IOE 511/MATH 562 - Continuous Optimization Methods
Project Problems - Winter 2023

Problem 1 `P1_quad_10_10`:

$$f(x) = \frac{1}{2}x^T Qx + q^T x$$

where $q \in \mathbb{R}^n$ and $Q \in \mathbb{R}^{n\times n}$. A randomly generated convex quadratic function. Dimension $n = 10$; Condition number $\kappa = 10$. Starting Point: `rng(0); x_0=20*rand(10,1)-10`.

(Functions provided: function, gradient, Hessian)

Problem 2 `P2_quad_10_1000`:

$$f(x) = \frac{1}{2}x^T Qx + q^T x$$

where $q \in \mathbb{R}^n$ and $Q \in \mathbb{R}^{n\times n}$. A randomly generated convex quadratic function. Dimension $n = 10$; Condition number $\kappa = 1000$. Starting Point: `rng(0); x_0=20*rand(10,1)-10`.

(Functions provided: function)

Problem 3 `P3_quad_1000_10`:

$$f(x) = \frac{1}{2}x^T Qx + q^T x$$

where $q \in \mathbb{R}^n$ and $Q \in \mathbb{R}^{n\times n}$. A randomly generated convex quadratic function. Dimension $n = 1000$; Condition number $\kappa = 10$. Starting Point: `rng(0); x_0=20*rand(1000,1)-10`.

(Functions provided: function)

Problem 4 `P4_quad_1000_1000`:

$$f(x) = \frac{1}{2}x^T Qx + q^T x$$

where $q \in \mathbb{R}^n$ and $Q \in \mathbb{R}^{n\times n}$. A randomly generated convex quadratic function. Dimension $n = 1000$; Condition number $\kappa = 1000$. Starting Point: `rng(0); x_0=20*rand(1000,1)-10`.

(Functions provided: function)

Problem 5 `P5_quartic_1`:

$$f(x) = \frac{1}{2}x^T x + \frac{\sigma}{4}(x^T Qx)^2$$

where $\sigma = 10^{-4}$ and $A \in \mathbb{R}^{n\times n}$. The dimension $n = 4$, and the matrix A is given by

$$Q = \begin{bmatrix} 5 & 1 & 0 & 0.5 \\ 1 & 4 & 0.5 & 0 \\ 0 & 0.5 & 3 & 0 \\ 0.5 & 0 & 0 & 2 \end{bmatrix}.$$

Starting Point: `x_0=[cos(70) sin(70) cos(70) sin(70)]`^{T}.

(Functions provided: function)

Problem 6 `P6_quartic_2`:

$$f(x) = \frac{1}{2}x^T x + \frac{\sigma}{4}(x^T Q x)^2$$

where $\sigma = 10^4$ and $A \in \mathbb{R}^{n\times n}$. The dimension $n = 4$, and the matrix A is given by

$$Q = \begin{bmatrix} 5 & 1 & 0 & 0.5 \\ 1 & 4 & 0.5 & 0 \\ 0 & 0.5 & 3 & 0 \\ 0.5 & 0 & 0 & 2 \end{bmatrix}.$$

Starting Point: `x_0=[cos(70) sin(70) cos(70) sin(70)]`^{T}.

(Functions provided: function)

Problem 7 `Rosenbrock_2`:

$$f(x) = (1 - x_{[1]})^2 + 100(x_{[2]} - x_{[1]}^2)^2, \quad \text{where } x = [x_{[1]}\ x_{[2]}]^T \in \mathbb{R}^2.$$

Dimension $n = 2$. Starting Point: `x_0=[-1.2 1]`^{T}.

(Functions provided: none–this function was studied in HW2)

Problem 8 `Rosenbrock_100`:

$$f(x) = \sum_{i=1}^{99} [(1 - x_{[i]})^2 + 100(x_{[i+1]} - x_{[i]}^2)^2], \quad \text{where } x \in \mathbb{R}^{100}.$$

Dimension $n = 100$. Starting Point: `x_0=[-1.2 1 ··· 1]`^{T}.

(Functions provided: none)

Problem 9 `DataFit_2`:

$$f(x) = \sum_{i=1}^{3} (y_{[i]} - x_{[1]}(1 - x_{[2]}^i))^2, \quad \text{where } x = [x_{[1]}\ x_{[2]}]^T \in \mathbb{R}^2,$$

where $y = [1.5\ 2.25\ 2.625]^T$. Dimension $n = 2$. Starting Point: `x_0=[1 1]`^{T}.

(Functions provided: none–this function was studied in HW3)

Problem 10 `Exponential_10`:

$$f(x) = \frac{\exp(x_{[1]}) - 1}{\exp(x_{[1]}) + 1} + 0.1\exp(-x_{[1]}) + \sum_{i=2}^{10} (x_{[i]} - 1)^4, \quad \text{where } x = [x_{[1]}\ x_{[2]}\ \cdots\ x_{[10]}]^T \in \mathbb{R}^{10}.$$

Dimension $n = 10$. Starting Point: `x_0=[1 0 ··· 0]`^{T}.

(Functions provided: none–this function was studied in HW3)

Problem 11 `Exponential_1000`:

$$f(x) = \frac{\exp(x_{[1]}) - 1}{\exp(x_{[1]}) + 1} + 0.1\exp(-x_{[1]}) + \sum_{i=2}^{100} (x_{[i]} - 1)^4, \quad \text{where } x = [x_{[1]}\ x_{[2]}\ \cdots\ x_{[100]}]^T \in \mathbb{R}^{100}.$$

Dimension $n = 100$. Starting Point: `x_0=[1 0 ··· 0]`^{T}.

(Functions provided: none–this function was studied in HW3)

Problem 12 `Genhumps_5`:

$$f(x) = \sum_{i=1}^{4} \sin(2x_{[i]})^2 \sin(2x_{[i+1]})^2 + 0.05(x_{[i]}^2 + x_{[i+1]}^2), \quad \text{where } x = [x_{[1]}\ x_{[2]}\ \cdots\ x_{[5]}]^T \in \mathbb{R}^5.$$

Dimension $n = 5$. Starting Point: `x_0=[-506.2 506.2 ··· 506.2]`^{T}.

(Functions provided: function, gradient, Hessian)

Constrained Optimization Problems

Problems:

1. `Problem_1`

$$\begin{aligned} \min_{x\in\mathbb{R}^2} \;& x_1 + x_2 \\ \text{s.t.} \;& x_1^2 + x_2^2 - 2 = 0 \end{aligned}$$

Starting Point: $x_0 = [2\ 2]^T$.

Optimal Solution: $x^* = [-1\ \ -1]^T$.

Details: $n = 2, m = 1$.

2. `Problem_2`

$$\begin{aligned} \min_{x\in\mathbb{R}^2} \;& e^{x_1x_2x_3x_4x_5} - \tfrac{1}{2}(x_1^3 + x_2^3 + 1)^2 \\ \text{s.t.} \;& x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 - 10 = 0 \\ & x_2x_3 - 5x_4x_5 = 0 \\ & x_1^3 + x_2^3 + 1 = 0 \end{aligned}$$

Starting Point: $x_0 = [-1.8\ 1.7\ 1.9\ \ -0.8\ \ -0.8]^T$.

Optimal Solution: $x^* = [-1.71\ 1.59\ 1.82\ \ -0.763\ \ -0.763]^T$.

Details: $n = 5, m = 3$.