Counting Sort

Sort an array of n integers, where each integer is in the range [0…k]

n = 8   k = 5

```
        0  1  2  3  4  5  6  7
a:    | 2| 5| 3| 0| 2| 3| 0| 3|
```

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```
      0 1 2 3 4 5 6 7
a:  | 2| 5| 3| 0| 2| 3| 0| 3|
```

```
      0 1 2 3 4 5
c:  | 0| 0| 0| 0| 0| 0|
```

$c[i]$: number of elements equal to i

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a: 2  5  3  0  2  3  0  3
```

```
   0  1  2  3  4  5
c: 2  0  2  3  0  1
```

$c[i]$: number of elements equal to i

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    0 1 2 3 4 5 6 7
a:  2 5 3 0 2 3 0 3
```

```
    0 1 2 3 4 5
c:  2 0 2 3 0 1
```

Update array c
c[i] will be the number of elements equal or less than i
c[i] = c[i] + c[i - 1]

```
    0 1 2 3 4 5
c:  2 ? ? ? ? ?
```

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```
     0 1 2 3 4 5 6 7
a:  | 2 | 5 | 3 | 0 | 2 | 3 | 0 | 3 |
```

```
     0 1 2 3 4 5
c:  | 2 | 0 | 2 | 3 | 0 | 1 |
```

⬇ Update array c
c[i] will be the number of elements equal or less than i
c[i] = c[i] + c[i - 1]

```
     0 1 2 3 4 5
c:  | 2 | 2 | 4 | 7 | 7 | 8 |
```

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```
        0 1 2 3 4 5 6 7              0 1 2 3 4 5
a:    | 2 5 3 0 2 3 0 3 |      c:  | 2 2 4 7 7 8 |
```

c[i] : number of elements equal or less than i

```
        0 1 2 3 4 5 6 7
b:    | 0 0 0 0 0 0 0 0 |
```

sorted array

```
for(j = n – 1; j >= 0; j--)
  insert a[j] in position c[a[j]] – 1 of array b;
  c[a[j]] ← c[a[j]] – 1;
```

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```
       0 1 2 3 4 5 6 7                    0 1 2 3 4 5
  a:  | 2 | 5 | 3 | 0 | 2 | 3 | 0 | 3 |    c:  | 2 | 2 | 4 | 7 | 7 | 8 |
                                                            6
                              c[i] : number of elements equal or less than i


       0 1 2 3 4 5 6 7
  b:  | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 0 |
```

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```
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a:  | 2| 5| 3| 0| 2| 3| 0| 3|    c: | 2| 2| 4| 6| 7| 8|
                                       1
                                   c[i] : number of elements equal or less than i
```

```
      0 1 2 3 4 5 6 7
b:  | 0| 0| 0| 0| 0| 0| 3| 0|
```

sorted array

```
for(j = n – 1; j >= 0; j--)
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```
        0 1 2 3 4 5 6 7                    0 1 2 3 4 5
a:     │2│5│3│0│2│3│0│3│         c:      │1│2│4│6│7│8│
        └─┴─┴─┴─┴─┴─┴─┴─┘                 └─┴─┴─┴─┴─┴─┘
                                                   /
                                                  5

                                c[i] : number of elements equal or less than i


        0 1 2 3 4 5 6 7
b:     │0│0│0│0│0│3│3│0│
        └─┴─┴─┴─┴─┴─┴─┴─┘

 sorted array

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```
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a:    | 2| 5| 3| 0| 2| 3| 0| 3|       c:  | 1| 2| 4| 5| 7| 8|
                                                    3
```

c[i] : number of elements equal or less than i

```
         0  1  2  3  4  5  6  7
b:    | 0| 0| 0| 2| 0| 3| 3| 0|
```

sorted array

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    insert a[j] in position c[a[j]] – 1 of array b;
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Sort an array of n integers, where each integer is in the range [0…k]

n = 8   k = 5

```
       0 1 2 3 4 5 6 7              0 1 2 3 4 5
a:    | 2 5 3 0 2 3 0 3 |      c:  | 1 2 3 5 7 8 |
                                     0

                              c[i] : number of elements equal or less than i


       0 1 2 3 4 5 6 7
b:    | 0 0 0 2 0 3 3 0 |

  sorted array

    for(j = n – 1; j >= 0; j--)
      insert a[j] in position c[a[j]] – 1 of array b;
      c[a[j]] ← c[a[j]] – 1;
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Counting Sort

Sort an array of n integers, where each integer is in the range [0…k]

n = 8   k = 5

```
      0 1 2 3 4 5 6 7              0 1 2 3 4 5
a:  | 2 5 3 0 2 3 0 3 |      c: | 0 2 3 5 7 8 |
                                          4

              c[i] : number of elements equal or less than i
```

```
      0 1 2 3 4 5 6 7
b:  | 0 0 0 2 3 3 3 0 |
```

sorted array

```
for(j = n – 1; j >= 0; j--)
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a:  | 2 5 3 0 2 3 0 3 |      c:  | 0 2 3 4 7 8 |
                                                 7

                               c[i] : number of elements equal or less than i
```

```
      0 1 2 3 4 5 6 7
b:  | 0 0 0 2 3 3 3 5 |
```

sorted array

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for(j = n – 1; j >= 0; j--)
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Counting Sort

Sort an array of n integers, where each integer is in the range [0…k]

n = 8   k = 5

```
      0 1 2 3 4 5 6 7               0 1 2 3 4 5
a:  | 2 5 3 0 2 3 0 3 |      c:   | 0 2 3 4 7 7 |
                                          /
                                         2
                          c[i] : number of elements equal or less than i
```

```
      0 1 2 3 4 5 6 7
b:  | 0 0 2 2 3 3 3 5 |
```

sorted array

```
for(j = n - 1; j >= 0; j--)
   insert a[j] in position c[a[j]] - 1 of array b;
   c[a[j]] ← c[a[j]] - 1;
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Counting Sort

Sort an array of n integers, where each integer is in the range [0...k]

```
        0 1 2 3 4 5 6 7
a:      2 5 3 0 2 3 0 3

        0 1 2 3 4 5
c:      2 0 2 3 0 1

            ↓

        0 1 2 3 4 5
c:      2 2 4 7 7 8

        0 1 2 3 4 5 6 7
b:      0 0 2 2 3 3 3 5
```

```
1   void countingSort(int a[], int b[], int n, int k)
2   {
3       int c[k + 1];
4
5       for(int i = 0; i <= k; i++)
6           c[i] = 0;
7
8       for(int j = 0; j < n; j++)
9           c[a[j]]++;
10
11      // c[i]: number of elements equal to i
12
13      for(int i = 1; i <= k; i++)
14          c[i] = c[i] + c[i-1];
15
16      // c[i]: number of elements less or equal to i
17
18      for(int j = n - 1; j >= 0; j--)
19      {
20          b[c[a[j]] - 1] = a[j];
21          c[a[j]]--;
22      }
23  }
```

Counting Sort

Sort an array of n integers, where each integer is in the range [0…k]

O(n) provided that k is less or equal than n

```
   0 1 2 3 4 5 6 7
a: 2 5 3 0 2 3 0 3

   0 1 2 3 4 5
c: 2 0 2 3 0 1

   ↓

   0 1 2 3 4 5
c: 2 2 4 7 7 8

   0 1 2 3 4 5 6 7
b: 0 0 2 2 3 3 3 5
```

```c
1   void countingSort(int a[], int b[], int n, int k)
2   {
3       int c[k + 1];
4
5       for(int i = 0; i <= k; i++)
6           c[i] = 0;
7
8       for(int j = 0; j < n; j++)
9           c[a[j]]++;
10
11      // c[i]: number of elements equal to i
12
13      for(int i = 1; i <= k; i++)
14          c[i] = c[i] + c[i-1];
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18      for(int j = n - 1; j >= 0; j--)
19      {
20          b[c[a[j]] - 1] = a[j];
21          c[a[j]]--;
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```