a *smaller* knapsack with capacity $x-w_i$

Value of item i

where item i is the item whose value maximizes this formula, such that its weight $w_i \leq x$

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where item i is the item whose value maximizes this formula, such that its weight $w_i \leq x$

$$K(x) = \max_i(K(x-w_i) + v_i)$$

and K(x)=0 if there is no item i such that its weight $w_i \leq x$

(Unbounded) Knapsack Problem – pseudocode

K[x]: optimal value for a knapsack of capacity x

```
unboundedKnapsack(w, n, weights, values)
{
    K[0] = 0;

    for x = 1,...,w:
        K[x] = 0;
        for i = 1,...,n:
            if(w_i <= x):
                K[x] = max(K[x],K[x-w_i] + v_i);

    return K[w];
}
```

(Unbounded) Knapsack Problem – pseudocode

K[x]: optimal value for a knapsack of capacity x

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unboundedKnapsack(w, n, weights, values)
{
    K[0] = 0;
    ITEMS[0] = ∅;

    for x = 1,...,w:
        K[x] = 0;
        ITEMS[x] = ∅;
        for i = 1,...,n:
            if(w_i <= x):
                K[x] = max(K[x],K[x-w_i] + v_i);
            if(K[x] was updated):
                ITEMS[x] = ITEMS[x-w_i] U { item i };

    return K[w];
}
```