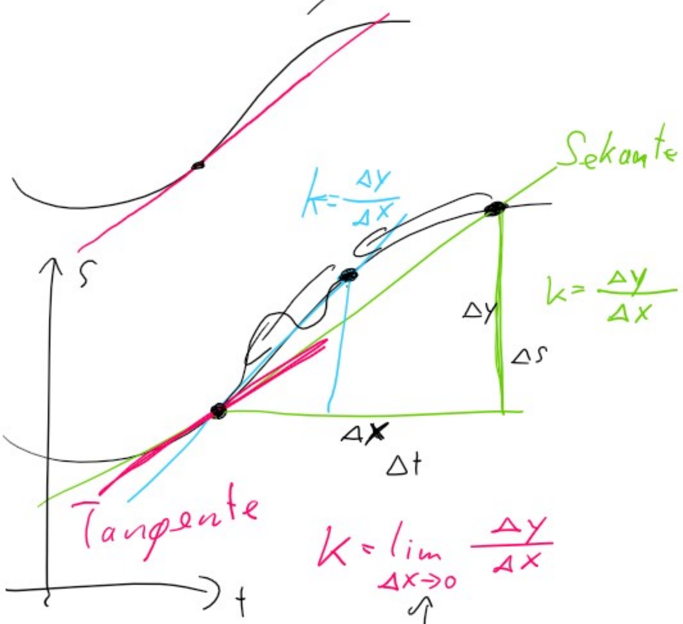
Mittwoch, 2. Dezember 2020 11:53

3 Differentialrechnung

Newton

Leibniz



$K = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$

$s = 500\ km$

$h = 5h$

$\varnothing v = 100\ \frac{km}{h}$

$v_{1h} = \frac{80\ km}{1\ h}$

$v_{57.min} = \frac{2{,}1\ km}{1\ min}$

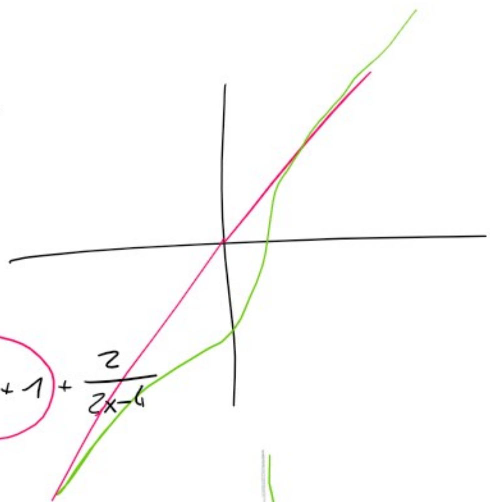
$v(t) = \frac{\Delta s}{\Delta t}$ $\Delta s = 0\,m$, $\Delta t = 0\,s$

$v(t) = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$

2.22) c) $3x^3 : (x^2+1) = 3x + \frac{-3x}{x^2+1}$

$-3x^3 \mp 3x$

$-3x$ R.



d) $\frac{x^2-2}{2(x-2)} = (x^2-2) : (2x-4) = \frac{1}{2}x + 1 + \frac{2}{2x-4}$

$-x^2 \pm 2x$

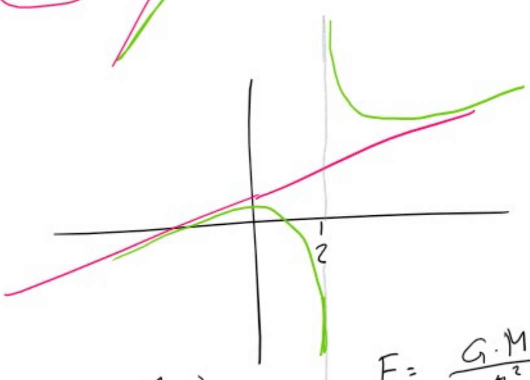
$2x - 2$

$-2x \pm 4$

2 R

$x - 2 = 0$

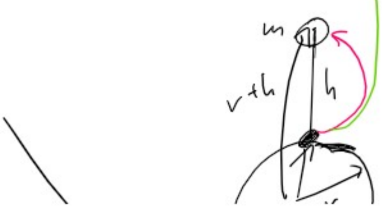
$x = 2$



$F = \frac{G \cdot M \cdot m}{r^2}$

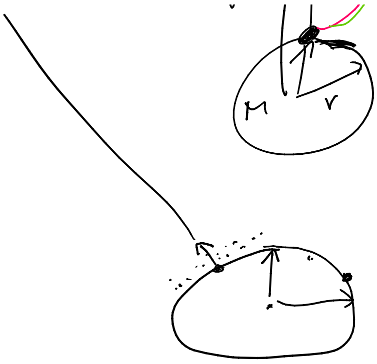
$W = m \cdot g \cdot h$

2.25)



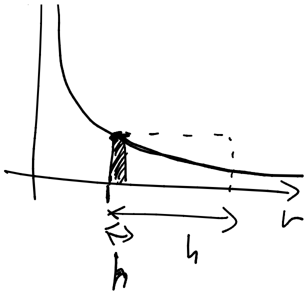
a) $W_{r,r+h} = GMm \cdot \left(\frac{1}{r} - \frac{1}{r+h}\right)$

$W_{r,\infty} = \lim_{h \to \infty} GMm \left(\frac{1}{r} - \frac{1}{r+h}\right) =$



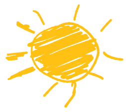
$$W_{r,\infty} = \lim_{h\to\infty} GMm\left(\frac{1}{r} - \frac{1}{r+h}\right)$$

$$= GM_m \cdot \frac{1}{r} = \ldots J$$

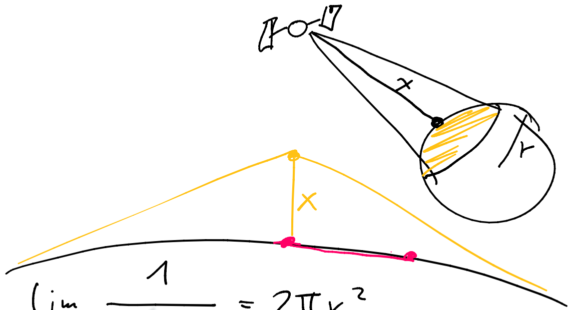


b) $W_{r,\infty} = \frac{m \cdot v^2}{2}$

$v = \ldots$



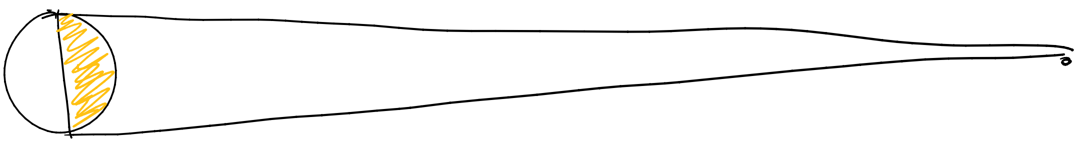
2.26) $A = \frac{2\pi r^2 x}{r+x}$



$$\lim_{x\to\infty} \frac{2\pi r^2 x}{r+x} =$$

$$= 2\pi r^2 \lim_{x\to\infty} \frac{x}{r+x} = 2\pi r^2 \lim_{x\to\infty} \frac{1}{\frac{r}{x}+1} = 2\pi r^2$$

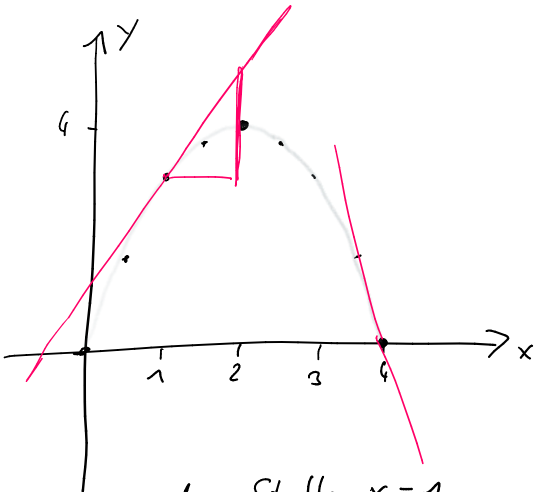
Kugeloberfläche: $4\pi r^2$



Beispiel 3.4: Graphisch Differenzieren

$f(x) = -x^2+4x$

x	$f(x)$
0	0
0,5	1,75
1	3
1,5	3,75
2	4
2,5	3,75
3	3
3,5	1,75
4	0

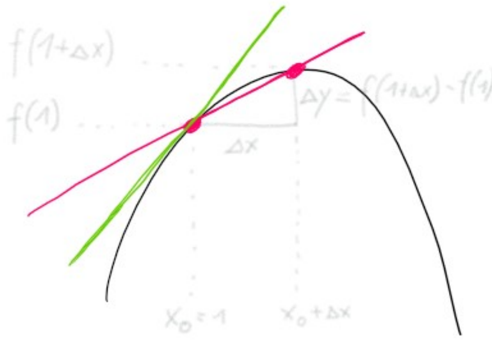


gesucht: Tangentensteigung an der Stelle $x_0 = 1$

⊛ graphisch Lineal anlegen und Steigung ablesen: $\approx 2;\ 2;\ 1,5;\ 1,6;\ 2;$

⊛ rechnerisch: wählen zweiten Punkt, der Δx neben x_0 liegt und legen eine Sekante durch die zwei Punkte, ihre Steigung

$f(1+\Delta x)$



liegt und legen eine Sekante durch die zwei Punkte, ihre Steigung ist

$$\frac{\Delta y}{\Delta x} = \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x} \ldots$$

... Sekantensteigung, Differenzenquotient, durchschnittliche Änderungsrate, wenn s über t … Durchschnittsgeschwindigkeit

$f(x) = -x^2 + 4x$

$$\left.\frac{\Delta y}{\Delta x}\right|_{x=1} = \frac{f(1+\Delta x) - f(1)}{\Delta x} =$$

$$= \frac{-(1+\Delta x)^2 + 4\cdot(1+\Delta x) - (-1^2 + 4\cdot 1)}{\Delta x} =$$

$$= \frac{-(1 + 2\Delta x + \Delta x^2) + 4 + 4\Delta x + 1 - 4}{\Delta x} =$$

$$= \frac{-\Delta x^2 + 2\Delta x + 0}{\Delta x} = \frac{\cancel{\Delta x}\cdot(-\Delta x + 2)}{\cancel{\Delta x}} = 2 - \Delta x$$

Grenzübergang $\lim\limits_{\Delta x \to 0}$ („Punkte aufeinanderschieben"):

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x} = \frac{dy}{dx} \ldots$$

„dy nach dx"

... Tangentensteigung, Differential, momentane Änderungsrate, wenn s über t … Momentangeschwindigkeit

$$f'(1) = \lim_{\Delta x \to 0} \frac{f(1+\Delta x) - f(1)}{\Delta x} = \ldots = \lim_{\Delta x \to 0} 2 - \Delta x = 2$$

Tangentensteigung bei $x_0 = 4$:

$$f'(4) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(4+\Delta x) - f(4)}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{-(4+\Delta x)^2 + 4\cdot(4+\Delta x) - (-4^2 + 4\cdot 4)}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{-(16 + 8\Delta x + \Delta x^2) + 16 + 4\Delta x - (-16 + 16)}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{-8\Delta x - \Delta x^2 + 4\Delta x}{\Delta x} = \lim_{\Delta x \to 0} \frac{-\Delta x^2 - 4\Delta x}{\Delta x} = \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}\cdot(-\Delta x - 4)}{\cancel{\Delta x}} =$$

$$= \lim_{\Delta x \to 0} -4 - \Delta x = -4$$

$\lim_{\Delta x \to 0} \frac{\quad}{\Delta x} = \lim_{\Delta x \to 0} \frac{\quad}{\Delta x} = \lim_{\Delta x \to 0} \frac{\quad}{\cancel{\Delta x}} =$

$= \lim_{\Delta x \to 0} -4 - \Delta x = -4$

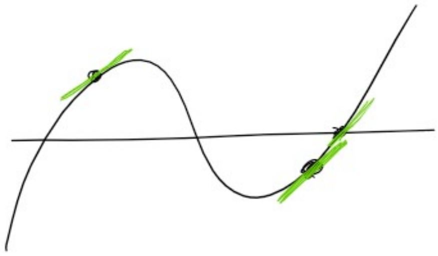
Möchte man allgemein die Tangentensteigung an der Stelle x_0 berechnen:

$$f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$$

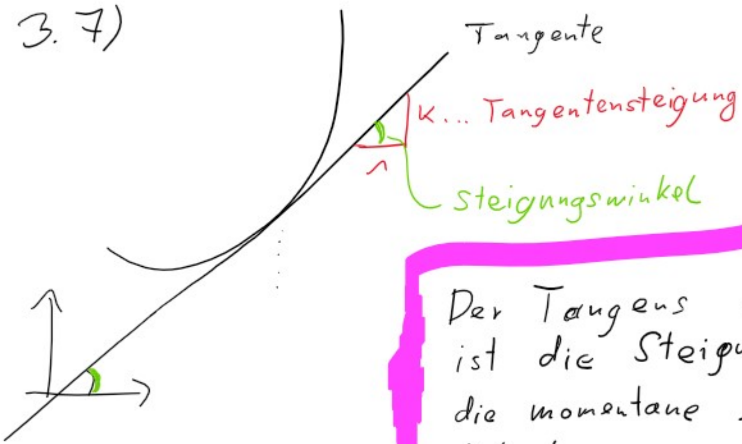
M. HÜ:

3.6
3.7
3.8 b graphisch, rechnerisch
3.9
3.11

3.6)



3.7)



$\tan(\varphi) = \frac{k}{1} = k$

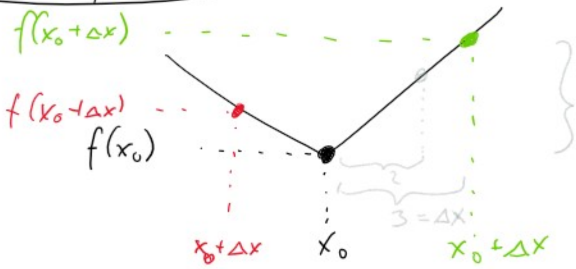
Der Tangens des Steigungswinkels ist die Steigung, der Differentialquotient, die momentane Änderungsrate, die Ableitung.

Differenzierbarkeit von f an der Stelle x_0:

- Es gibt dort eine Tangente.
- Der Differential quotient

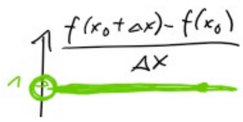
$$\lim_{\Delta x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x} \quad \text{existiert.}$$

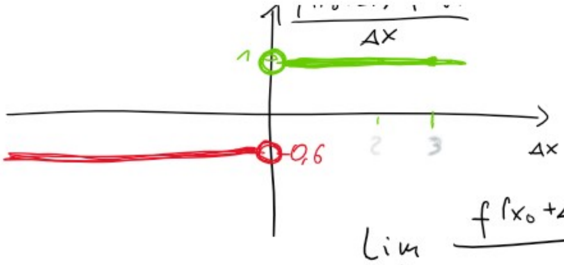
Gegenbeispiel:



$\frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$ ist die Steigung des rechten Astes

$\frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$ ist die Steigung des linken Astes





Steigung des linken Astes

$$\lim_{\Delta x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$$ existiert nicht!

Bsp. von oben: $f(x) = -x^2 + 4x$

Tangentensteigung bei x_0 allgemein (nicht $x_0 = 1$, $x_0 = 4$)

$$f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\overbrace{-(x_0+\Delta x)^2 + 4\cdot(x_0+\Delta x)}^{f(x_0+\Delta x)} - \overbrace{(-x_0^2+4\cdot x_0)}^{f(x_0)}}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{-(\cancel{x_0^2} + 2x_0\Delta x + \Delta x^2) + \cancel{4x_0} + 4\Delta x + \cancel{x_0^2} - \cancel{4x_0}}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}\cdot(-2x_0 - \overset{0}{\Delta x} + 4)}{\cancel{\Delta x}} = \underline{\underline{-2x_0 + 4}}$$

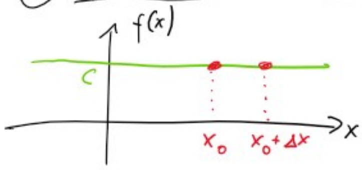
$f'(1) = -2\cdot 1 + 4 = 2$

$f'(4) = -2\cdot 4 + 4 = -4$

3.2 Ableitung elementarer Funktionen

① Konstante Funktion

$f(x) = c$



$$f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x} =$$

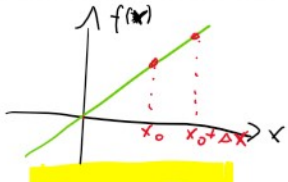
$$= \lim_{\Delta x \to 0} \frac{c - c}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{0}{\Delta x} = \lim_{\Delta x \to 0} 0 = \underline{\underline{0}}$$

$$\boxed{\begin{aligned} f(x) &= c \\ f'(x) &= 0 \end{aligned}}$$

② Lineare Funktion

$f(x) = x$



$$f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x} =$$

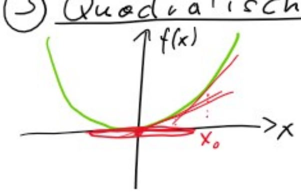
$$= \lim_{\Delta x \to 0} \frac{x_0 + \Delta x - x_0}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x} = \lim_{\Delta x \to 0} 1 = 1$$

$$\boxed{\begin{aligned} f(x) &= x \\ f'(x) &= 1 \end{aligned}}$$

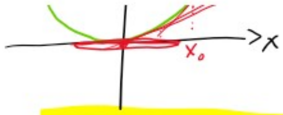
③ Quadratische Funktion

$f(x) = x^2$



$$f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x} =$$

$$= \lim_{\Delta \to 0} \frac{(x_0+\Delta x)^2 - x_0^2}{\Delta x} =$$



$$f(x) = x^2$$
$$f'(x) = 2x$$

$$= \lim_{\Delta \to 0} \frac{(x_0+\Delta x)^2 - x_0^2}{\Delta x} =$$
$$= \lim_{\Delta x \to 0} \frac{\cancel{x_0^2} + 2x_0\Delta x + \Delta x^2 - \cancel{x_0^2}}{\Delta x} =$$
$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x} \cdot (2x_0 + \Delta x)}{\cancel{\Delta x}} =$$
$$= 2 \cdot x_0$$

④ Potenzfunktionen

$$f(x) = x^n$$
$$f'(x) = n \cdot x^{n-1}$$

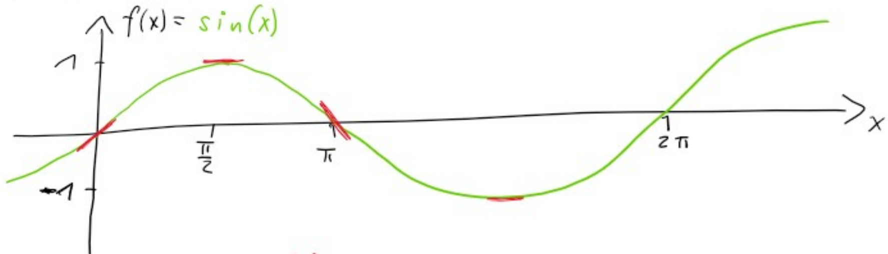
$$f(x) = x^3$$
$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x} =$$
$$= \lim_{\Delta x \to 0} \frac{(x_0+\Delta x)^3 - x_0^3}{\Delta x} =$$
$$= \lim_{\Delta x \to 0} \frac{\cancel{x_0^3} + 3x_0^2\Delta x + 3x_0\Delta x^2 + \Delta x^3 - \cancel{x_0^3}}{\Delta x} =$$
$$= \lim_{\Delta x \to 0} \frac{\Delta x \cdot (3x_0^2 + 3x_0\Delta x + \Delta x^2)}{\Delta x} =$$
$$= \lim_{\Delta x \to 0} 3x_0^2 + 3x_0\underbrace{\Delta x}_{\to 0} + \underbrace{\Delta x^2}_{\to 0} = 3x_0^2$$

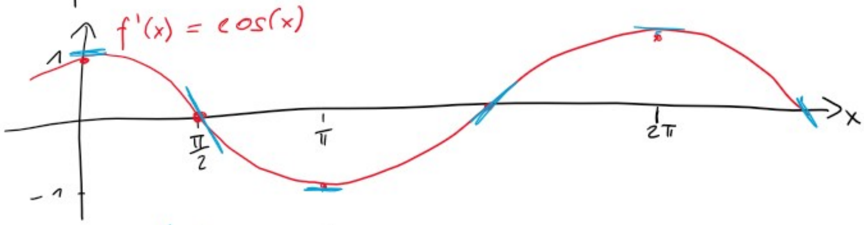
$$f(x) = x^4$$
$$f'(x) = 4x^3$$

Beispiele:

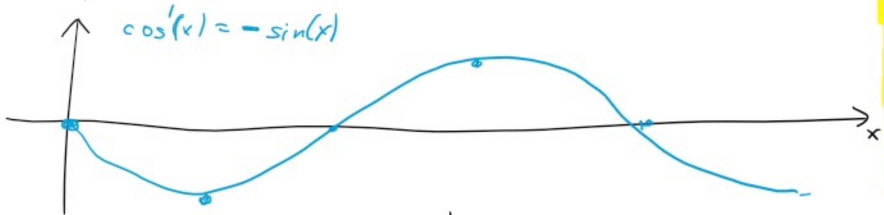
- $f(x) = x^{-7}$
  $f'(x) = -7 \cdot x^{-8}$
- $f(x) = \frac{1}{x} = x^{-1}$
  $f'(x) = -1 \cdot x^{-2} = -\frac{1}{x^2}$
- $f(x) = \sqrt{x} = x^{\frac{1}{2}}$
  $f'(x) = \frac{1}{2} \cdot x^{-\frac{1}{2}} = \frac{1}{2 \cdot \sqrt{x}}$

⑤ Winkelfunktionen:





$$f(x) = \sin(x)$$
$$f'(x) = \cos(x)$$



$$f(x) = \cos(x)$$
$$f'(x) = -\sin(x)$$

12. HÜ:

3.18 alle
3.19 d
3.20 b
3.21

mit Regel, nicht Differentialquotient

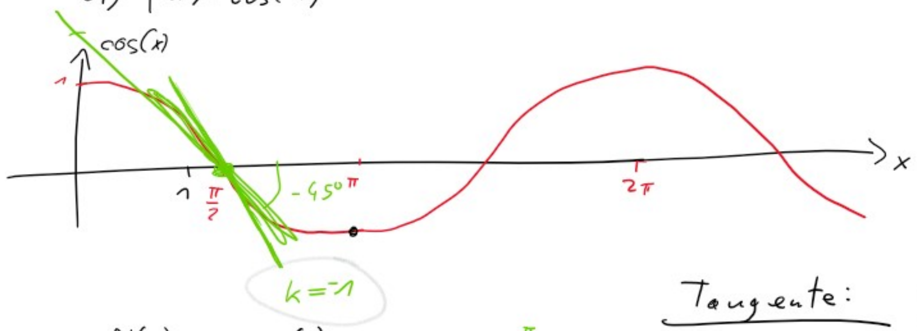
3.18) a) $f(x) = \sqrt{x \cdot \sqrt[3]{x}} = \left(x \cdot x^{\frac{1}{3}}\right)^{\frac{1}{2}} = x^{\frac{1}{2}} \cdot x^{\frac{1}{3} \cdot \frac{1}{2}} = x^{\frac{1}{2}} \cdot x^{\frac{1}{6}} = x^{\frac{1}{2}+\frac{1}{6}} = x^{\frac{4}{6}} = x^{\frac{2}{3}}$

3.21

3.18) c) $f(x)=\sqrt{x\cdot\sqrt[3]{x}}=\left(x\cdot x^{\frac{1}{3}}\right)^{\frac{1}{2}}=x^{\frac{1}{2}}\cdot x^{\frac{1}{3}\cdot\frac{1}{2}}=x^{\frac{1}{2}}\cdot x^{\frac{1}{6}}=x^{\frac{1}{2}+\frac{1}{6}}=x^{\frac{4}{6}}=x^{\frac{2}{3}}$

$f'(x)=\frac{2}{3}\cdot x^{-\frac{1}{3}}=\frac{2}{3\cdot\sqrt[3]{x}}$

3.19)* d) $f(x)=\cos(x)$



$f'(x)=-\sin(x)$ $x_0=\frac{\pi}{2}$

$f'(\frac{\pi}{2})=-\sin(\frac{\pi}{2})=\underline{\underline{-1=k}}$

Tangente: $y=k\cdot x+d$

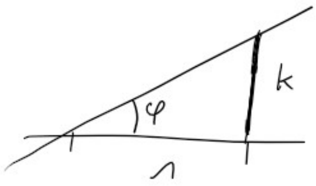
$y=-x+d$

geht durch $(\frac{\pi}{2}/0)$:

$0=-\frac{\pi}{2}+d$

$d=\frac{\pi}{2}$

$\underline{\underline{y=-x+\frac{\pi}{2}}}$



$\tan(\varphi)=\frac{k}{1}=k=-1$

$\varphi=\arctan(-1)=\underline{\underline{-45°}}$

3.20) b)

$f(x)=\frac{1}{\sqrt{x}}=x^{-\frac{1}{2}}$ $\alpha=-30°$

$f'(x)=-\frac{1}{2}x^{-\frac{3}{2}}=$ $k=\tan(-30°)=...$

$x=$

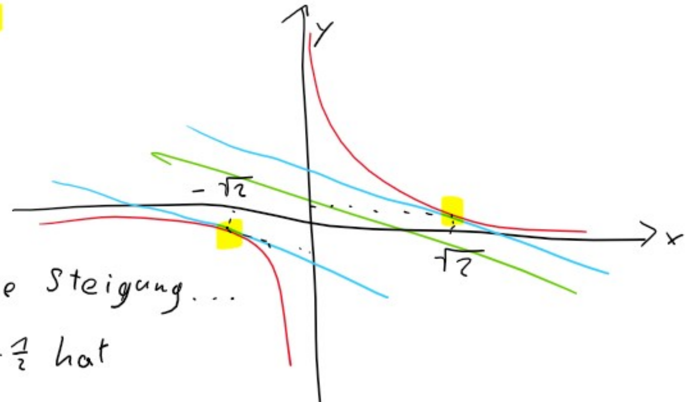
3.21) $y=\frac{1}{x}=x^{-1}$

$y=-\frac{x}{2}+3$

parallel ... gleiche Steigung ...

... rot Steigung $-\frac{1}{2}$ hat

$y'=-1\cdot x^{-2}=-\frac{1}{x^2}=-\frac{1}{2}\quad|\cdot(-1)/(\ )^{-1}$



3.16 d)

$x^2=2$

$x_{1,2}=\pm\sqrt{2}$

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6) Exponentialfunktion

$f(x)=e^x$

$f'(x)=e^x$

$f'(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}=$

$=\lim_{\Delta x\to 0}\frac{e^{x+\Delta x}-e^x}{\Delta x}=$

$f(x)=e^x$ $e^x\cdot e^{\Delta x}-e^x$

$f'(x) = e$

$= \lim\limits_{\Delta x \to 0} \frac{\ldots - \ldots}{\Delta x} =$

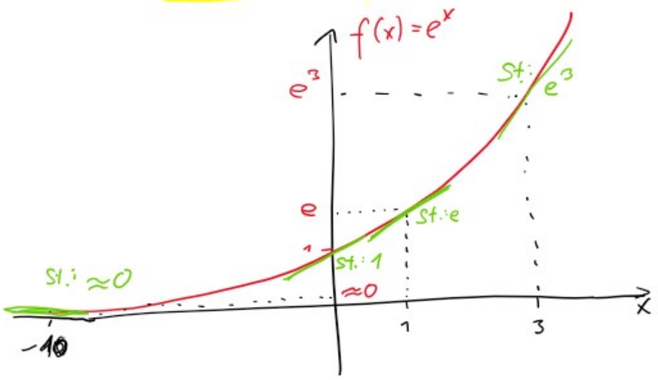
$= \lim\limits_{\Delta x \to 0} \frac{e^x \cdot e^{\Delta x} - e^x}{\Delta x} =$

$= \lim\limits_{\Delta x \to 0} e^x \cdot \frac{e^{\Delta x} - 1}{\Delta x} =$

$= e^x \cdot \lim\limits_{\Delta x \to 0} \frac{e^{\Delta x} - 1}{\Delta x} =$

$= e^x \cdot \underbrace{\left[\lim\limits_{x \to 0} \frac{e^x - 1}{x}\right]}_{=1} = e^x$

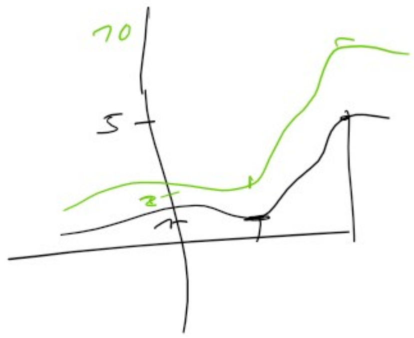
S. 66, 2.14 b



$f(x) = x^n$
$f'(x) = n \cdot x^{n-1}$

7) Logarithmusfunktion

$f(x) = \ln(x)$
$f'(x) = \frac{1}{x}$



3.3) Ableitungsregeln

1 Faktorregel

$f(x) = c \cdot g(x)$
$f'(x) = c \cdot g'(x)$

3.22) e) $f(x) = \sqrt{\frac{3x}{2}} = \left(\frac{3x}{2}\right)^{\frac{1}{2}} = \left(\frac{3}{2}\right)^{\frac{1}{2}} \cdot x^{\frac{1}{2}}$

$f'(x) = \left(\frac{3}{2}\right)^{\frac{1}{2}} \cdot \frac{1}{2} \cdot x^{-\frac{1}{2}} = \sqrt{\frac{3}{2}} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{x}} = \sqrt{\frac{3}{8}} \cdot \frac{1}{\sqrt{x}} = \sqrt{\frac{3}{8x}}$

b) $f(x) = e^{x+1} = e^x \cdot e^1$

$f'(x) = e \cdot e^x = e^{x+1}$

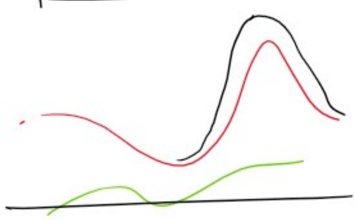
Potenzfunktion:
$(x^2)' = 2x$
Exponentialfkt.:
$(e^x)' = e^x$

d) $f(x) = x^1 \cdot \ln(2) = 0{,}69 \cdot x$

$f'(x) = \ln(2) \cdot 1 \cdot x^0 = \ln(2)$

f) $f(x) = 2 \cdot \ln(x)$

$f'(x) = 2 \cdot \frac{1}{x}$



2 Summenregel

$f(x) = g(x) \pm h(x)$
$f'(x) = g'(x) \pm h'(x)$

$$\boxed{f'(x) = g'(x) \pm h'(x)}$$

3.23)

a) $f(x) = x^3 + 4$

$f'(x) = 3x^2 + 0 = 3x^2$

b) $f(x) = -x^2 + 4x$

$f'(x) = -2x^1 + 4 \cdot 1 = -2x + 4$

c) $f(t) = 2 \cdot \sin(t) + 3\cos(t)$

$f'(t) = 2 \cdot \cos(t) - 3 \cdot \sin(t)$

d) $f(x) = \frac{x-3}{3} = \frac{x}{3} - \frac{3}{3} = \frac{1}{3} \cdot x - 1$

$f'(x) = \frac{1}{3} \cdot 1 - 0 = \frac{1}{3}$

e) $f(t) = \ln(2 \cdot t) = \ln(2) + \ln(t)$

$f'(t) = 0 + \frac{1}{t} = \frac{1}{t}$

f) $f(x) = x \cdot (3x - 5) = 3x^2 - 5x$

$f'(x) = 3 \cdot 2x - 5 = \underline{\underline{6x - 5}}$

③ Produktregel

$$\boxed{\begin{aligned} f(x) &= u(x) \cdot v(x) \\ f'(x) &= u'(x) \cdot v(x) + u(x) \cdot v'(x) \end{aligned}}$$

Beispiele:

a) $f(x) = \underbrace{\sin(x)}_{u} \cdot \underbrace{x^3}_{v}$

$u(x) = \sin(x)$ $\quad v(x) = x^3$

$u'(x) = \cos(x)$ $\quad v'(x) = 3x^2$

$f'(x) = \underbrace{\cos(x)}_{u'} \cdot \underbrace{x^3}_{v} + \underbrace{\sin(x)}_{u} \cdot \underbrace{3x^2}_{v'}$

b) $f(x) = \underbrace{\cos(x)}_{u} \cdot \underbrace{e^x}_{v}$

$f'(x) = \underbrace{-\sin(x)}_{u'} \cdot \underbrace{e^x}_{v} + \underbrace{\cos(x)}_{u} \cdot \underbrace{e^x}_{v'}$

c) $f(x) = \underbrace{x^2}_{u} \cdot \underbrace{x^3}_{v} \longrightarrow f(x) = x^5$

$f'(x) = \underline{\underline{5 \cdot x^4}}$

$f'(x) = \underbrace{2x}_{u'} \cdot \underbrace{x^3}_{v} + \underbrace{x^2}_{u} \cdot \underbrace{3x^2}_{v'} = 2x^4 + 3x^4 = \underline{\underline{5x^4}}$

$u(x) = x^2$ $\quad v(x) = x^3$

$u'(x) = 2x$ $\quad v'(x) = 3x^2$

d) $f(x) = \underline{(x^2 - 2x + 1)} \cdot (3x + 1) \longrightarrow f(x) = 3x^3 - 5x^2 + x + 1$

$f'(x) = \underline{9x^2 - 10x + 1}$

$f'(x) = (2x - 2) \cdot (3x + 1) + (x^2 - 2x + 1) \cdot 3 =$

$= 6x^2 + 2x - 6x - 2 + 3x^2 - 6x + 3 =$

$= \underline{\underline{9x^2 - 10x + 1}}$

$u(x) = x^2 - 2x + 1$ $\quad v(x) = 3x + 1$

$u'(x) = 2x - 2$ $\quad v'(x) = 3$

3.27)

d) $f(x) = \ln(\sqrt{2x}) = \ln\left((2x)^{\frac{1}{2}}\right) =$

$= \frac{1}{2} \cdot \ln(2x) = \frac{1}{2} \cdot [\ln(2) + \ln(x)] =$

$= \frac{1}{2} \cdot \ln(2) + \frac{1}{2} \cdot \ln(x)$

$(\ln(x))' = \frac{1}{x}$

$\rightarrow \ln(a^b) = b \cdot \ln(a)$

$\rightarrow \ln(a \cdot b) = \ln(a) + \ln(b)$

$= \frac{1}{2}\cdot \ln(2) + \frac{1}{2}\cdot \ln(x)$

$\rightarrow \ln(a^b) = b\cdot \ln(a)$

$\rightarrow \ln(a\cdot b) = \ln(a) + \ln(b)$

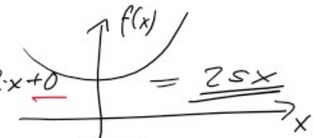
$\ln(a^3) = \ln(a\cdot a\cdot a) = 3\cdot \ln(a)$

$f'(x) = 0 + \frac{1}{2}\cdot \frac{1}{x} = \frac{1}{2x}$

3.29) f) $f(x) = x + 3\cdot \lg(x)$ $\qquad (\lg(x))' = \frac{1}{x\cdot \ln(10)}$

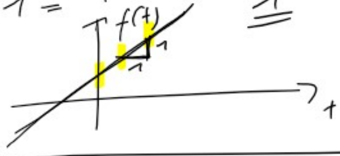
3.31)

a) $f(x) = s\cdot x^2 + t$ $\qquad f'(x) = s\cdot 2x + 0 = \underline{\underline{2sx}}$



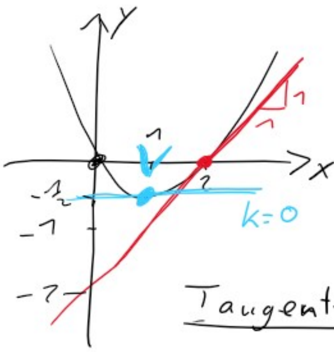
b) $f(s) = s\cdot x^2 + t$ $\qquad f'(s) = 1\cdot x^2 + 0 = \underline{\underline{x^2}}$

c) $f(t) = s\cdot x^2 + t$ $\qquad f'(t) = 0 + 1 = \underline{\underline{1}}$



3.33) $y = \frac{1}{2}x^2 - x = x\cdot(\frac{1}{2}x - 1)$ Parabel

a) slope 1 ... Steigung 1 ... $f'(x) = 1$



$f(x) = \frac{1}{2}x^2 - x$

$f'(x) = \frac{1}{2}\cdot 2x - 1 =$

$= x - 1$

$x - 1 = 1 \quad |+1$

$x = 2$

Taugente: $y = k x + d$, $k = 1$, $d = ?$

geht durch Punkt $(2/0)$:

$f(2) = 0$

$0 = 1\cdot 2 + d$

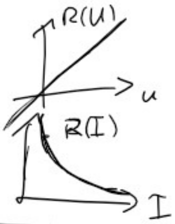
$d = -2$

$\underline{\underline{y = 1\cdot x - 2}}$

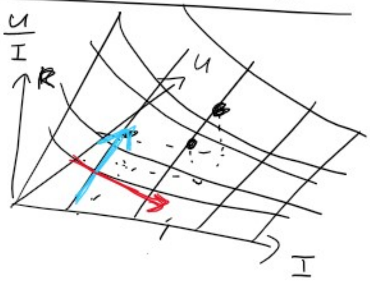
$R = \frac{U}{I}$

$R(U) = \frac{U}{I}$

$R(I) = \frac{U}{I}$



$R(U,I) = \frac{U}{I}$



b) vertex... Scheitel $\rightarrow$ dort ist $f'(x) = 0$

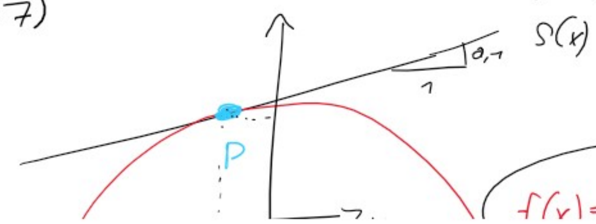
$f'(x) = x - 1 \stackrel{!}{=} 0 \quad |+1$

$\underline{\underline{x = 1}}$

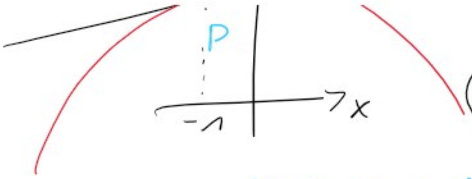
$f(1) = \frac{1}{2}\cdot 1^2 - 1 = -\frac{1}{2}$

$V = (1 / -\frac{1}{2})$

3.37)



$f(x) = -\frac{1}{...}\cdot x^2$



$f(x) = -\frac{1}{20} \cdot x^2$

P liegt auf f(x), liegt auf s(x).

f(x), s(x) haben in P die gleiche Steigung,

also 10% = 0,1

ges.: jenes x, wo f(x) die Steigung 0,1 hat

$$f'(x) = -\frac{1}{20} \cdot 2x = -\frac{1}{10} x \overset{!}{=} 0{,}1$$

$$x = -1$$

y-Koord.: $f(-1) = -\frac{1}{20} \cdot (-1)^2 = -\frac{1}{20} = -0{,}05$

$$P = (-1/-0{,}05)$$

$$s(x) = \underset{0{,}1}{\underset{\shortparallel}{k}} \cdot x + d$$

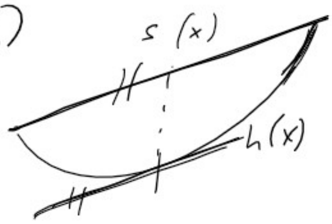
P einsetzen: $-0{,}05 = 0{,}1 \cdot (-1) + d$

$$0{,}1 - 0{,}05 = d$$

$$d = 0{,}05$$

$$s(x) = 0{,}1 \cdot x + 0{,}05$$

3.38)



3.(17) b) $f(x) = \underbrace{(x-1)^2}_{u} \cdot \underbrace{\sqrt{x}}_{v}$

$u = (x-1)^2 = x^2 - 2x + 1 \qquad v = \sqrt{x} = x^{\frac{1}{2}}$

$u' = 2x - 2 \qquad v' = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$$f'(x) = \underbrace{(2x-2)}_{u'} \cdot \underbrace{\sqrt{x}}_{v} + \underbrace{(x^2-2x+1)}_{u} \underbrace{\frac{1}{2\sqrt{x}}}_{v'} =$$

$$= (2x-2) \cdot x^{\frac{1}{2}} + \frac{(x^2-2x+1)}{2} \cdot x^{-\frac{1}{2}} = 2x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + \frac{1}{2} x^{\frac{3}{2}} - x^{\frac{1}{2}} + \frac{1}{2} x^{-\frac{1}{2}}$$

g) $f(x) = x \cdot \ln(x^2) = \underbrace{x \cdot 2}_{u} \cdot \underbrace{\ln(x)}_{v}$

$$f'(x) = \underbrace{2}_{u'} \cdot \underbrace{\ln(x)}_{v} + \underbrace{2\cancel{x}}_{u} \cdot \underbrace{\frac{1}{\cancel{x}}}_{v'} = \underline{\underline{2 \cdot \ln(x) + 2}}$$

h) $f(x) = (1+x^2) \cdot \ln\left(\frac{x}{2}\right) = \underbrace{(1+x^2)}_{u} \cdot \underbrace{(\ln(x) - \ln(2))}_{v}$

$\boxed{1' = 0}$

$u' = 2x$

$v' = \frac{1}{x} - 0$

$$f'(x) = \underbrace{2x}_{u'} \cdot \underbrace{(\ln(x) - \ln(2))}_{v} + \underbrace{(1+x^2)}_{u} \cdot \underbrace{\frac{1}{x}}_{v'}$$

3.48)

a) $f(x) = \underbrace{x \cdot e^x}_{u} \cdot \underbrace{\sin(x)}_{v}$

$u = x \cdot e^x \qquad v = \sin(x)$

$u' = 1 \cdot e^x + x \cdot e^x \qquad v' = \cos(x)$

$$f'(x) = (1 e^x + x \cdot e^x) \cdot \sin(x) + x \cdot e^x \cdot \cos(x)$$

4) Quotientenregel

$$f(x)=\frac{u(x)}{v(x)}$$

$$f'(x)=\frac{u'(x)\,v(x)-u(x)\,v'(x)}{(v(x))^2}$$

$$\boxed{\left(\frac{u}{v}\right)'=\frac{u'v-uv'}{v^2}}$$

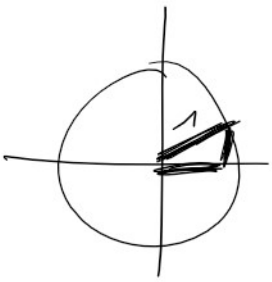
3.46) a) $f(x)=\frac{x^2-1}{x}$ $\qquad u=x^2-1 \quad v=x$ $\qquad u'=2x \quad v'=1$

$$f'(x)=\frac{2x\cdot x-(x^2-1)\,1}{x^2}=\frac{2x^2-(x^2-1)}{x^2}=\frac{x^2+1}{x^2}=1+\frac{1}{x^2}$$

3.50) c) $f(x)=\frac{x+1}{x-1}$ $\qquad u=x+1 \quad v=x-1$ $\qquad u'=1 \quad v'=1$

$$f'(x)=\frac{1\,(x-1)-(x+1)\,1}{(x-1)^2}=\frac{x-1-x-1}{(x-1)^2}=\frac{-2}{(x-1)^2}$$

g) $f(x)=\frac{\sin(x)}{\sin(x)+\cos(x)}$ $\qquad u=\sin(x) \quad v=\sin(x)+\cos(x)$ $\qquad u'=\cos(x) \quad v'=\cos(x)-\sin(x)$



$$f'(x)=\frac{\cos(x)\cdot(\sin(x)+\cos(x))-\sin(x)\cdot(\cos(x)-\sin(x))}{(\sin(x)+\cos(x))^2}=$$

$$=\frac{\cancel{\cos(x)\cdot\sin(x)}+\cos^2(x)-\cancel{\sin(x)\cos(x)}+\sin^2(x)}{(\qquad)^2}=$$

$$=\frac{1}{(\sin(x)+\cos(x))^2}$$

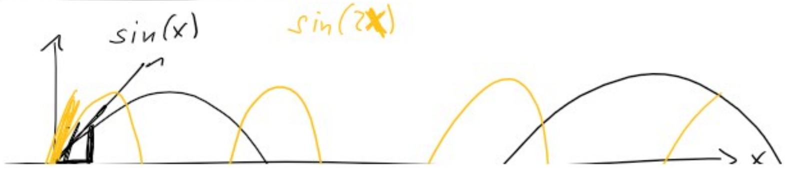
Ableitung von tan(x):

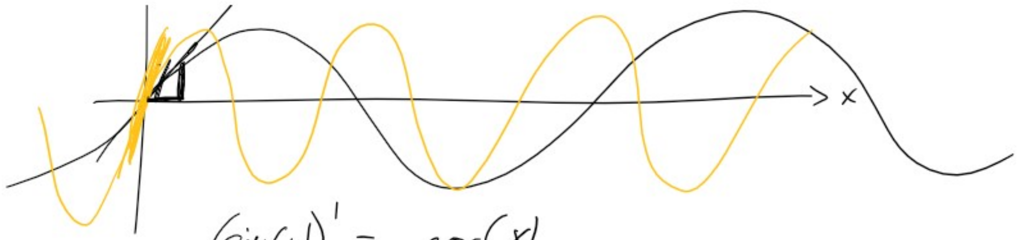
$$\tan(x)=\frac{\sin(x)}{\cos(x)}$$

$$(\tan(x))'=\frac{\cos(x)\cdot\cos(x)-\sin(x)\cdot(-\sin(x))}{(\cos(x))^2}=\frac{\cos^2(x)+\sin^2(x)}{(\cos(x))^2}=$$

$$=\frac{1}{\cos^2(x)}$$

5.) Kettenregel





$$(\sin(x))' = \cos(x)$$

$$(\sin(2x))' = 2\cdot\cos(2x)$$

Ableitung

$$\boxed{(f(g(x)))' = \underbrace{f'(g(x))}_{\text{äußere Abl.}} \cdot \underbrace{g'(x)}_{\text{innere Ableitung}}}$$

3.59)

b) $h(x) = \sqrt{5x+1} = (5x+1)^{\frac{1}{2}}$

$$h'(x) = \underbrace{\frac{1}{2}\cdot(5x+1)^{-\frac{1}{2}}}_{\text{äußere Abl.}} \cdot 5$$

$f(\bullet) = \bullet^{\frac{1}{2}}$

$f'(\bullet) = \frac{1}{2}\bullet^{-\frac{1}{2}}$

$g(x) = 5x+1$

$g'(x) = 5$