CSE 321: Algorithms
Solutions for Homework 5

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Perihan Mirkelam, Scientific Preparation

1. For optimal solution, we should sort jobs according to "weight per times". The greatest "weight per time" job belongs to most important client for us. So that, we are willing to satisfy this precious clients first.

   Firstly jobs are sorted by using merge sort so that sorting takes $n \log n$ time.

   Calculating weighted sum of completion time takes $\frac{n(n+1)}{2}$:

   So if we want to calculate completion time too. Our algorithm takes $\mathcal{O}(n^2)$

2. (a) The algorithm doesn't offer an optimal solution when the cost of changing location is greater than the cost difference between operations in two cities. ($M > |N_i - S_i|$)

   For example,
   Let $n = 3$, $M = 10$
   NY costs $= \{2, 6, 1\}$
   SF costs $= \{3, 5, 2\}$

   ALG plan: $[NY, SF, NY]$
   ALG total cost $= 2 + 5 + 10 + 1 + 10 = 38$ (Change location twice)

   OPT plan: $[NY, NY, NY]$
   OPT total cost $= 2 + 6 + 1 = 9$

   (b) Each ith month, there is 4 possible case we should care about:

   - Previous month was at NY, current month is at NY
   - Previous month was at SF, current month is at NY (add change location cost)
   - Previous month was at SF, current month is at SF
   - Previous month was at NY, current month is at SF (add change location cost)

   We seperate them into two cases according to current month. Then we choose minimum of minimum of them.

   The algorithm takes approximately $3n$ times. Therefore time complexity is: $\mathcal{O}(n)$