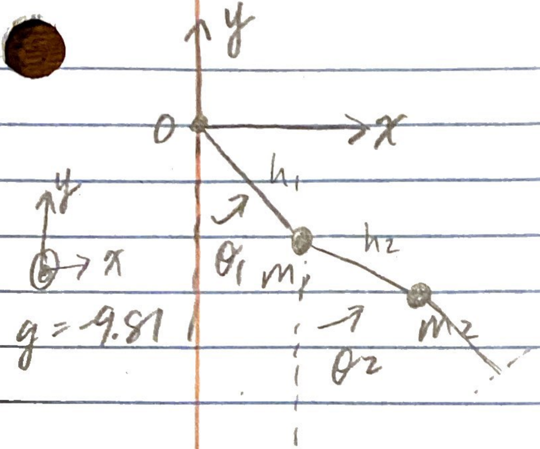
Sruti Vutukury 2/16/2022



N Link Pendulum

- lagrange w/ minimal coordinates
  $\theta_1, \theta_2, \theta_3, \ldots$
  $\dot\theta_1, \dot\theta_2, \dot\theta_3, \ldots$
- pt. mass pendulum, no rigid body motion
- no friction @ joints, no drag

1. derive EOMs wrt minimal coords using lagrange
2.

```
driver fn
    ICs = [      ]
    derivation_fn()
    [t_out, z_out] = ode45(@RHS, tspan, ICs)
      plotting_fn
      animate_fn
end
```

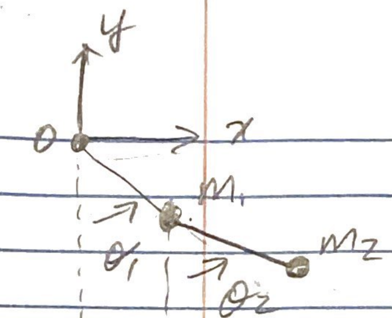
- symbolic vars?
- fsolve?
- [M, b] = mass Matrix (not linear)

$z = Mx + b$
$z - b/M = x$

```
dz = RHS(z)
    dz(1:n) = z(n+1:end)
    for i = 1:n
        dz(n+i)        % propogate state
    end
end
```

```
derivation_fn()
    PE total = 0
    KE total = 0
    for i = 1:n
        KE link = ...  , PE link = ...
        KE total = KE total + KE link
        PE total = PE tot + PE link
    end
    L = PE total - KE total
    for i = 1:n
        eqn(i) = subs(diff(L, thetas)) = 0
    end
end
```

← [new eqns, new vars] = reduceDiffOrder(eqns, vars)



$$\boxed{x_n = \sum_{i=1}^{n} h_i \sin\theta_i}$$

$$\boxed{y_n = -\sum_{i=1}^{n} h_i \cos\theta_i}$$

$x_1 = h_1 \sin\theta_1$

$x_2 = x_1 + h_2 \sin\theta_2$

$x_3 = x_1 + x_2 + h_3 \sin\theta_3$

$\vdots$

$x_n = x_1 + \dots + x_{n-1} + h_n \sin\theta_n$

$y_1 = -h_1 \cos\theta_1$

$y_2 = y_1 - h_2 \cos\theta_2$

$\vdots$

$y_n = y_1 - \dots - y_{n-1} - h_n \cos\theta_n$

$\dot{x}_1 = h_1 \cos\theta_1 \dot{\theta}_1$

$\dot{x}_2 = \dot{x}_1 + h_2 \cos\theta_2 \dot{\theta}_2$

$\vdots$

$\dot{x}_n = \dot{x}_1 + \dots + \dot{x}_{n-1} + h_n \cos\theta_n \dot{\theta}_n$

$\dot{y}_1 = h_1 \sin\theta_1 \dot{\theta}_1$

$\dot{y}_2 = \dot{y}_1 + h_2 \sin\theta_2 \dot{\theta}_2$

$\vdots$

$\dot{y}_n = \dot{y}_1 + \dots + \dot{y}_{n-1} + h_n \sin\theta_n \dot{\theta}_n$

$PE_1 = m_1 g y_1 = -m_1 g h_1 \cos\theta$

$PE_2 = m_2 g y_2 = m_2 g (y_1 - h_2 \cos\theta_2)$

$$\boxed{PE_n = -m_n g \sum_{i=1}^{n} h_i \cos\theta_i}$$

$PE_T = m_1 g y_1 + m_2 g y_2 + \dots$

$= m_1 g(-h_1 \cos\theta_1) + m_2 g(-h_1 \cos\theta_1 - h_2 \cos\theta_2) + \dots$

$KE = \frac{1}{2} m_1 \underbrace{(\dot{x}_1^2 + \dot{y}_1^2)}_{\text{scalar}} + \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2)$

speed x

speed y

$KE_1 = \frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2)$

$= \frac{1}{2} m_1 ((h_1 \sin\theta_1 \dot{\theta}_1)^2 + (h_1 \cos\theta_1 \dot{\theta}_1)^2)$

$KE_2 = \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2)$

$= \frac{1}{2} m_2 \left((h_1 \sin\theta_1 \dot{\theta}_1 + h_2 \sin\theta_2 \dot{\theta}_2)^2 + (h_1 \cos\theta_1 \dot{\theta}_1 - h_2 \cos\theta_2 \dot{\theta}_2)^2 \right)$

$$\boxed{KE_n = \frac{1}{2} m_n \left(\left(\sum_{i=1}^{n} h_i \cos\theta_i \dot{\theta}_i\right)^2 + \left(\sum_{i=1}^{n} h_i \sin\theta_i \dot{\theta}_i\right)^2 \right)}$$

each link

$L_1 = E_{k1} - E_{p1} = 0$

$L_2 = E_{k2} - E_{p2} = 0$

$\vdots$

$L_n = E_{kn} - E_{pn} = 0$

$$\Rightarrow \left[\begin{array}{l} \frac{d}{dt}\left(\frac{\partial L_1}{\partial \dot{\theta}_1}\right) - \frac{\partial L_1}{\partial \theta_1} = 0 \\ \frac{d}{dt}\left(\frac{\partial L_2}{\partial \dot{\theta}_2}\right) - \frac{\partial L_2}{\partial \theta_2} = 0 \\ \vdots \\ \frac{d}{dt}\left(\frac{\partial L_n}{\partial \dot{\theta}_n}\right) - \frac{\partial L_n}{\partial \theta_n} = 0 \end{array}\right]$$

$\Downarrow$ solve via MATLAB

	θ_1	θ_2	θ_3	$\dot{\theta}_1$	$\dot{\theta}_2$	$\dot{\theta}_3$
t_1						
t_2						
t_3						

$L = E_k - E_p = 0$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_1}\right) - \frac{\partial L}{\partial q_1} = 0$$

$E_k(q_s, \dot{q}_s)$ — q_s in min. coords

$E_p(q_s, \dot{q}_s)$ — $q = \theta_1, \theta_2, \theta_3, \ldots, \theta_n$

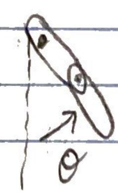
$\dot{q} = \dot{\theta}_1, \dot{\theta}_2, \ldots, \dot{\theta}_n$

discussion

- EOMs checked against double pendulum and triple pendulum eoms found online ✓
- unstable for large θ
- variations: nlinks, ICs
- checks: energy, postion, velocity plots, animation

Future

- rigid body links to add rotational motion



- add friction @ joints
- add drag
- more initial conditions
- try other ways to solve: DAE min, DAE maximal coord, ~~Lagrange~~