Which Is the Most Beautiful?

David Wells

According to Aristotle, "Those who assert that the mathematical sciences say nothing of the beautiful are in error. The chief forms of beauty are order, commensurability and precision" [1].

G. H. Hardy in a famous passage asserted: "A mathematician, like a painter or a poet, is a maker of patterns. . . . The mathematician's patterns, like the painter's or poet's, must be beautiful. . . . Beauty is the first test: there is no permanent place in the world for ugly mathematics" [2].

Von Neumann wrote: "I think it is correct to say that [the mathematician's] criteria of selection, and also those of success, are mainly aesthetical" [3].

Poincaré reflected a similar opinion when he wrote, "It is true aesthetic feeling which all mathematicians recognise. . . . The useful combinations are precisely the most beautiful" [4].

Weyl claimed: "My work always tried to unite the truth with the beautiful, but when I had to choose one or the other, I usually chose the beautiful" [5].

Morris Kline emphasised a different perspective: "Much research for new proofs of theorems already correctly established is undertaken simply because the existing proofs have no aesthetic appeal" [6].

Beauty does seem to be an essential, if little discussed, aspect of mathematics and the work of mathematicians. Yet no one can say precisely of what beauty in mathematics consists, and professional mathematicians will not necessarily agree on their definitions of mathematical beauty, on their practical judgments of which theorems, proofs, concepts, or strategies are the most beautiful, or on the role their personal feelings for mathematical beauty play in their own work.

This questionnaire is a simple attempt to gather some data on the preferences of readers of the *Mathematical Intelligencer*. The least you are asked to do is to give each of these theorems a score of 0 through 10 for beauty. (The most beautiful theorems score the highest marks.)

Notice that you are not asked to judge between different proofs of the same theorem, or between theorems and proofs. I appreciate that readers are bound to be influenced in their judgments by their knowledge, or lack of knowledge, of particular proofs.

Given constraints of space, and readers' time, I decided to focus on 24 different and varied theorems, all relatively easy to understand as statements, rather than a small handful of detailed proofs or other aspects of mathematical beauty.

Meta-responses—for example, that this questionnaire is impossible to answer, or that particular theorems cannot be ranked with merely a number—are welcome and will be considered as valid as straightforward rankings of the theorems. Additional comments of any kind will of course also be most welcome.

I hope to report on readers' views in a future issue of the *Mathematical Intelligencer.*

References

(I am indebted for references 1, 4, and 6 to an excellent paper by Harold Osborne, Mathematical Beauty and Physical Science, *British Journal of Aesthetics,* Vol. 24 (Autumn 1984), p. 291).

[1] Aristotle, *Metaphysics,* XIII 3.107b.
[2] G. H. Hardy, *A Mathematician's Apology,* Cambridge University Press (1940), p. 25.
[3] John von Neumann, *The Mathematician,* in *The Works of the Mind,* edited by Heywood and Nef, University of Chicago Press, reprinted in J. R. Newman, *The World of Mathematics,* Vol. 4, George Allen & Unwin (1960), p. 2053.
[4] H. Poincaré, *Science and Method,* trans. F. Maitland, Dover Press (n.d.), p. 59.
[5] Quoted by Tito Tonietti from a letter written by Hermann Weyl to Freeman Dyson, *The Mathematical Intelligencer,* Vol. 7 (1985), No. 4, p. 8.
[6] Morris Kline, *Mathematics in Western Culture,* George Allen & Unwin (1954), p. 470.

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Beauty in Mathematics

Instructions

Please photocopy this page, and give each theorem a score for beauty between 0 and 10, inclusive, using the boxes provided. Then send the completed questionnaire, with any additional comments, to:
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A Euler's formula for a polyhedron: $V + F = E + 2$. ☐

B Any square matrix satisfies its characteristic equation. ☐

C $\dfrac{5\{(1 - x^5)(1 - x^{10})(1 - x^{15}) \ldots\}^5}{\{(1 - x)(1 - x^2)(1 - x^3)(1 - x^4) \ldots\}^6}$ ☐
$= p(4) + p(9)x + p(14)x^2 + \ldots,$
where $p(n)$ is the number of partitions of n.

D The number of primes is infinite. ☐

E There is no rational number whose square is 2. ☐

F Every prime of the form $4n + 1$ is the sum of two integral squares in exactly one way. ☐

G $1 + 1/2^2 + 1/3^2 + 1/4^2 + 1/5^2 + \ldots = \pi^2/6$. ☐

H $\dfrac{1}{2 \times 3 \times 4} - \dfrac{1}{4 \times 5 \times 6} + \dfrac{1}{6 \times 7 \times 8} - \ldots\ldots = \dfrac{\pi - 3}{4}$. ☐

I π is transcendental. ☐

J Every number greater than 77 is the sum of integers, the sum of whose reciprocals is 1. ☐

K The maximum area of a quadrilateral with sides a, b, c, d is $\{(s-a)(s-b)(s-c)(s-d)\}^{1/2}$, where s is half the perimeter. ☐

L There is no equilateral triangle whose vertices are plane lattice points. ☐

M At any party, there is a pair of people who have the same number of friends present. ☐

N The number of partitions of an integer into odd integers is equal to the number of partitions into distinct integers. ☐

O If the points of the plane are each coloured red, yellow, or blue, there is a pair of points of the same colour of mutual distance unity. ☐

P Every plane map can be coloured with 4 colours. ☐

Q A continuous mapping of the closed unit disk into itself has a fixed point. ☐

R Write down the multiples of root 2, ignoring fractional parts, and underneath the numbers missing from the first sequence. ☐

1	2	4	5	7	8	9	11	12	...
3	6	10	13	17	20	23	27	30	...

The difference is $2n$ in the nth place.

S A regular icosahedron inscribed in a regular octagon divides the edges in the Golden Ratio. ☐

T The number of representations of an odd number as the sum of 4 squares is 8 times the sum of its divisors; of an even number, 24 times the sum of its odd divisors. ☐

U The word problem for groups is unsolvable. ☐

V The order of a subgroup divides the order of the group. ☐

W $e^{i\pi} = -1$. ☐

X There are 5 regular polyhedra. ☐