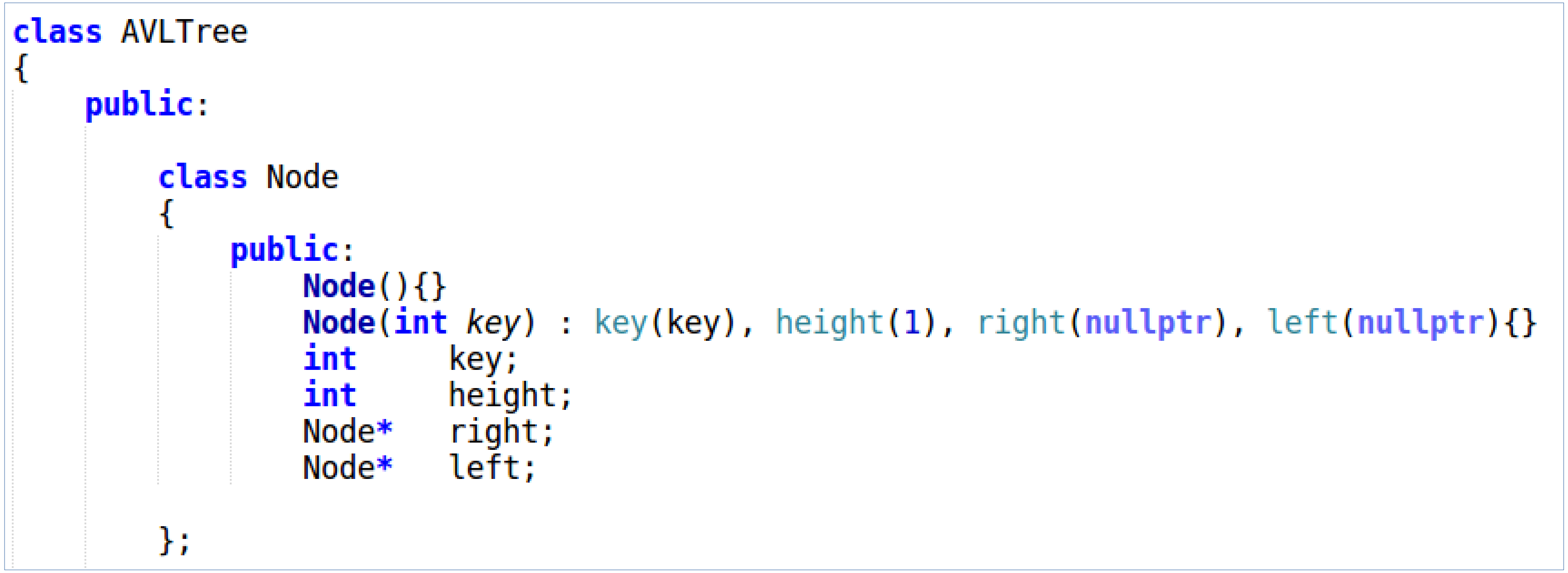
AVL – “self-balancing” binary tree

The heights h of the two child subtrees of any node x differ at most by one.

```
Balance Factor bf(x) = (x→left→height) – (x→right→height)

bf(x) must be -1, 0, or 1
```

After an insertion/deletion, if the heights of left and right subtrees of node x differ more than one, the tree must automatically rebalance.



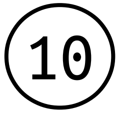
```
class AVLTree
{
    public:

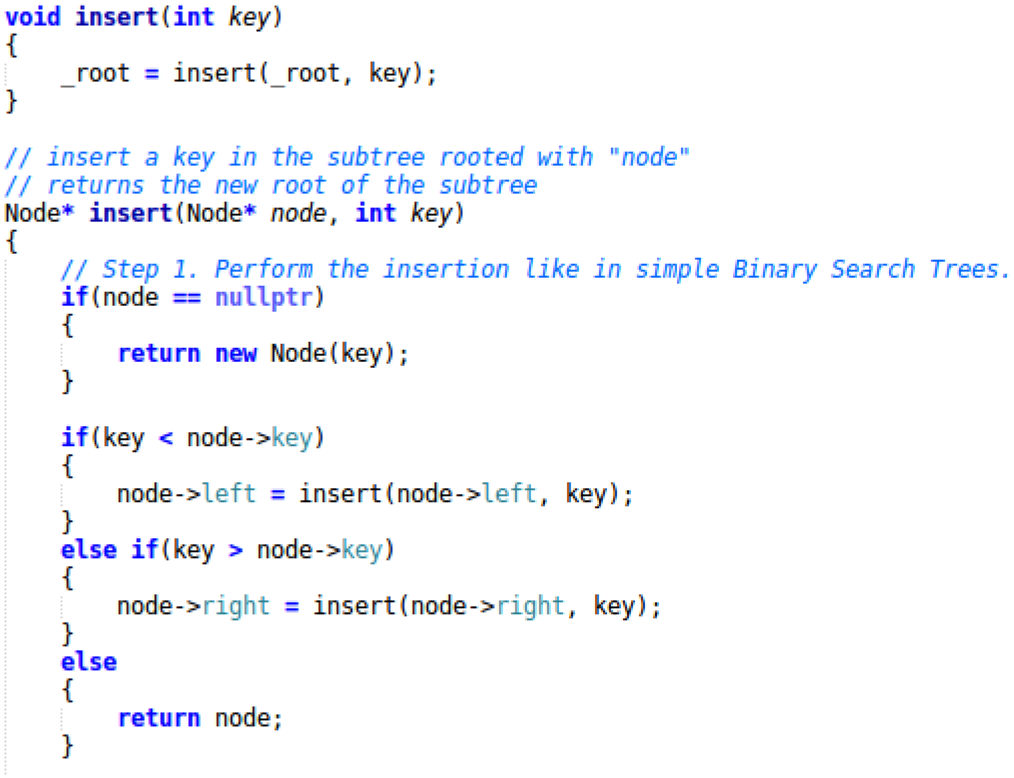
        class Node
        {
            public:
                Node(){}
                Node(int key) : key(key), height(1), right(nullptr), left(nullptr){}
                int     key;
                int     height;
                Node*   right;
                Node*   left;

        };
```

AVL

Insert 10 20 30





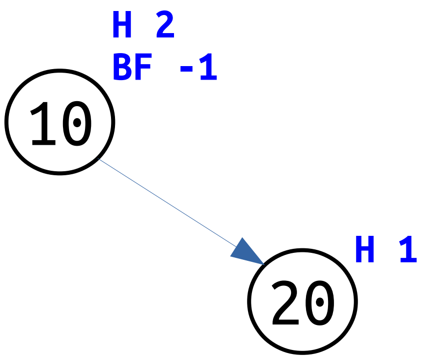
```
void insert(int key)
{
    _root = insert(_root, key);
}

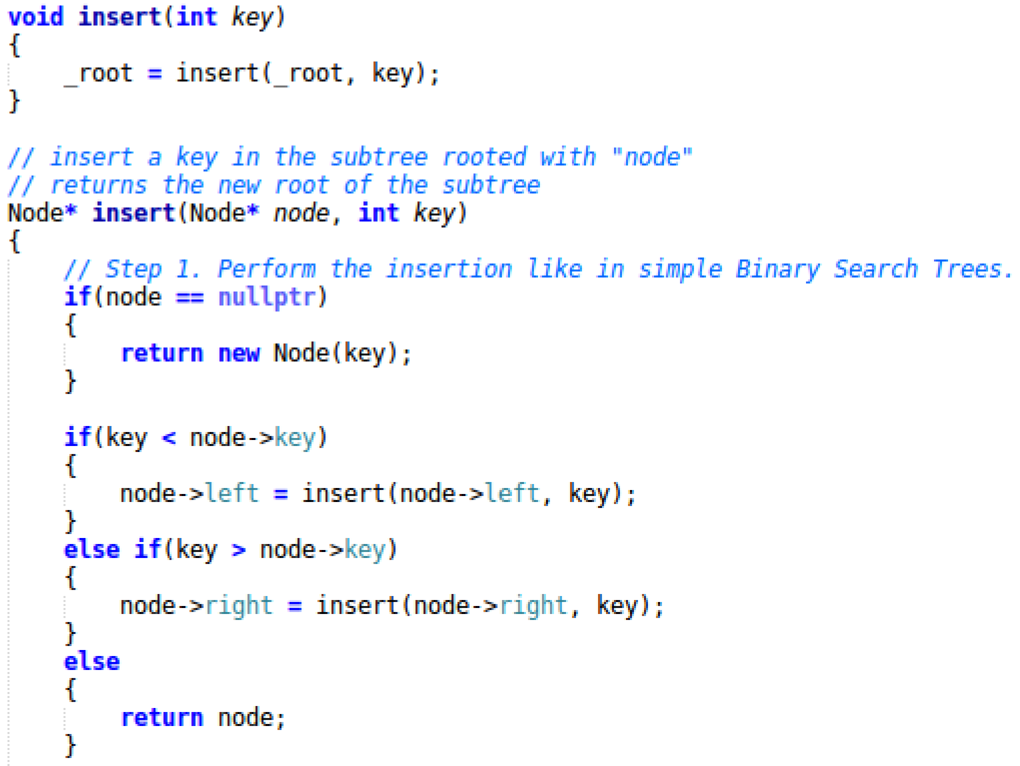
// insert a key in the subtree rooted with "node"
// returns the new root of the subtree
Node* insert(Node* node, int key)
{
    // Step 1. Perform the insertion like in simple Binary Search Trees.
    if(node == nullptr)
    {
        return new Node(key);
    }

    if(key < node->key)
    {
        node->left = insert(node->left, key);
    }
    else if(key > node->key)
    {
        node->right = insert(node->right, key);
    }
    else
    {
        return node;
    }
```

AVL

Insert 10 20 30





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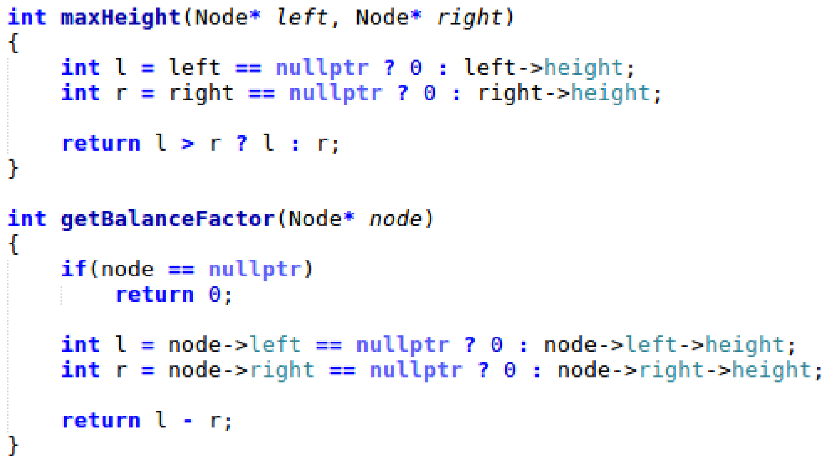
    }
    else
    {
        return node;
    }
```

```
// Step 2. Update height of this ancestor
node->height = 1 + maxHeight(node->left,node->right);

// Step 3. Check the balance factor of this ancestor
int bf = getBalanceFactor(node);
```



```
int maxHeight(Node* left, Node* right)
{
    int l = left == nullptr ? 0 : left->height;
    int r = right == nullptr ? 0 : right->height;

    return l > r ? l : r;
}

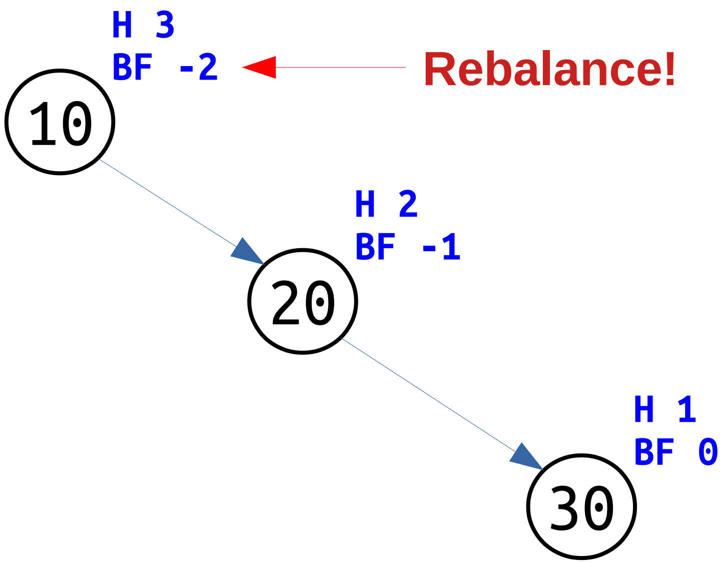
int getBalanceFactor(Node* node)
{
    if(node == nullptr)
        return 0;

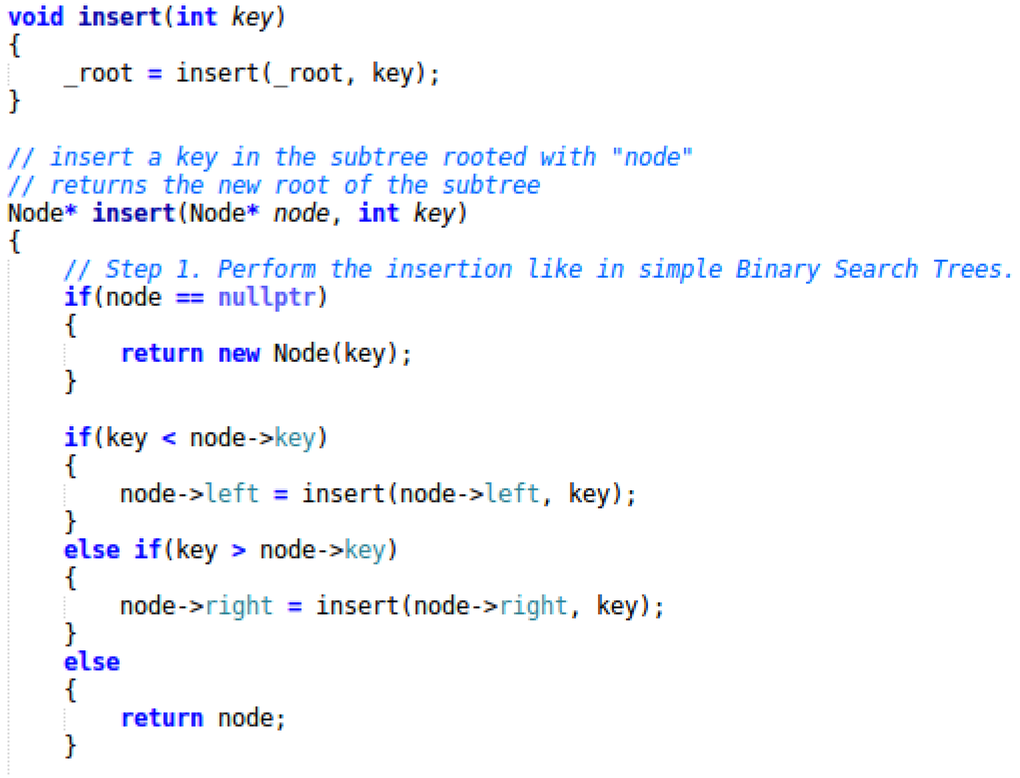
    int l = node->left == nullptr ? 0 : node->left->height;
    int r = node->right == nullptr ? 0 : node->right->height;

    return l - r;
}
```

AVL

Insert 10 20 30





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int bf = getBalanceFactor(node);
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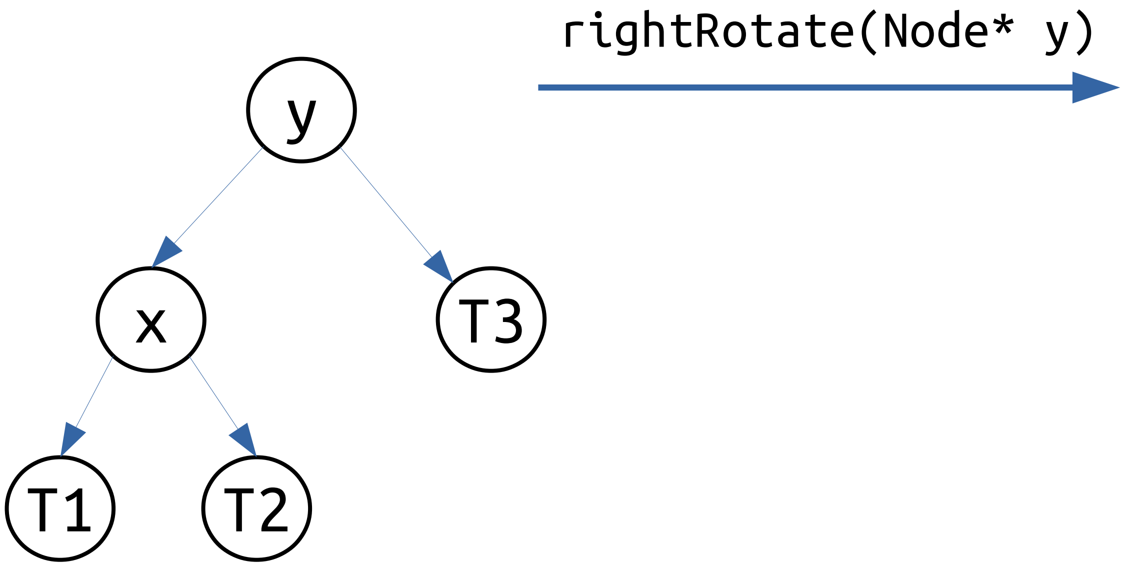
```
// Step 4. If the subtree is unbalanced, check in which case we are, and balance!
```

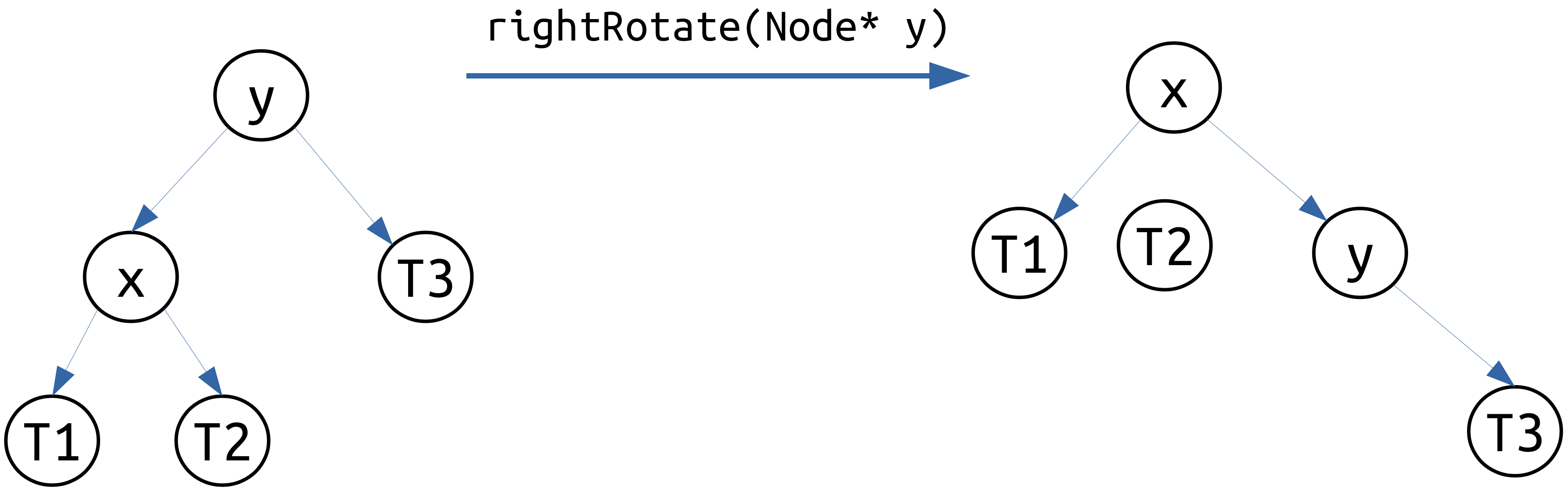
AVL - Rotations



rightRotate(Node* y)

```
Node* rightRotate(Node *y)
//1.  Let x be the left child of y
//2.  Let t2 be the right child of x
--- rotation step ---
//3. The right child of x will be y
//4.  The left child of y will be T2
--- update heights ---
//5. height of y =(max height of his children) + 1
//6. height of x =(max height of x's children) + 1
return x;
```

AVL - Rotations



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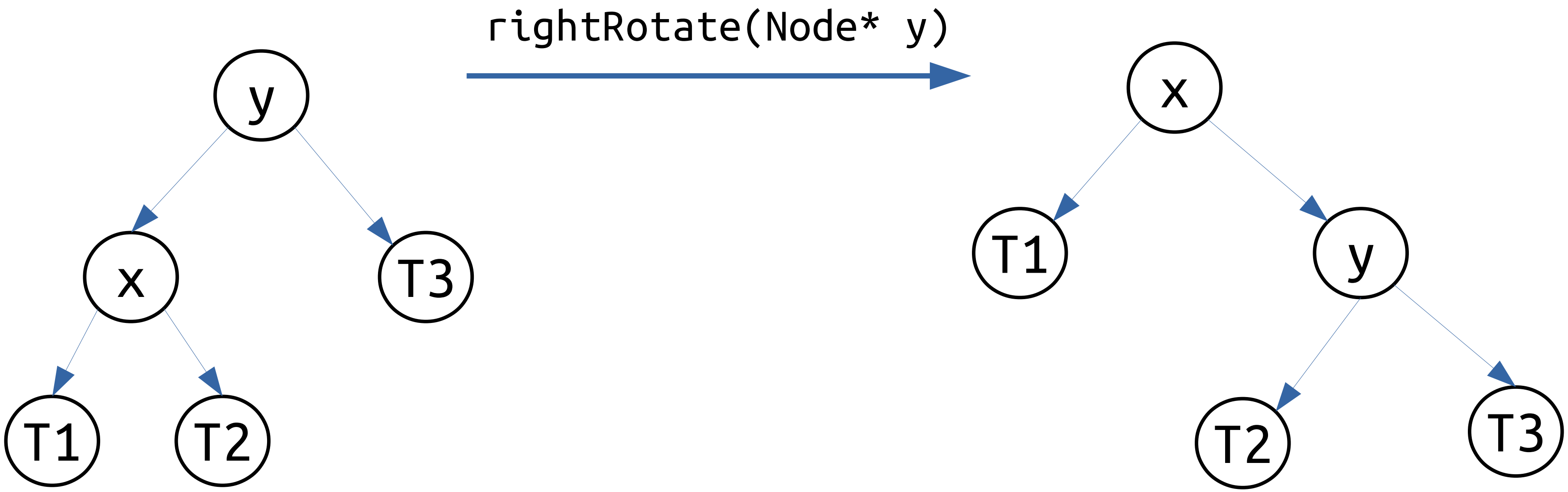
AVL - Rotations



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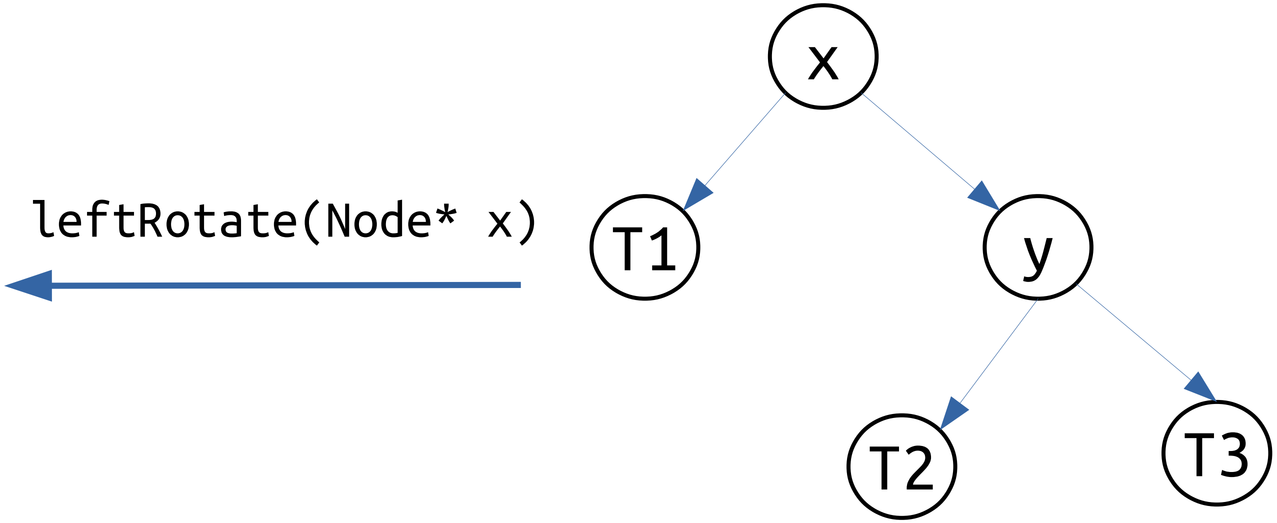
AVL - Rotations



leftRotate(Node* x)

```
Node* leftRotate(Node *x)
//1.  Let y be the right child of x

//2.  Let t2 be the left child of x

--- rotation step ---

//3. The left child of y will be x

//4.  The right child of x will be t2

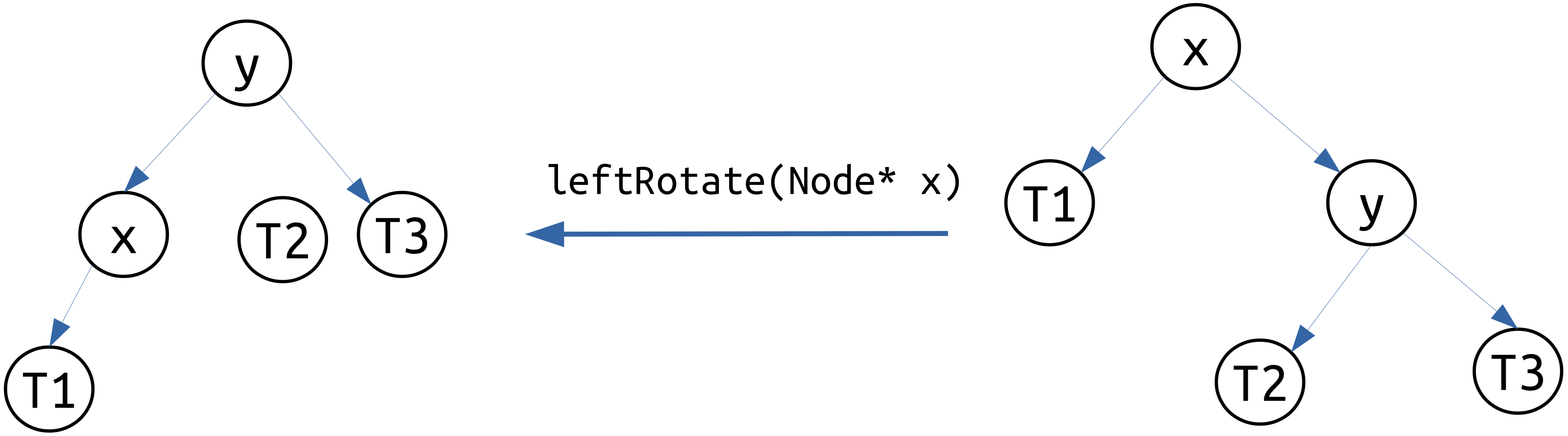
--- update heights ---

//5. height of x =(max height of x’s children) + 1

//6. height of y =(max height of y’s children) + 1

return y;
```

AVL - Rotations



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Node* leftRotate(Node *x)

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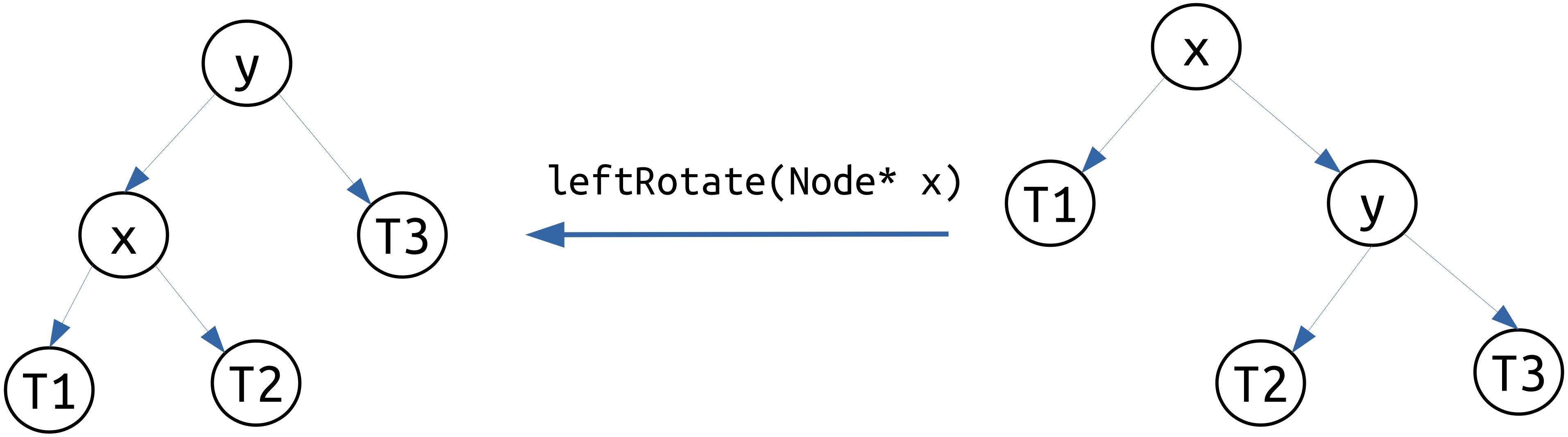
return y;
```

AVL - Rotations



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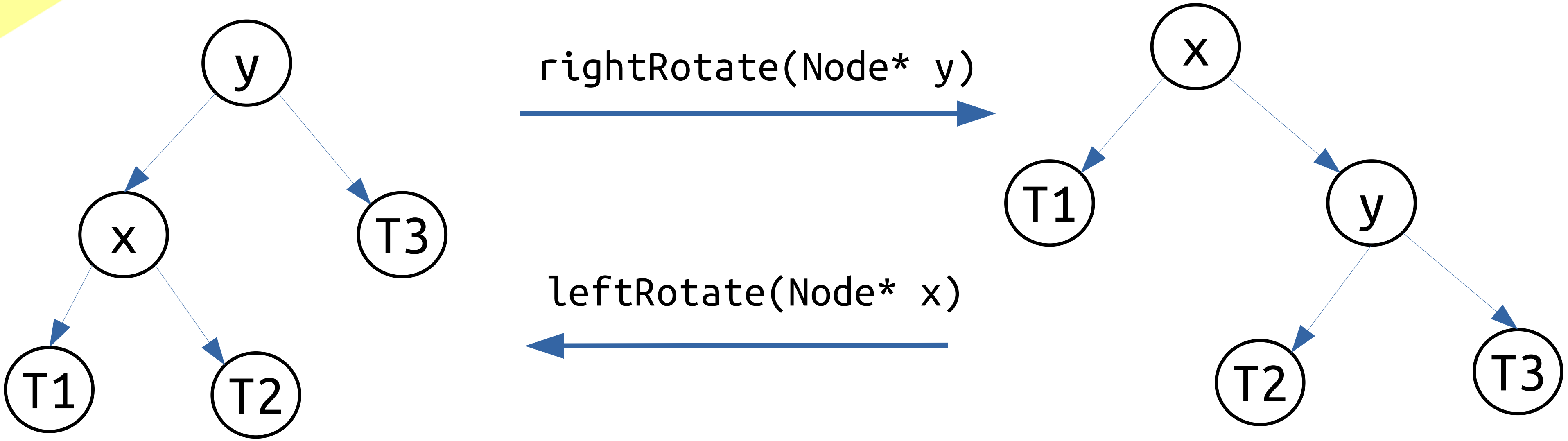
--- update heights ---
//5. height of x =(max height of x’s children) + 1
//6. height of y =(max height of y’s children) + 1
return y;
```

Important

AVL - Rotations



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--- update heights ---

//5. height of y =(max height of y’s children) + 1

//6. height of x =(max height of x’s children) + 1

return x;
```

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Node* leftRotate(Node *x)
//1.  Let y be the right child of x

//2.  Let t2 be the left child of x

--- rotation step ---

//3. The left child of y will be x

//4.  The right child of x will be t2

--- update heights ---

//5. height of x =(max height of x’s children) + 1

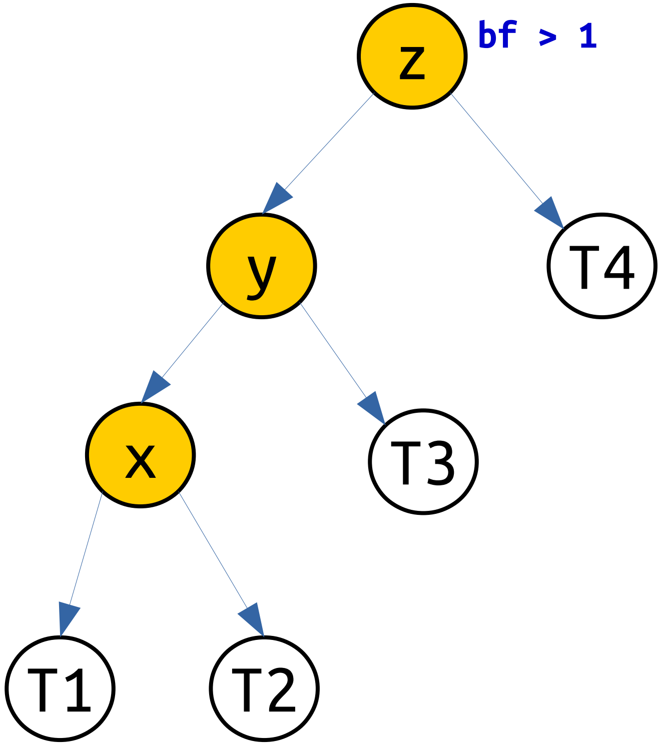
//6. height of y =(max height of y’s children) + 1

return y;
```

AVL - Cases

Case 1. LEFT LEFT

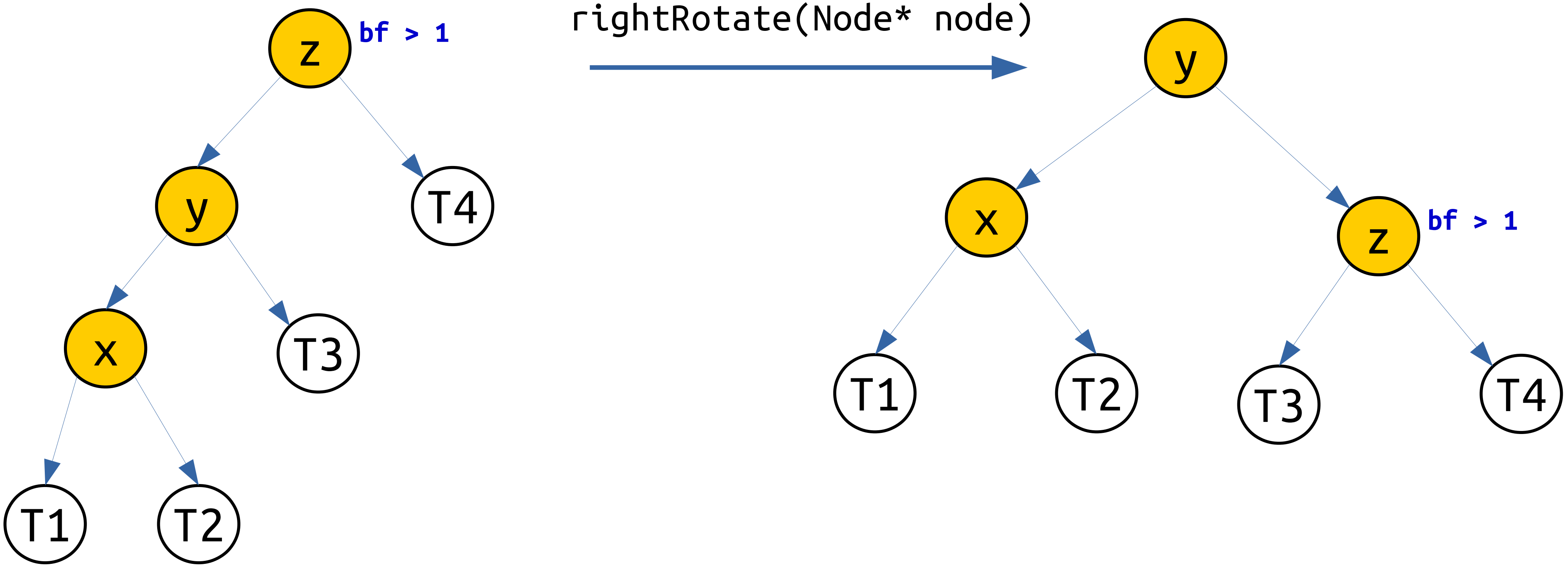
`bf > 1 and key < node->left->key`



AVL - Cases

Case 1. LEFT LEFT

bf > 1 and key < node->left->key



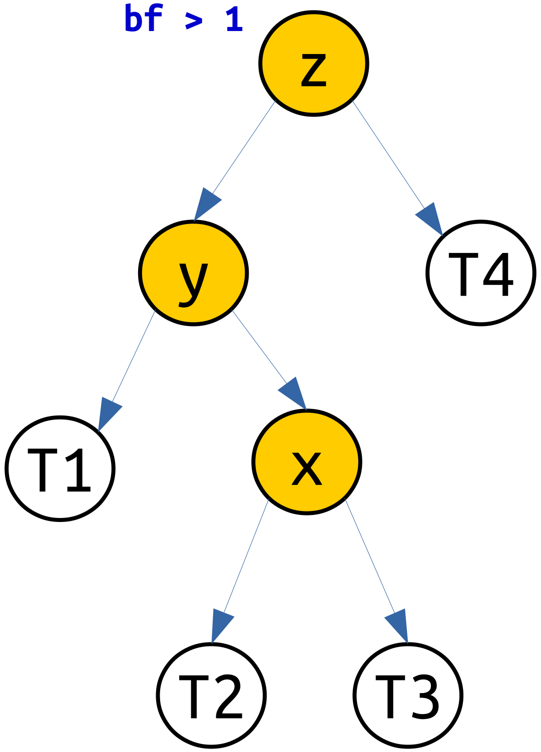
rightRotate(Node* node)

AVL - Cases

Case 2. LEFT RIGHT

bf > 1 and key > node→left->key

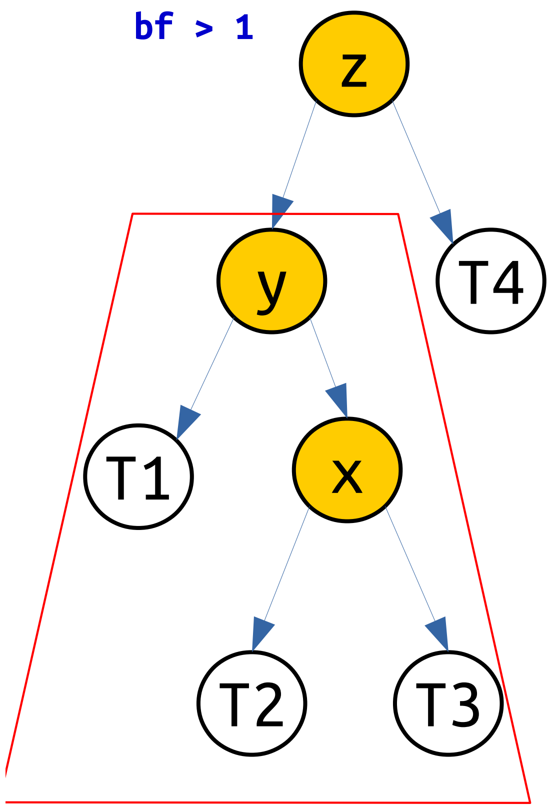
bf > 1



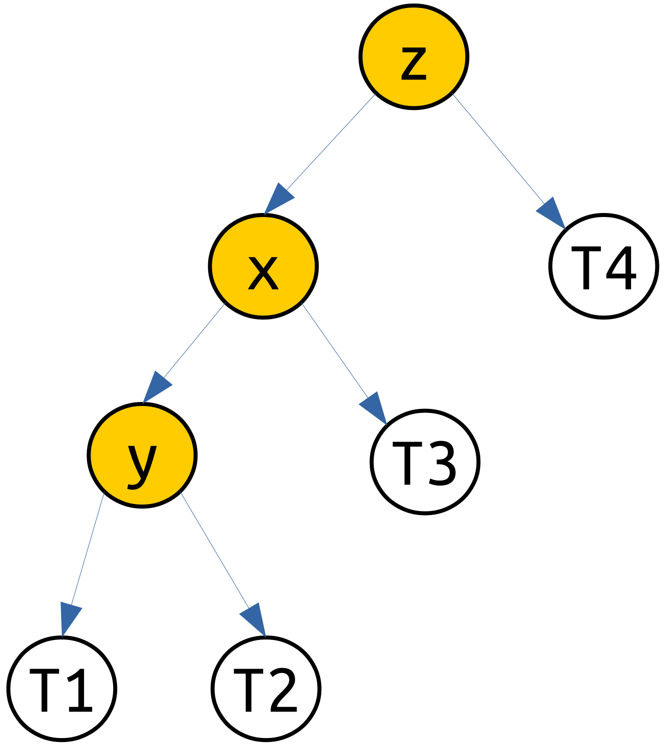
AVL - Cases

Case 2. LEFT RIGHT

bf > 1 and key > node→left->key



leftRotate(node->left)



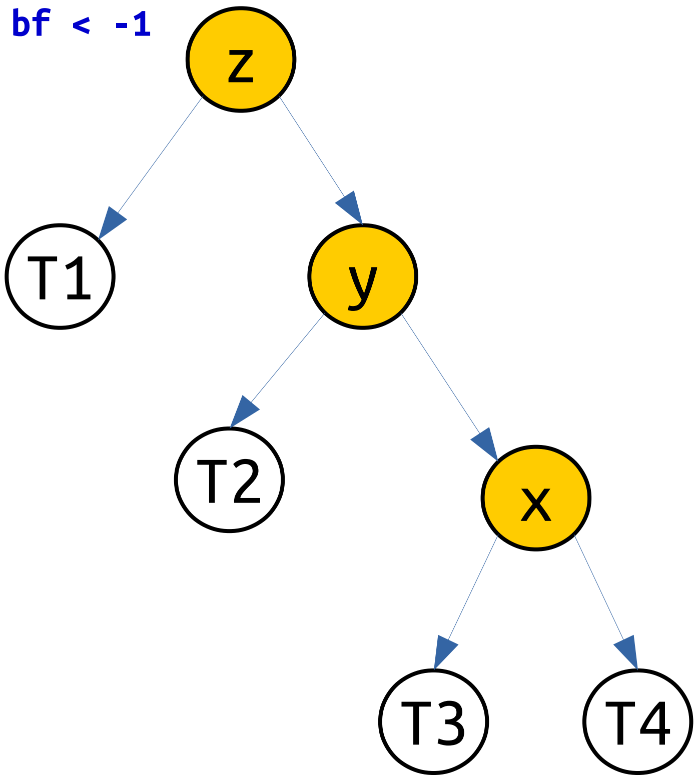
Now, this looks like Case 1. LEFT LEFT

Do right rotation on node z.

AVL - Cases

Case 3. RIGHT RIGHT

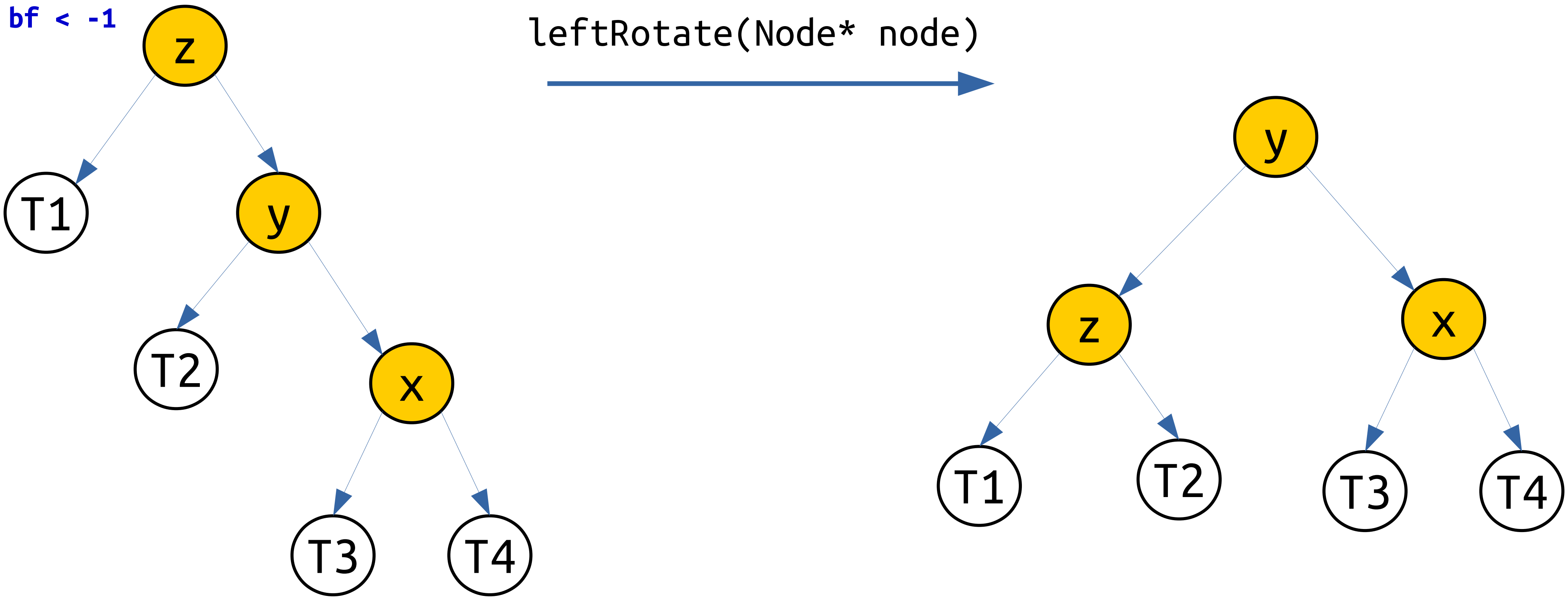
bf < -1 and key > node→right->key



AVL - Cases

Case 3. RIGHT RIGHT

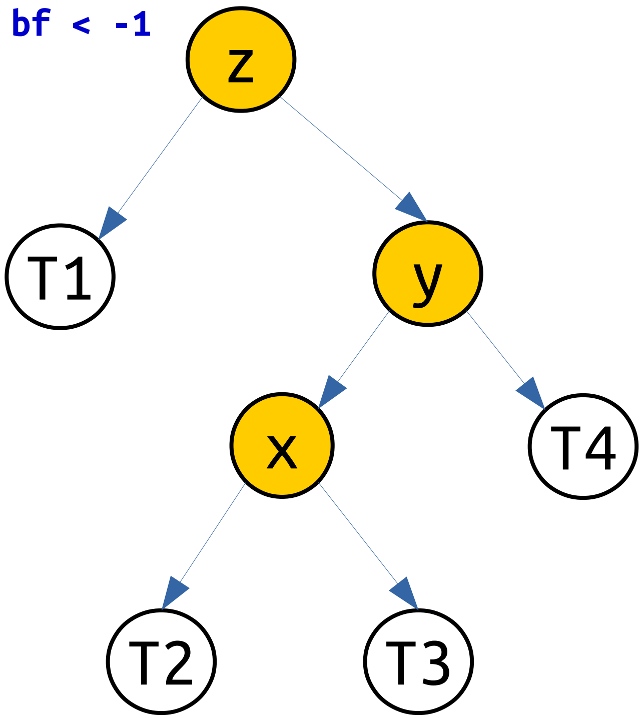
bf < -1 and key > node→right->key



AVL - Cases

Case 4. RIGHT LEFT

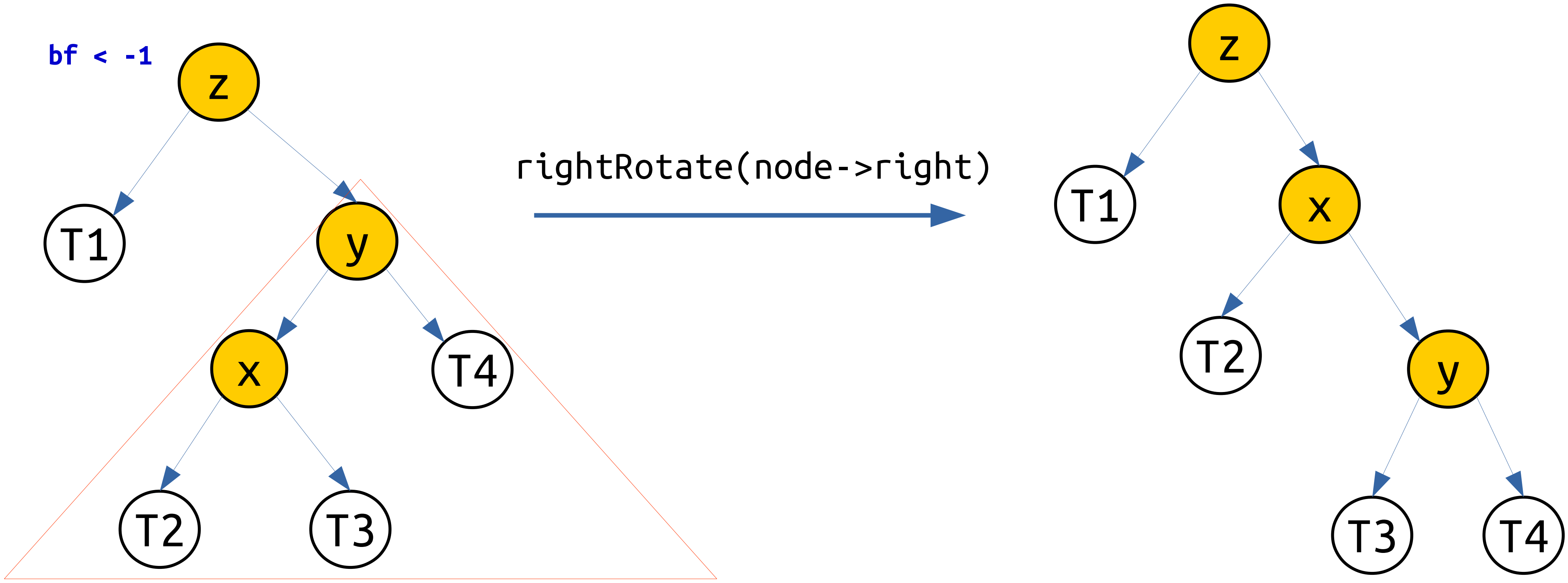
bf < -1 and key < node→right->key



AVL - Cases

Case 4. RIGHT LEFT

bf < -1 and key < node→right->key

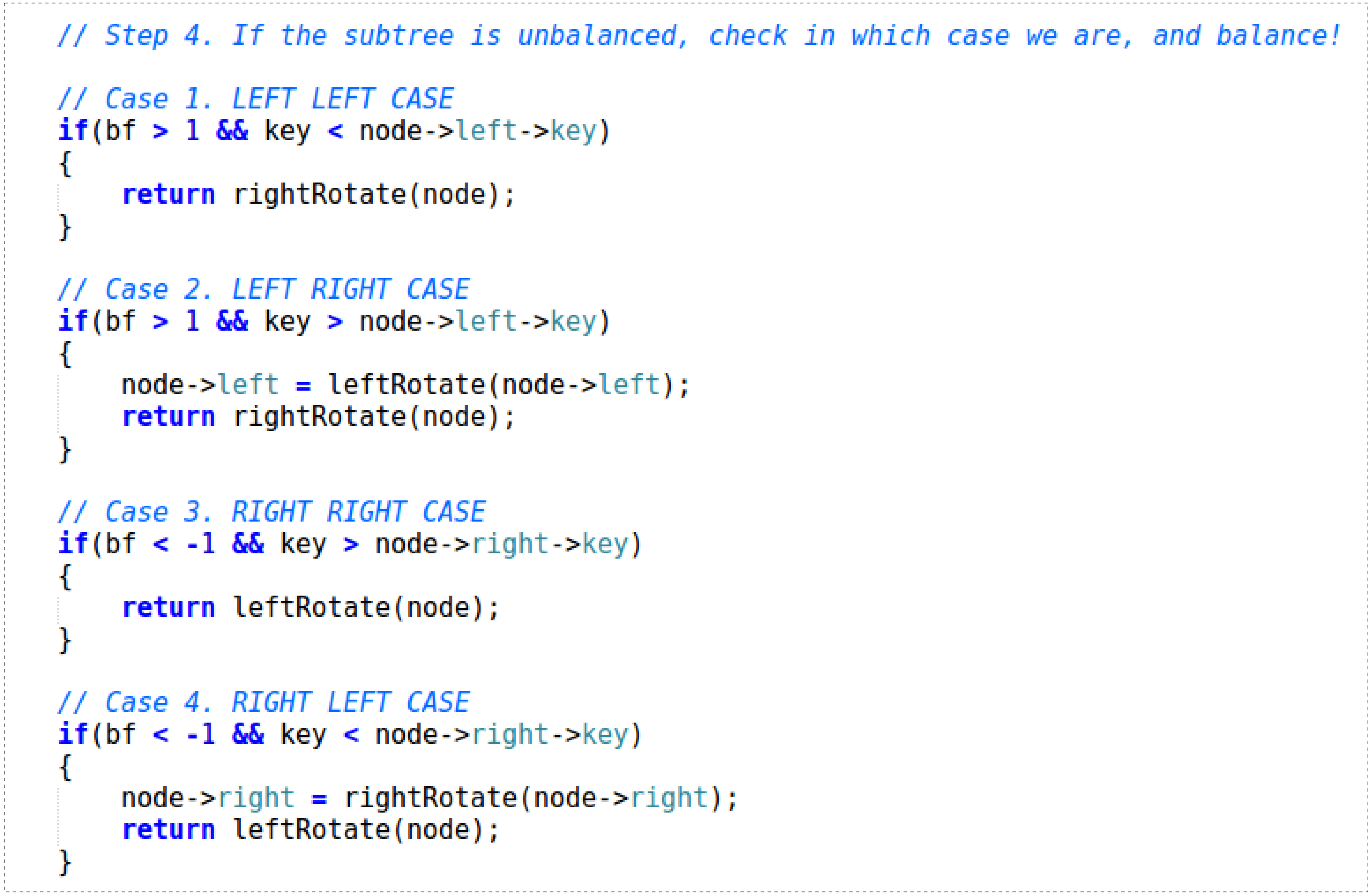


rightRotate(node->right)

Now, this looks like Case 3. RIGHT RIGHT

Do left rotation on node z.

AVL - Cases



```
// Step 4. If the subtree is unbalanced, check in which case we are, and balance!

// Case 1. LEFT LEFT CASE
if(bf > 1 && key < node->left->key)
{
    return rightRotate(node);
}

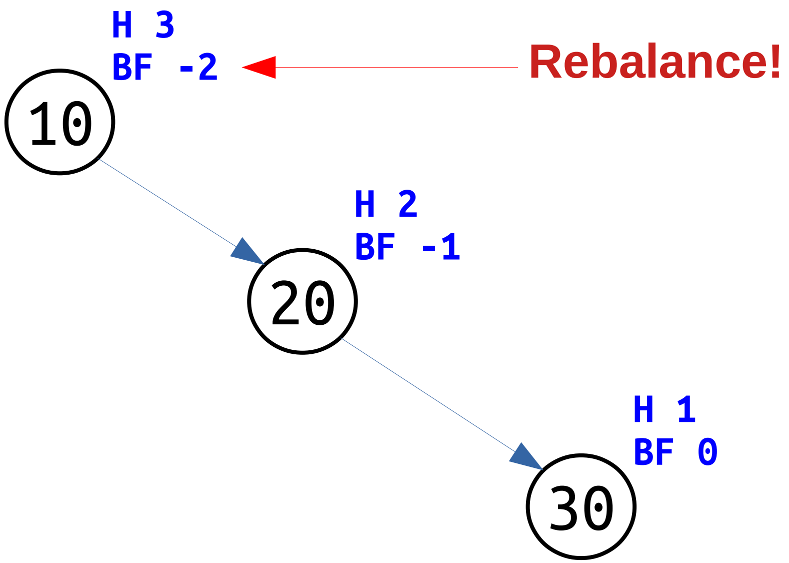
// Case 2. LEFT RIGHT CASE
if(bf > 1 && key > node->left->key)
{
    node->left = leftRotate(node->left);
    return rightRotate(node);
}

// Case 3. RIGHT RIGHT CASE
if(bf < -1 && key > node->right->key)
{
    return leftRotate(node);
}

// Case 4. RIGHT LEFT CASE
if(bf < -1 && key < node->right->key)
{
    node->right = rightRotate(node->right);
    return leftRotate(node);
}
```

AVL

Insert 30



AVL

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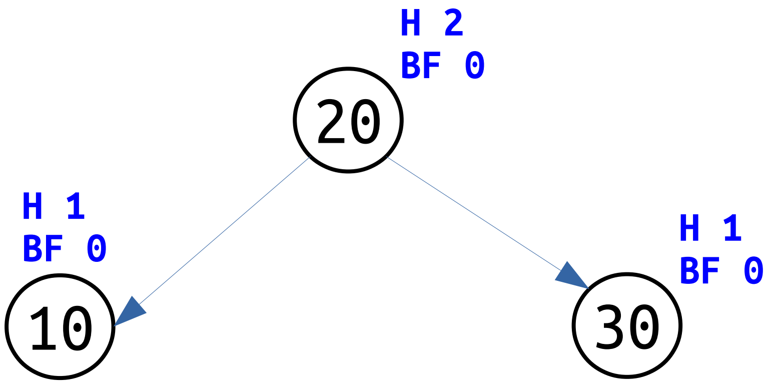
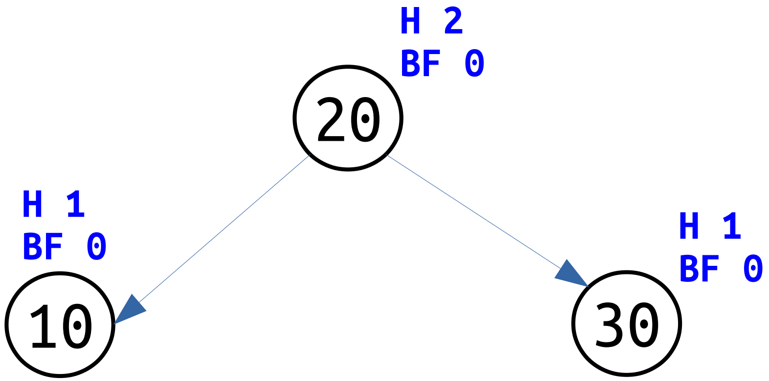


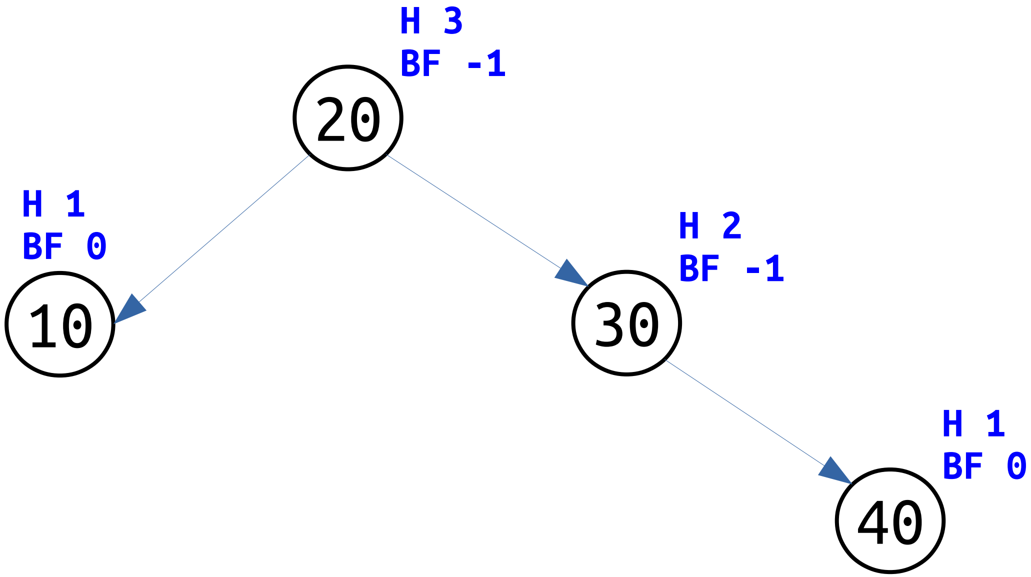
AVL

Insert 40



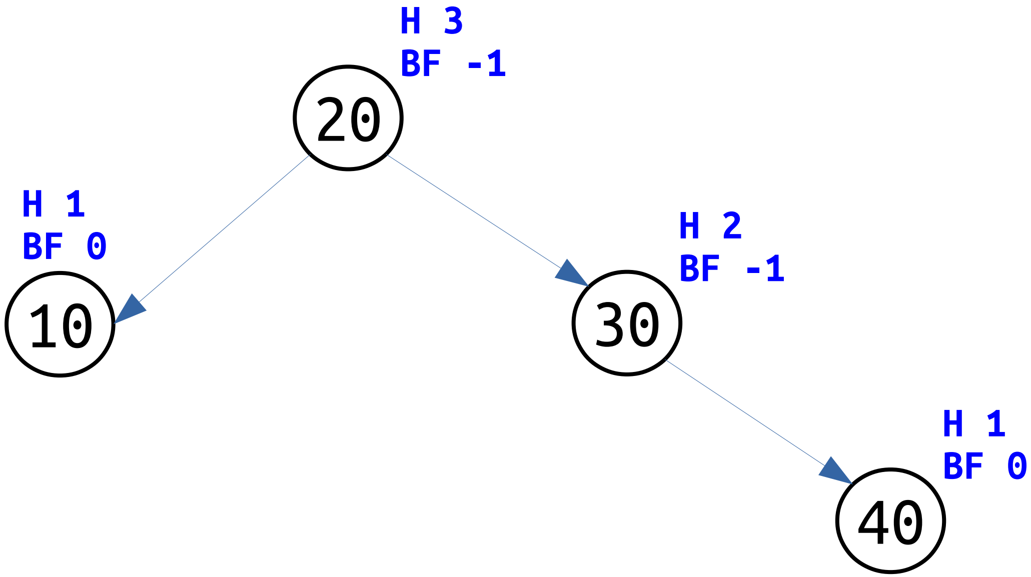
AVL

Insert 40



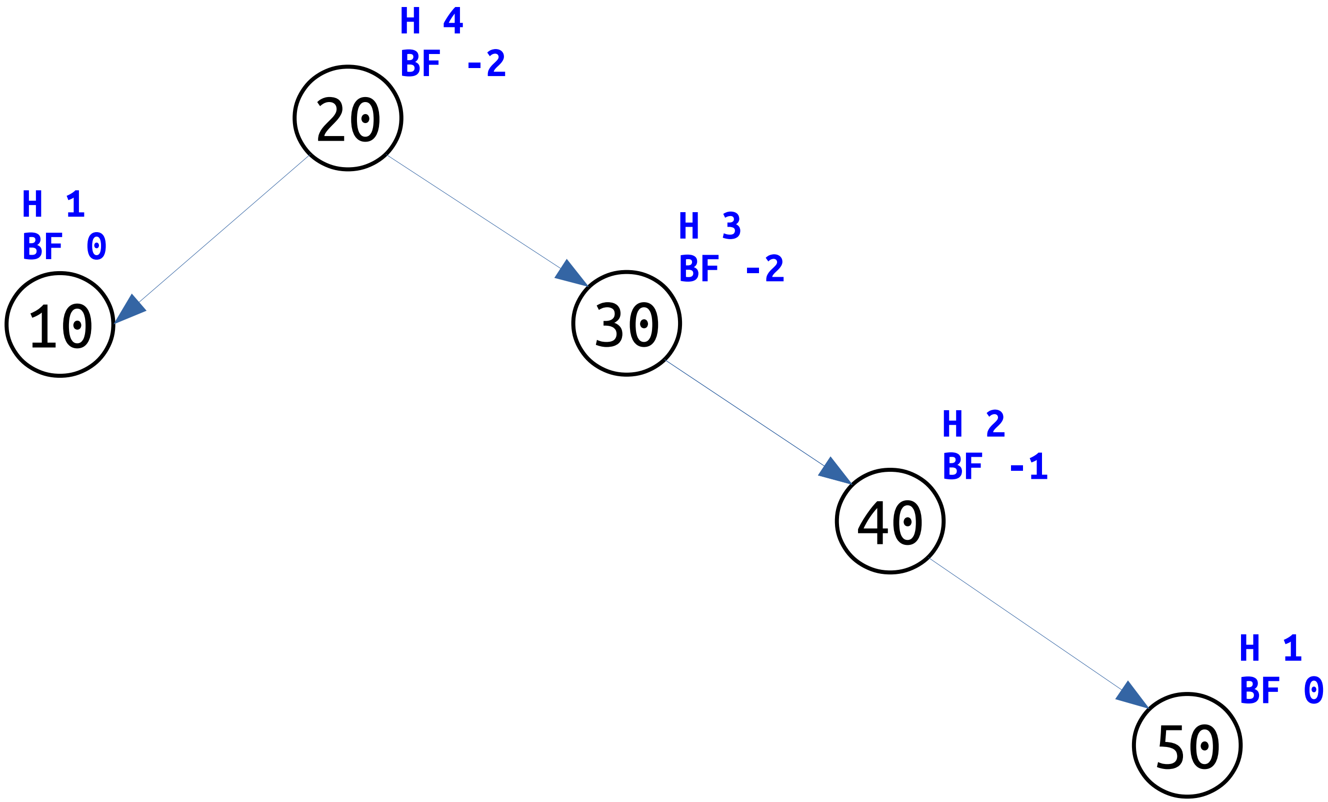
AVL

Insert 50



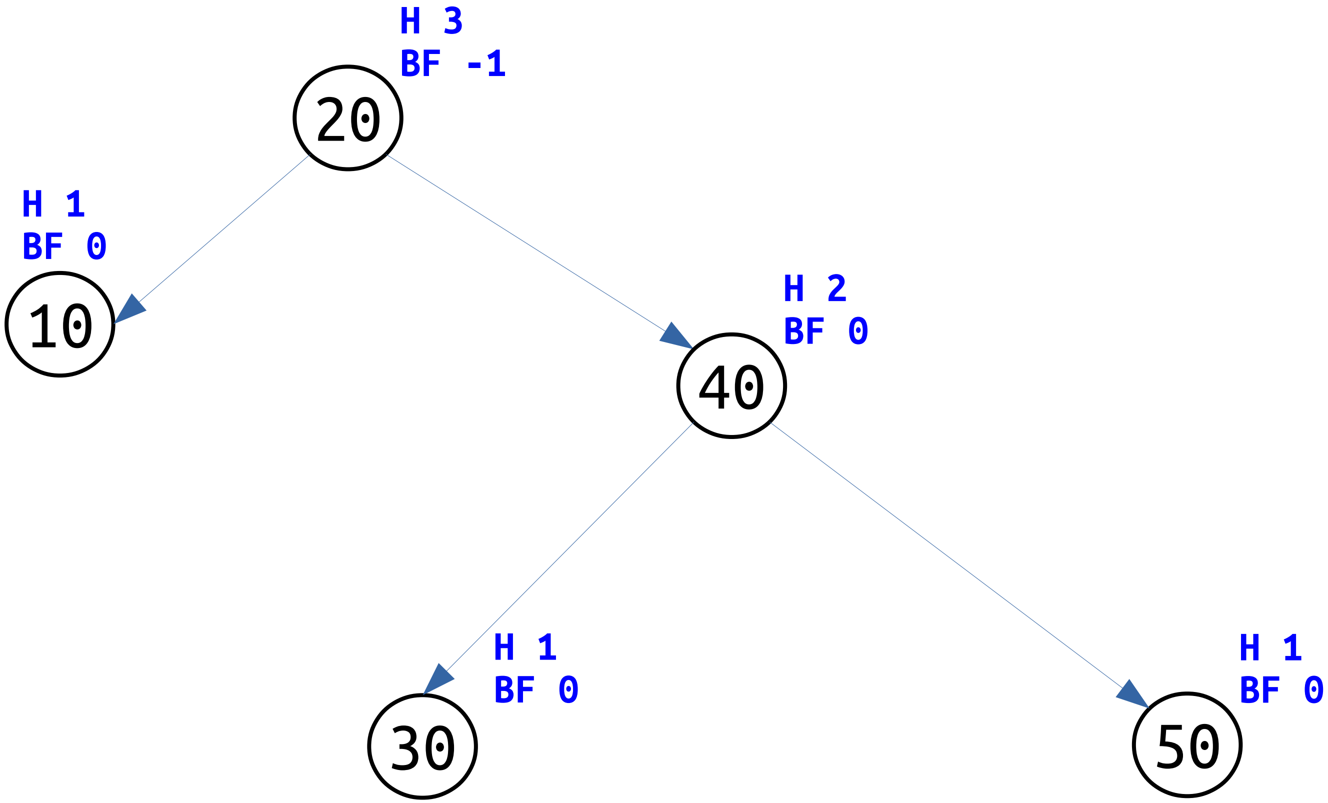
AVL

Insert 50



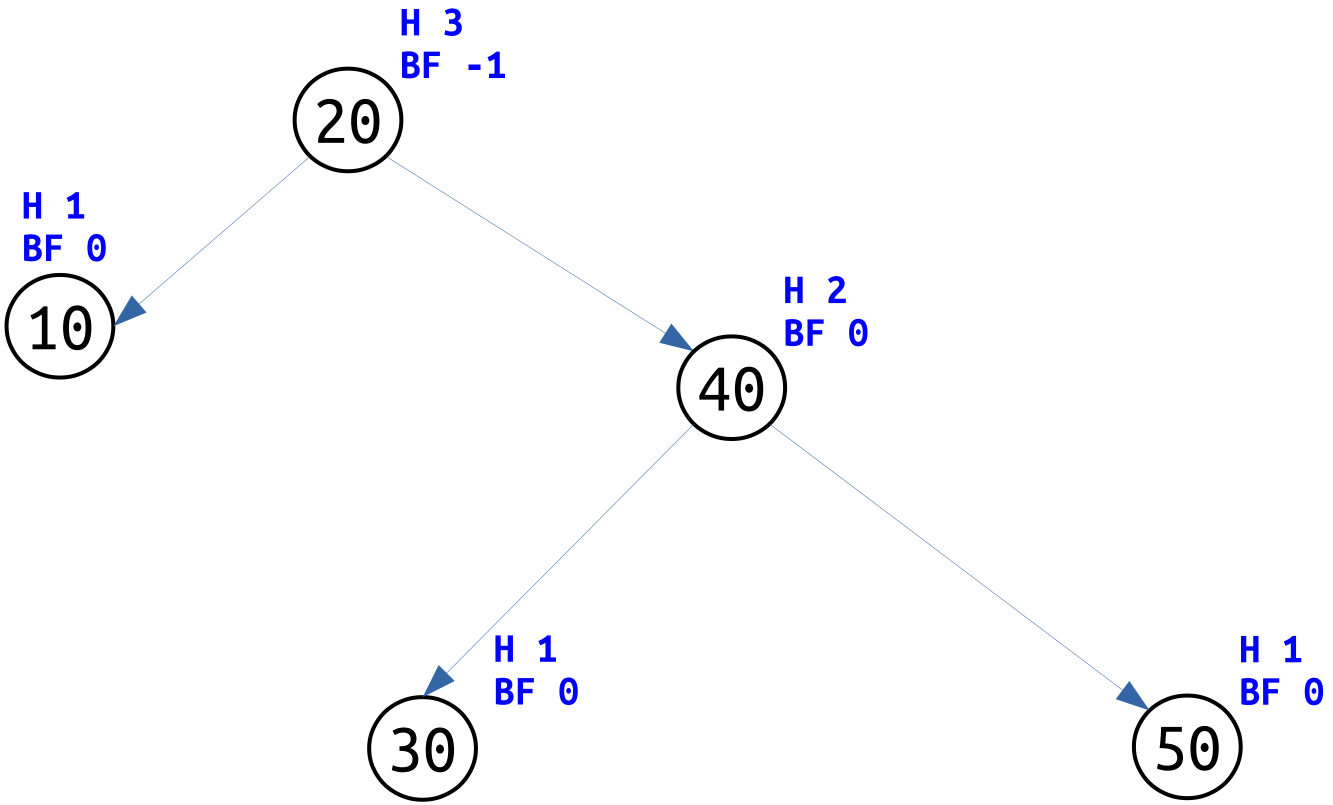
AVL

Insert 50



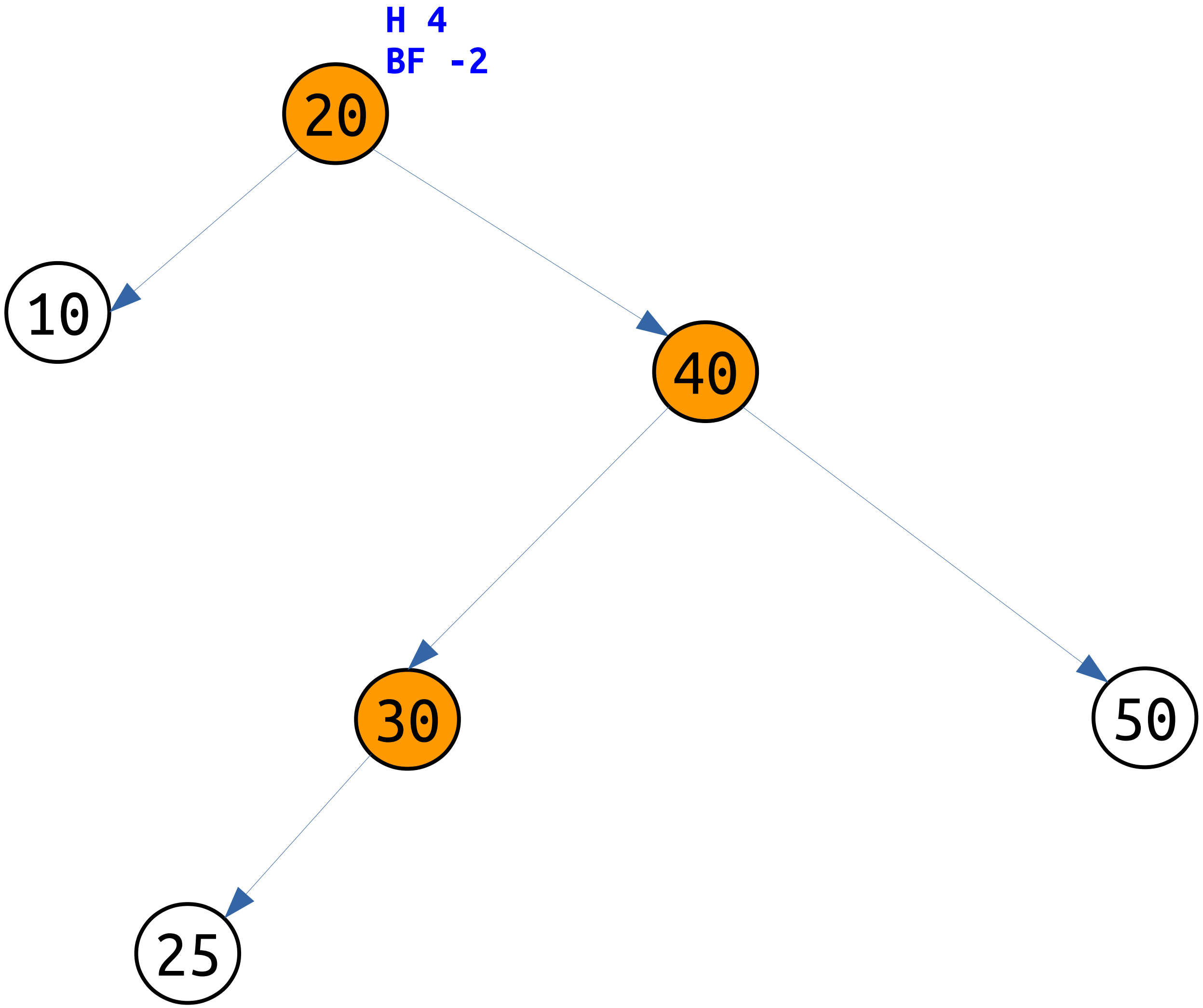
AVL

Insert 25



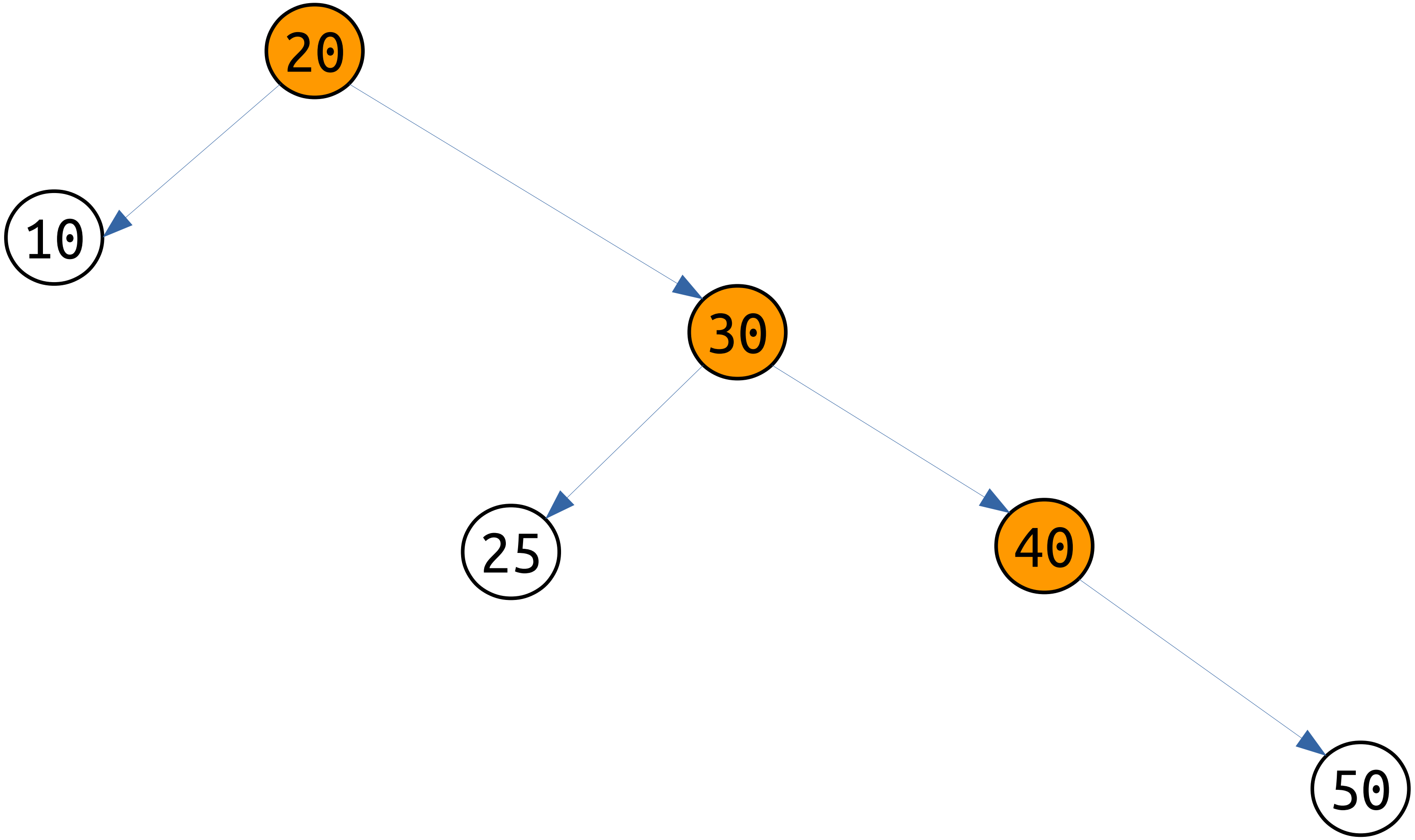
AVL

Insert 25



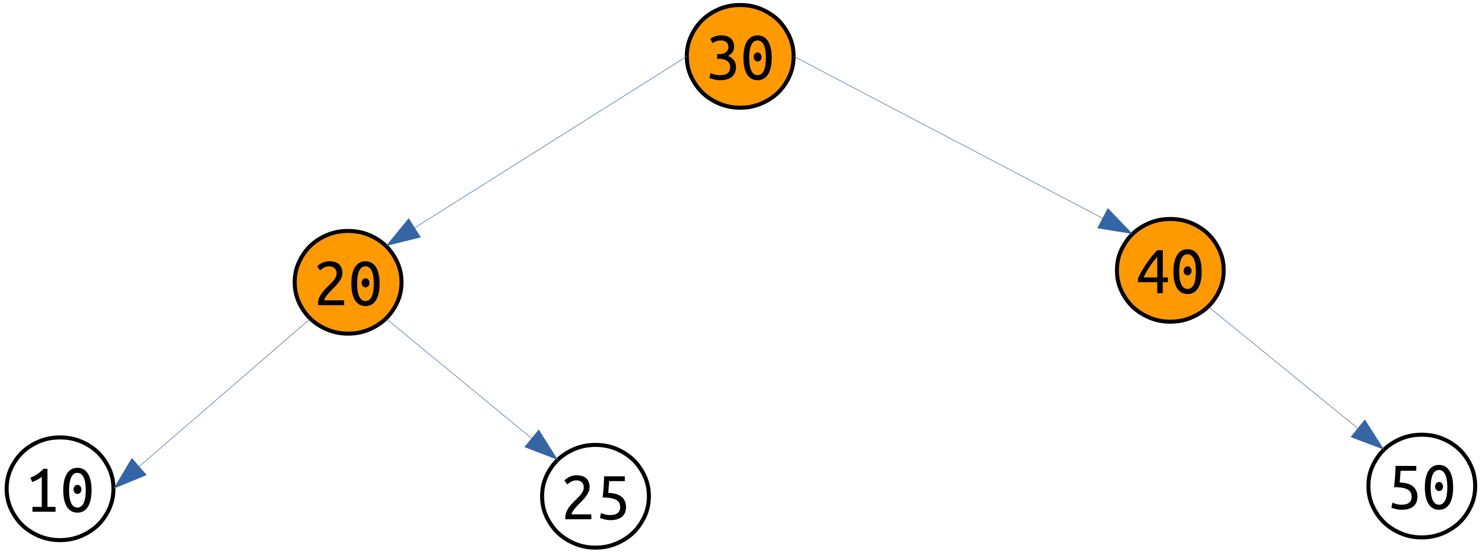
AVL

Insert 25



AVL

Insert 25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